Physikalisch-Technische Bundesanstalt
National Metrology Institute

Accurate Rotational Speed Measurement for Determining the Mechanical Power and Efficiency of Electrical Machines

Song, Zihang[1]; Weidinger, Paula[2]; Kumme, Rolf[3]

[1] Physikalisch-Technische Bundesanstalt, Braunschweig, Germany. https://orcid.org/ 0000-0001-8153-1206

[2] Physikalisch-Technische Bundesanstalt, Braunschweig, Germany. https://orcid.org/0000-0003-2978-0672

[3] Physikalisch-Technische Bundesanstalt, Braunschweig, Germany.

DOI: 10.7795/EMPIR.19ENG08.CA.20230831A

Year of publication: 2023

Acknowledgement of funding

Project 19ENG08 has received funding from the EMPIR programme co-financed by the Participating States from the European Union's Horizon 2020 research and innovation programme.

Suggestion for the quotation of the reference

Z. Song *et al.*, 2023. Accurate rotational speed measurement for determining the mechanical power and efficiency of electrical machines. DOI: 10.7795/EMPIR.19ENG08.CA.20230831A

Pre-print version of

10.23919/ICEMS52562.2021.9634265

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.

Accurate Rotational Speed Measurement for Determining the Mechanical Power and Efficiency of Electrical Machines

Zihang Song
Department 1.2 Solid Mechanics
Physikalisch-Technische Bundesanstalt
Braunschweig, Germany
zihang.song@ptb.de

Paula Weidinger
Department 1.2 Solid Mechanics
Physikalisch-Technische Bundesanstalt
Braunschweig, Germany
paula.weidinger@ptb.de

Rolf Kumme
Department 1.2 Solid Mechanics
Physikalisch-Technische Bundesanstalt
Braunschweig, Germany
rolf.kumme@ptb.de

***Abstract*—Accurately measuring the mechanical power of electrical machines is essential for determining their efficiency. Conventionally, the machines' rotational speed is measured over a short time interval with a high measurement uncertainty. In this paper, we propose a rotational speed determination based on the angle covered within a longer time interval. This method is shown to achieve a higher accuracy for the determination of mechanical power and total efficiency with negligible systematic effect.**

***Keywords*—rotational speed measurement, encoder, mechanical power measurement, efficiency determination**

I. Introduction

In test benches for electrical machines, rotary encoders are the most frequently used rotational speed transducers. Here, the rotational speed n in min^{-1} is not determined via direct speed measurement but as an averaged speed for a short period of time Δt by measuring the angle covered. A metrological analysis of an optical encoder was conducted in [1]. In this paper, however, we propose to determine the rotational speed separately over a longer time interval to improve the accuracy of the mechanical power determination.

II. Rotational Speed Measurement

Rotary encoders can be divided into two types:

- an absolute encoder measures the angular position of the shaft using the binary code marked on its disk.
- an incremental encoder induces a frequency signal proportional to the rotational speed. Here, the changes in the angular position are represented by the number of pulses. Thus, the rotational speed is measured by counting the number of pulses for a certain length of time or by measuring the length of each pulse period.

For both encoder types, the rotational speed n in min^{-1} is not determined via direct speed measurement but as an averaged speed for a certain length of time Δt by measuring the angle covered $(\phi_2 - \phi_1)$:

$$n = \frac{\phi_2 - \phi_1}{\Delta t} \cdot \frac{60}{360\,°}. \quad (1)$$

The corresponding standard measurement uncertainty (MU) σ_{n} of the rotational speed can be estimated based on the standard MU contributions due to the angle measurement σ_{ϕ} and the time measurement σ_{t}:

$${\sigma_{\mathrm{n}}}^2 = \left(\frac{\partial n}{\partial \phi_1} \cdot \sigma_{\phi}\right)^2 + \left(\frac{\partial n}{\partial \phi_2} \cdot \sigma_{\phi}\right)^2 + \left(\frac{\partial n}{\partial \Delta t} \cdot \sigma_{\mathrm{t}}\right)^2 \quad (2)$$

$$\sigma_{\mathrm{n}} = \frac{60}{360°} \cdot \sqrt{2\left(\frac{\sigma_{\phi}}{\Delta t}\right)^2 + \left(\frac{\phi_2 - \phi_1}{\Delta t^2} \cdot \sigma_{\mathrm{t}}\right)^2}. \quad (3)$$

