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SENSITIVITY EVALUATION OF MEASUREMENT UNCERTAINTY CONTRIBUTIONS OF SPECTRAL DATA FOR CALCULATED INTEGRAL QUANTITIES

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Abstract

Integrating spectral data (spectral responsivities of photometers or spectral distributions of light sources) to calculate quantities such as tristimulus values is straightforward at first sight. However, estimating the measurement uncertainty of these integrated quantities is challenging. When calculating integral photometric quantities, some uncertainty contributions from the spectral data transfer to the final results, some "cancel out", some "average out" and others increase or decrease their weight by correlation.

The spectral quantities are usually assumed to be uncorrelated when deriving other quantities by integration, which is typically not justified. Using Monte Carlo simulations (MCS) and orthogonal functions, this paper shows that the neglection of partial spectral correlations may lead to a significant underestimation of the measurement uncertainty of these integral quantities, and how to use this information for better estimation.

Keywords: Photometry, Colorimetry, Measurement Uncertainty, Monte Carlo Simulation, Correlation

1 Introduction

Integration of spectral data is critical for calculating photometric quantities, non-visual effects, and photobiological safety. The main calculations to determine the value of the measurand, i.e., by weighted sums of spectral responsivities of photometers or spectral distributions (SDs), or similar, over the spectral range, are relatively simple, but calculating estimates for the associated measurement uncertainty immediately becomes complicated and challenging (CIE 198-SP2:2018, 2018). The correlation of uncertainty components at different wavelengths can significantly contribute to the combined uncertainty of the integrated quantities (Kärhä et al., 2017, Schmähling et al., 2018).

Based on Monte Carlo simulations (MCS) (JCGM 102:2011, 2011), and the application of orthogonal basis functions to describe correlations, a basic approach for the estimation of dependencies is developed, which allows a better insight into the origin of those significant contributions to the measurement uncertainty caused by correlations in the spectral data to be integrated.

The main contribution of this work is that, in addition to the fully-correlated errors on the wavelength and responsivity scales, the partially correlated contributions are also investigated. For this purpose, a method that describes correlations through orthogonal basis functions was used in this work.

2 Methods

The proposed basic approach does not describe a specific measurement technique, and the impact of correlated spectral data on the calculated spectrally integrated quantities does not consider the physical background of a particular measurement setup or device under test (DUT) in a first step. Instead, predetermined input SDs are used as the outcome of fictitious measurements, whose data fluctuate around their mean values within given uncertainties. While the basis function approach accomplishes the (correlated) fluctuation, the calculation of the

desired output quantity is done by a classical MCS. In this way, calculating the spectrally integrated quantity of interest is decoupled from the measurement process and becomes universally valid. For the basic approach of the simulation, the variation of the wavelength and the variation of the amplitude of the signal are evaluated separately.

On the one hand, the basic approach makes it possible to understand the main contributions of the spectral measurement uncertainties to the estimated combined measurement uncertainty and consider them for specific cases. On the other hand, this approach identifies possible correlations between the output quantities. For instance, the overlap of the CIE colour-matching functions (CIE15:2018, 2018) in the wavelength range generates correlations between the tristimulus values. In a second step, one can look for parts of physical models which can be modelled with the different parameters of the basic approach model (see section 2.3).

According to the methodology to be presented, the uncertainty contributions of the spectral data are represented as values with specific probability distributions, divided into additive and multiplicative components, and, for both categories, in uncorrelated, fully-correlated and partially correlated values. The partially correlated values are modelled by orthogonal basis functions (Kärhä et al., 2017) or by applying known covariance matrices from measurement results.

2.1 Model for uncertainty contributions

A vector representation describes spectral data, where vectors are represented with bold symbols, whereas scalar have italic symbols. For spectral data with N_λ elements (e.g., from 360 nm to 830 nm in 5 nm steps → N_λ = 95), two vectors are used:

- Amplitude vector: $\mathbf{S} = [S_0, S_1, S_2, \dots, S_{N_\lambda - 1}]$
- Wavelength vector: $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \lambda_2, \dots, \lambda_{N_\lambda - 1}]$

where the variation of the values of the elements in the vectors can be originated either from a wavelength jitter, signal change, or both. Both vectors are used to calculate the spectrally integrated quantity of interest (see section 2.1.6). However, to better understand their influence, they will be simulated independently in the first step.

2.1.1 Amplitude vector

For the basic approach introduced, the following model is used for the signal or amplitude uncertainty.

$$\boldsymbol{S}^{\mathrm{r}} = \boldsymbol{S} \cdot \boldsymbol{k}_{\mathrm{Sm\text{-}b}}(1 + \boldsymbol{S}_{\mathrm{a\text{-}c}} + \boldsymbol{S}_{\mathrm{a\text{-}uc}} + \boldsymbol{S}_{\mathrm{a\text{-}b}}) \tag{1}$$

where

- $\boldsymbol{S}^{\mathrm{r}}$ is the random variable for the amplitude scale in the MCS (vector);
- $\boldsymbol{S}$ is the nominal amplitude value of the amplitude (vector);
- $\boldsymbol{k}_{\mathrm{Sm\text{-}b}}$ is the multiplicative uncertainty component modelled with the basis function approach (vector, systematic and random, e.g. amplification);
- $\boldsymbol{S}_{\mathrm{a\text{-}c}}$ is the additive fully-correlated* uncertainty component (vector, systematic, e.g. order sorting filters);
- $\boldsymbol{S}_{\mathrm{a\text{-}uc}}$ is the additive un-correlated uncertainty component (vector, random);
- $\boldsymbol{S}_{\mathrm{a\text{-}b}}$ is the additive uncertainty component modelled with the basis function approach (vector, systematic).

Formally the parameter $k_{\mathrm{Sm\text{-}c}}$ and $k_{\mathrm{Sm\text{-}uc}}$ are missing in the model, but the information is included in the basis function approach.

* In the fully correlated case only one random number is generated for all elements in the vector.

2.1.2 Wavelength vector

For the basic approach introduced, the following model is used for the wavelength uncertainty:

$$\boldsymbol{\lambda}^{r} = k_{\lambda\text{m-c}} \cdot (\boldsymbol{\lambda} + \boldsymbol{\lambda}_{\text{a-c}} + \boldsymbol{\lambda}_{\text{a-uc}} + \boldsymbol{\lambda}_{\text{a-b}}) \quad (2)$$

where

- $\boldsymbol{\lambda}^{\text{r}}$ is the random variable for the wavelength in the MCS (vector);
- $k_{\lambda\text{m-c}}$ is the fully-correlated factor for the wavelength scale (scalar, systematic, e.g., wavelength scale factor);
- $\boldsymbol{\lambda}$ is the nominal wavelength (vector);
- $\boldsymbol{\lambda}_{\text{a-c}}$ is the additive fully-correlated uncertainty component (vector (same value for all elements), systematic, e.g., offset);
- $\boldsymbol{\lambda}_{\text{a-uc}}$ is the additive uncorrelated random uncertainty or noise (vector, random, e.g., noise);
- $\boldsymbol{\lambda}_{\text{a-b}}$ is the additive uncertainty, modelled with the basis function approach (vector, systematic).

Factors other than a correlated factor are not helpful for the wavelength scale.

The different modelling of the wavelength and value vector is because wavelength errors are usually modelled as absolute quantities, and amplitude errors are usually modelled relatively.