During the manufacturing process for the encoder, the tolerance of the position and the width of the slots can contribute to the uncertainty of the angle measurement σ_{ϕ}. The uncertainty of the time measurement σ_{t} can be influenced by the triggering process [1] or the internal clock error. Here, we use an incremental encoder with 360 slots per revolution for the MU estimation:

TABLE I. Parameters of the Encoder

Encoder type	Number of slots	$2\sigma_{\phi}$	$2\sigma_t$
incremental	360	0.01 °	100 ns

Conventionally, the time interval Δt is measured between every two slots $(\phi_2 - \phi_1 = 1\,°)$. This ensures that the rotational speed is measured with the widest bandwidth, as this is important for dynamic speed control [2]. In terms of accuracy, however, a small Δt yields a high MU in the measurement result according to (3). This method is referred to as M1. With the assumption of a normal distribution, the expanded MU $(k = 2)$ of the rotational speed measurement using M1 can be expressed as:

$$U_{\mathrm{n,M1}} = 2\sigma_{\mathrm{n}} = 2n\sqrt{2{\sigma_{\phi}}^2 + (6n\sigma_{\mathrm{t}})^2}. \quad (4)$$

From the perspective of power determination, the measurement accuracy has a higher priority than the measurement bandwidth. In order to reduce the MU, a power analyser receives the rotational speed measured in M1 and averages the measurement signal over a certain period of time. This period Δt can generally be freely adjusted by the users. For the sake of simplification and ease of comparison, in this paper, Δt is set as the time needed for one shaft revolution (360 slots). This averaging method is referred to as M2 and can contribute to the MU reduction in contrast to M1 as expressed in (5). It is worth mentioning that only randomly distributed error sources can be reduced using this averaging method. If the measurement is coupled with a

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.

systematic error, the error sources would not be undermined using M2.

$$U_{\mathrm{n,M2}} = \frac{U_{\mathrm{n,M1}}}{\sqrt{360}} \tag{5}$$

As mentioned, M1 and M2 are currently the most commonly used methods for rotational speed measurement on test benches for electrical machines. To further improve the accuracy of rotational speed measurement, in this paper, we propose a new method (M3) in which the rotational speed is determined by measuring the angle covered $\Delta\phi$ over one shaft revolution ($\phi_2 - \phi_1 = 360\,°$) and by measuring the time needed. Using M3, the expanded MU can be further reduced as follows:

$$U_{\mathrm{n,M3}} = 2\sigma_{\mathrm{n}} = \frac{n}{180}\sqrt{2{\sigma_{\phi}}^2 + (6n\sigma_{\mathrm{t}})^2}. \tag{6}$$

Based on (4) to (6), a comparison of the expanded MU U_{n} and the relative expanded MU W_n among M1, M2 and M3 under different rotational speeds is presented in Fig. 1.

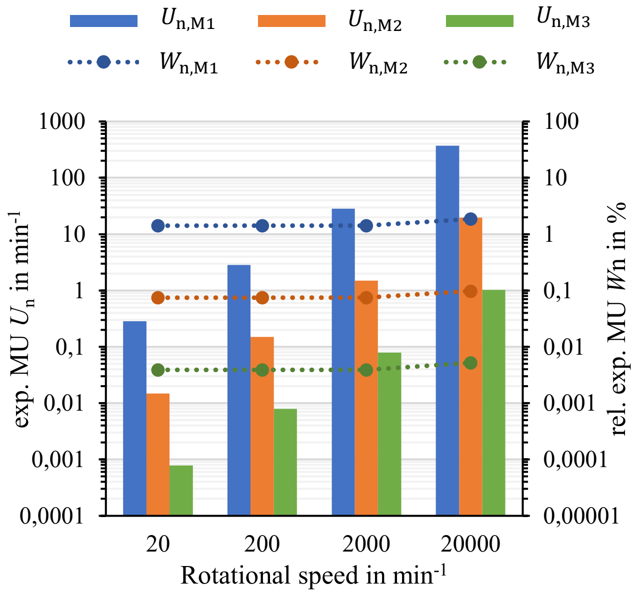


Fig. 1. A comparison of expanded MU U_{n} and relative expanded MU W_{n} of rotational speed measurement among different methods (M1, M2 and M3) under different rotational speeds.

As shown in the figure, the expanded MU has a nearly proportional relation to the rotational speed; therefore, the relative expanded MUs remain nearly constant in a wide rotational speed range. Compared to M1 and M2, the proposed method (M3) provides a significantly lower MU for accurate rotational speed measurement.

III. Ripples in Torque and Rotational Speed

In real test benches, the torque of electrical machines is always coupled with alternating components: the cogging torque is caused by changes in the magnetic reluctance between the rotor and the stator slots, while the torque ripples are due to back electromotive force harmonics [3]. Based on [3]–[5], the torque of electrical machines can be simplified as the superposition of a DC component M_0 and a sinusoidal wave with a specific frequency f_1 and the dominant amplitude M_1 compared to other frequency components:

$$M(t) = M_0 + M_1 \cdot \sin(\mu_1 \omega t), \tag{7}$$

where μ_1 is the harmonic order of the frequency f_1 with regard to the shaft angular velocity ω:

$$\mu_1 = \frac{2\pi f_1}{\omega}. \tag{8}$$

By neglecting the friction of the bearing and assuming the counter-torque from the load motor M_L to be constant, the dynamic of the rotating shaft can be described as:

$$J_s \frac{d\omega}{dt} = M(t) - M_L, \tag{9}$$

where J_s is the total moment of inertia. For a constant operating condition, the counter-torque should be equal to the DC component M_0. Thus, the shaft rotational speed can be derived as:

$$n = \frac{60}{2\pi}\omega = n_0 - n_1 \cos(\mu_1 \omega t) \tag{10}$$

$$n_1 = \frac{60}{2\pi} \cdot \frac{M_1}{J_s \mu_1 \omega}. \tag{11}$$

It can be concluded that the alternating torque component will result in an alternating rotational speed component with the same harmonic order and a -90 ° phase offset. Meanwhile, the amplitude of the alternating rotational speed component is much smaller due to the shaft inertia.

IV. Mechanical Power Determination

The standard method of calculating the mechanical power P_{m1} of electrical machines is to multiply the transient torque $M(t)$ by the transient rotational speed $n(t)$ and to average the product over a time interval ΔT (in the range of seconds):

$$P_{\mathrm{m1}} = \frac{1}{\Delta T} \cdot \frac{2\pi}{60} \int_0^{\Delta T} M(t) \cdot n(t)\, dt. \tag{12}$$

As mentioned in the previous section, the rotational speed can be determined with higher accuracy by measuring the angle covered over a longer time interval ($\Delta T = \Delta \mathrm{t}$). In this manner, (12) is re-expressed as:

$$P_{\mathrm{m2}} = \frac{1}{\Delta \mathrm{t}} \cdot \frac{2\pi}{60} \int_0^{\Delta \mathrm{t}} M(t)\, dt \cdot \frac{1}{\Delta \mathrm{t}} \int_0^{\Delta \mathrm{t}} n(t)\, dt. \tag{13}$$

Compared to (12), the torque and the rotational speed are averaged separately during Δt; their product is used to calculate the average mechanical power P_{m2}.

Mathematically speaking, this re-expression is only valid for constant torque and rotational speed. In real test benches, the torque and speed of electrical machines are always coupled with alternating components, as elaborated in Section III. Therefore, the applicability of (13) must first be proved by investigating its determination error $e_{\mathrm{P_m}\%}$ comparing to (12) in practice:

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.

$$e_{P_m\%} = \frac{P_{m2} - P_{m1}}{P_{m1}}. \quad (14)$$

The determination error $e_{P_m\%}$ results from the alternating components in torque and rotational speed. As elaborated in Section III, the alternating components of the torque and the speed can be simplified as sinusoidal waves. To efficiently describe the characteristics of the alternating components, three parameters regarding the torque $M(t)$ and the speed $n(t)$, as shown in Fig. 2, are investigated to characterise their individual influences on the determination error:

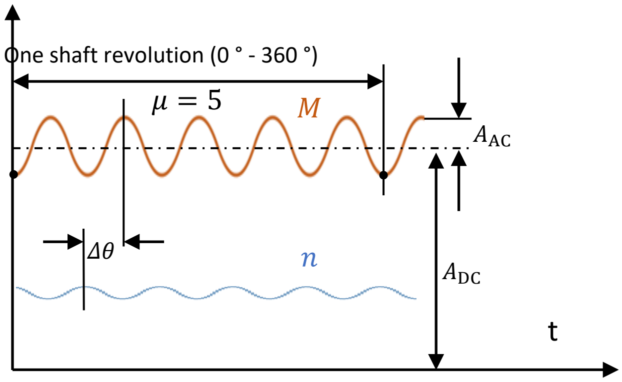


Fig. 2. The parameters of torque and rotational speed used to describe the characteristics of the alternating components as sinusoidal waves.

- **The alternating ratio r** indicates the amplitude of the alternating component with regard to the amplitude of the DC component in percentage:

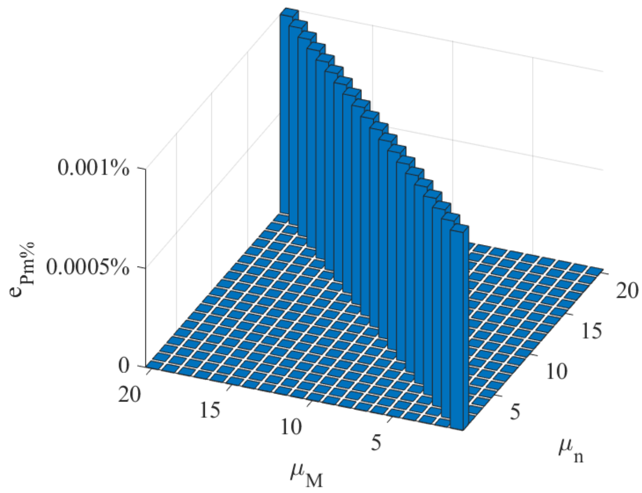
$$r = A_{AC}/A_{DC} \cdot 100\%. \quad (15)$$

- **The phase offset $\Delta\theta$** defines the phase difference between $M(t)$ and $n(t)$.
- **The harmonic order μ** is the harmonic order of the alternating frequency with regard to the shaft revolution frequency.

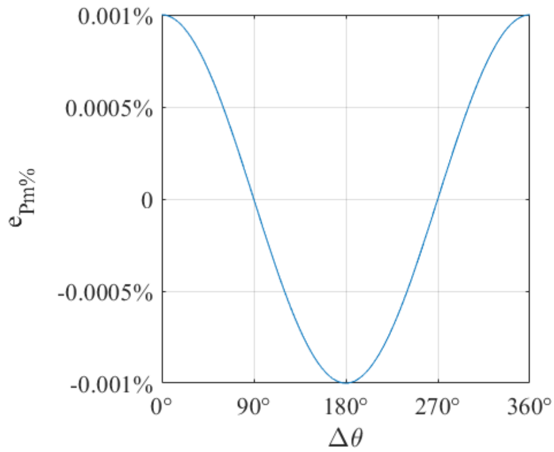
Other parameters such as the absolute magnitude of the DC component A_{DC} are shown to have no influence on the result, which means the determination error $e_{P_m\%}$ does not vary with the mean values of the torque and the speed.

The determination error $e_{P_m\%}$ for $M(t)$ and $n(t)$ with sinusoidal waveforms is presented in Fig. 3. It is apparent from (a) that a salient $e_{P_m\%}$ is present only when $M(t)$ and $n(t)$ have the same frequency ($\mu_M = \mu_n$). (b) shows that the maximum $e_{P_m\%}$ occurs when $M(t)$ and $n(t)$ are in phase or completely out of phase ($\Delta\theta = 0$ ° or 180 °).

According to (7) and (10), the alternating torque component results in an alternating rotational speed component with the same harmonic order but a -90 ° phase offset. Thus, theoretically speaking, the determination error $e_{P_m\%}$ is always zero for electrical machine test benches. In practice, however, the counter-torque from the load motor M_L in (9) is not necessarily a constant value. Therefore, the phase offset can shift from -90 ° in both directions. For a safer MU estimation, it is better to consider the worst-case scenario, where the harmonic orders are identical ($\mu_M = \mu_n$) and the phase offset is zero ($\Delta\theta = 0$ °) at the same time.

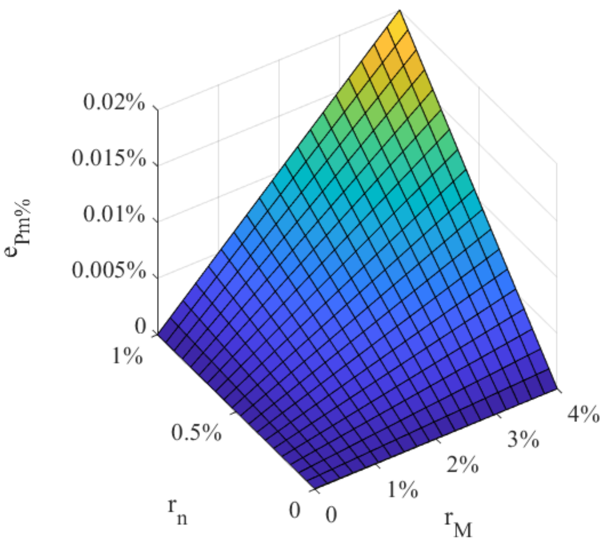


(a)



(b)

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.



(c)

Fig. 3. Determination error $e_{P_m\%}$ of mechanical power measurement resulting from torque and rotational speed ripples: (a) at different harmonic orders μ ($\Delta\theta = 0\,°$, $r_M = 2\,\%$, $r_n = 0.1\,\%$); (b) at different phase offsets $\Delta\theta$ ($\mu_M = \mu_n$, $r_M = 2\,\%$, $r_n = 0.1\,\%$); (c) at different alternating ratios r ($\mu_M = \mu_n$, $\Delta\theta = 0\,°$).

Finally, the determination error $e_{\mathrm{P_m}\%}$ with different alternating ratios is presented in Fig. 3 (c). It is comprehensible that higher alternating ratios r will result in a higher determination error $e_{\mathrm{P_m}\%}$. In test benches for electrical machines, the alternating torque ratio r_M resulting from the cogging torque and the torque ripples is strongly influenced by the electrical machine configuration and can be reduced by optimisation methods from either machine design or control perspectives [5]. The alternating torque ratio r_M of a high-value permanent magnet synchronous machine (PMSM) with a sinusoidal current supply is usually less than 1 % [6]. Furthermore, a well-designed optimisation method can further reduce the r_M of a PMSM from 2 % to 0.16 % [7]. In this paper, the condition $r_M < 2\,\%$ and $r_n < 0.1\,\%$ is considered typical of electrical machines and the corresponding determination error $e_{\mathrm{P_m}\%}$ is very low (0.001 %), which is negligible in mechanical power measurement. However, the torque's alternating ratio may also reach up to 25% under certain circumstances according to [8]. In this situation, the determination error should be considered carefully.

To obtain a clear understanding of the impact of this determination error $e_{\mathrm{P_m}\%}$ on the mechanical power measurement, the comparison of relative expanded MU W_n for rotational speed in Fig. 1 is extended with the torque measurement with a relative expanded MU W_M of 0.02 %. Here, the determination error $e_{\mathrm{P_m}\%} = 0.001\,\%$ is regarded as an uncorrected systematic effect which is independent of the torque and rotational speed uncertainties. According to the analysis in [9], the total relative expanded MU W_P ($k = 2$) for mechanical power measurement is determined using (16).

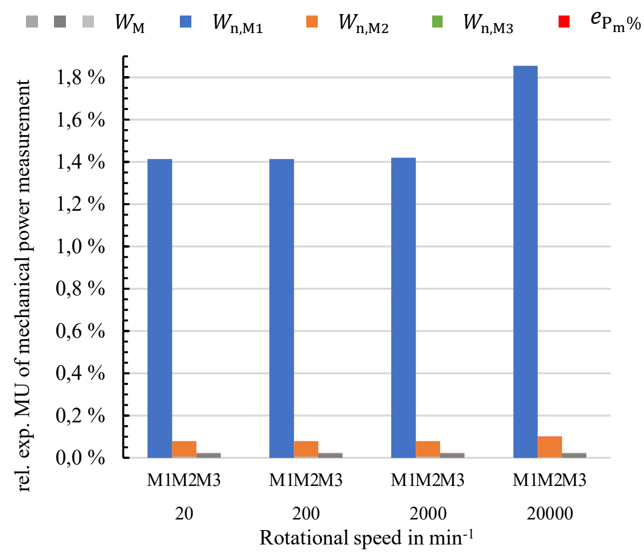
$$W_\mathrm{P} = 2\sqrt{\left(\frac{W_\mathrm{M}}{2}\right)^2 + \left(\frac{W_\mathrm{n}}{2}\right)^2 + {e_{\mathrm{Pm}\%}}^2} \qquad (16)$$

In Fig. 4, a comparison of the relative expanded MU W_P among M1, M2 and M3 under different rotational speed is presented, along with the contributions of all uncertainty sources (W_M, W_n and $e_{\mathrm{Pm}\%}$). At the same time, the contributions are listed below in Table II, where the contribution index indicates the individual contributions to the sum of squared MU in percentage.

TABLE II. Contributions of All Uncertainty Sources (W_M, W_N and $e_{\mathrm{PM}\%}$) to the Total Uncertainty of the Mechanical Power Measurement. The Contribution Index Indicates the Individual Contributions to the Sum of Squared MU in Percentage.

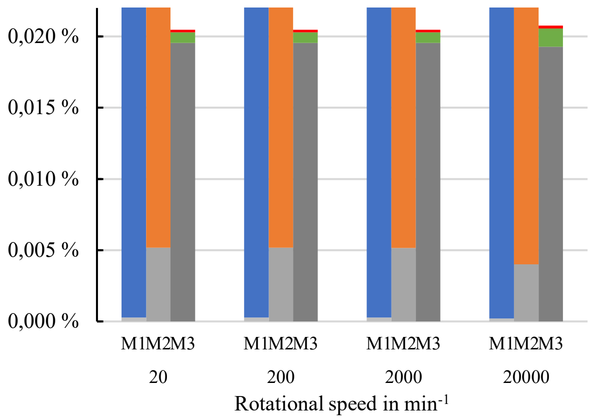
n [min^{-1}]	Method	W_P	Contribution index		
			W_n	W_M	$e_{\mathrm{Pm}\%}$
20	M1	1.4143 %	99.98 %	0.02 %	-
	M2	0.0772 %	93.21 %	6.72 %	0.07 %
	M3	0.0205 %	3.63 %	95.42 %	0.95 %
200	M1	1.4144 %	99.98 %	0.02 %	-
	M2	0.0772 %	93.21 %	6.72 %	0.07 %
	M3	0.0205 %	3.63 %	95.42 %	0.95 %
2000	M1	1.4194 %	99.98 %	0.02 %	-
	M2	0.0775 %	93.27 %	6.67 %	0.06 %
	M3	0.0205 %	3.63 %	95.42 %	0.95 %
20000	M1	1.8548 %	99.99 %	0.01 %	-
	M2	0.0998 %	95.95 %	4.01 %	0.04 %
	M3	0.0208 %	6.27 %	92.80 %	0.93 %

The results above show that the total MU of the mechanical power measurement can be significantly reduced with a more accurate rotational speed measurement method. Furthermore, the impact of the determination error due to the torque ripples is very low, which is negligible compared to the MU resulting from the torque and rotational speed measurements in a wide rotational speed range.



(a)

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.



(b)

Fig. 4. A comparison of the relative expanded MU W_P among M1, M2 and M3 under different rotational speeds, alongside the contributions of all uncertainty sources (W_M, W_n and $e_{Pm\%}$): (a) original vertical axis scale; (b) zoomed vertical axis scale.

V. Conclusion

Accurately measuring the rotational speed of electrical machines is essential for determining mechanical power. The rotational speed determination method proposed here is based on the angle covered within a longer time interval. Compared to conventional methods, this method shows a significant improvement in the accuracy of the rotational speed measurement. Using this method, torque and rotational speed are averaged separately and then multiplied to calculate the mechanical power. Because torque and rotational speed are not constant but coupled with ripples, the determination error occurs during this process. The analysis performed within the scope of this work shows that the determination error is negligible compared to the MU resulting from the torque and rotational speed measurements in a wide rotational speed range. In conclusion, the method proposed here has been shown to achieve higher accuracy for the rotational speed measurement and to enable a more accurate determination of mechanical power and total efficiency with negligible systematic effect than methods used to date.

Acknowledgments

The project 19ENG08 – WindEFCY has received funding from the EMPIR programme co-financed by the Participating States from the European Union's Horizon 2020 research and innovation programme. The input of all the project partners is gratefully acknowledged.

The lead author gratefully acknowledges the support of the Braunschweig International Graduate School of Metrology B-IGSM.

References

[1] A. Brüge and H. Pfeiffer, "A Standard for Rotatory Power Measurement," *ACTA IMEKO*, vol. 8, p. 48, Sep. 2019, doi: 10.21014/acta_imeko.v8i3.561.

[2] R. Petrella, M. Tursini, L. Peretti, and M. Zigliotto, "Speed measurement algorithms for low-resolution incremental encoder equipped drives: a comparative analysis," in *2007 International Aegean Conference on Electrical Machines and Power Electronics*, 2007, pp. 780–787, doi: 10.1109/ACEMP.2007.4510607.

[3] M.-H. Hwang, H.-S. Lee, and H.-R. Cha, "Analysis of Torque Ripple and Cogging Torque Reduction in Electric Vehicle Traction Platform Applying Rotor Notched Design," *Energies*, vol. 11, no. 11. 2018, doi: 10.3390/en11113053.

[4] R. Deng, J. Yang, S. Zheng, T. Gu, S. Zhuo, and F. Li, "Periodic Speed Ripple Suppression Based on Cascade Filter of IPMSM Drive in Air-Conditioners," in *2020 International Conference on Electrical Machines (ICEM)*, 2020, vol. 1, pp. 1199–1205, doi: 10.1109/ICEM49940.2020.9270759.

[5] W. Fei, P. C. K. Luk, J. X. Shen, B. Xia, and Y. Wang, "Permanent-Magnet Flux-Switching Integrated Starter Generator With Different Rotor Configurations for Cogging Torque and Torque Ripple Mitigations," *IEEE Trans. Ind. Appl.*, vol. 47, no. 3, pp. 1247–1256, 2011, doi: 10.1109/TIA.2011.2125750.

[6] A. Binder, "Permanentmagneterregte Synchronmaschinen (Permanent magnet synchronous machines)," in *Elektrische Maschinen und Antriebe (Electrical machines and drives)*, A. Binder, ed. Berlin, Heidelberg: Springer Berlin Heidelberg, 2017, p. 686.

[7] N. Vaks, S. Pekarek, and D. Horvath, "Feedback-based mitigation of torque harmonics in interior permanent magnet synchronous machines," in *2013 International Electric Machines & Drives Conference*, 2013, pp. 564–570, doi: 10.1109/IEMDC.2013.6556151.

[8] J. Cros, J. M. Vinassa, S. Clenet, S. Astier, and M. Lajoie-Mazenc, "A novel current control strategy in trapezoidal EMF actuators to minimize torque ripples due to phases commutations," in *1993 Fifth European Conference on Power Electronics and Applications*, 1993, pp. 266–271 vol.4.

[9] I. H. Lira and W. Wöger, "Evaluation of the uncertainty associated with a measurement result not corrected for systematic effects," *Meas. Sci. Technol.*, vol. 9, no. 6, pp. 1010–1011, 1998, doi: 10.1088/0957-0233/9/6/019.

This is an author-created, un-copyedited version of a Conference contribution published in 2021 24th International Conference on Electrical Machines and Systems (ICEMS), 2021, 10.23919/ICEMS52562.2021.9634265. The present version is available for open access, DOI: 10.7795/EMPIR.19ENG08.CA.20230831A.