2.1.3 Combination of amplitude and wavelength scale

Pairs of wavelength and amplitude vectors together are necessary to define the spectral information $\mathbf{X}^{\text{r}}$ for further calculations:

$$\boldsymbol{X}^{r}_{\text{SD}} = (\boldsymbol{\lambda}^{r}, \boldsymbol{S}^{\text{r}}) \quad (3)$$

For comparison using actual measurements of spectral irradiance from PTB, the random vector $\boldsymbol{S}^{r}_{\text{PTB}}$ is drawn from a multivariate normal distribution using the mean vector and the given covariance matrix (see Figure 3 and Figure 4). In this case, the uncertainty information of the wavelength scale is already included in the covariance matrix, and the nominal wavelength scale can be used for the value pairs:

$$\boldsymbol{X}^{r}_{\text{SD,PTB}} = (\boldsymbol{\lambda}, \boldsymbol{S}^{r}_{\text{PTB}}) \quad (4)$$

2.1.4 Model parameter

The basic approach uses additive (subscript 'a') and multiplicative (subscript 'm') model parameters linked to the values as uncertainty components.

Furthermore, the uncertainty contributions will be implemented during the simulation as follows:

- Uncorrelated (subscript 'uc'): Every vector element represents a different realisation of a random process in each of the N_{MC} MC trials. A vector with random elements is drawn in each trial. Variations of the vector elements are independent.
- Correlated (subscript 'c'): All vector element variations behave the same way in an MC step. The random number is drawn once for every trial only.
- Basis function (subscript ' b'): The correlated variations of spectral data points are simulated by the variations of the vector elements with a periodic spectral dependence. The different vectors for the trials are calculated according to the basis function approach introduced by (Kärhä et al., 2017) with Fourier or Chebyshev basis functions as described by (Vaskuri et al., 2018). See section 2.5 for details.

2.1.5 Input quantities

Using the classical Monte Carlo procedure, spectral information is required to calculate photometric quantities like colour coordinates, correlated colour temperatures, etc. As examples of spectral data, Figure 1 shows the $V(\boldsymbol{\lambda})$-curve and the spectral responsivity of a photometer, and Figure 2 shows two examples of spectral distributions of light sources.

The simulated approach is compared with real measurement data from an FEL lamp at PTB operating at a CCT of about 3077 K (see Figure 3 and Figure 4), including covariance matrixes. The measured relative spectral distribution of the FEL lamp is very close to the relative spectral distribution of the blackbody radiator at the same CCT used for the basis function approach (see Figure 2).

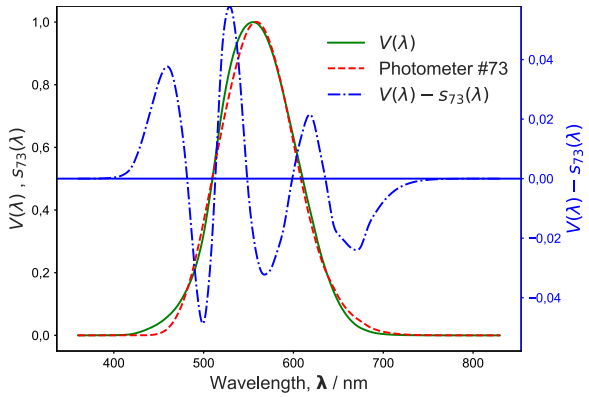

Figure 1 – *V(λ)* and sample detector spectral responsivity

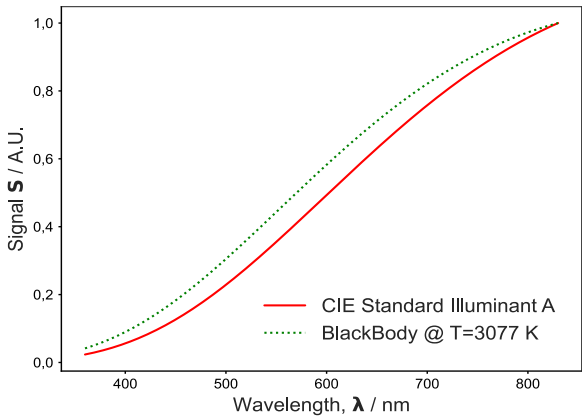

Figure 2 – Examples of spectral distributions of light sources

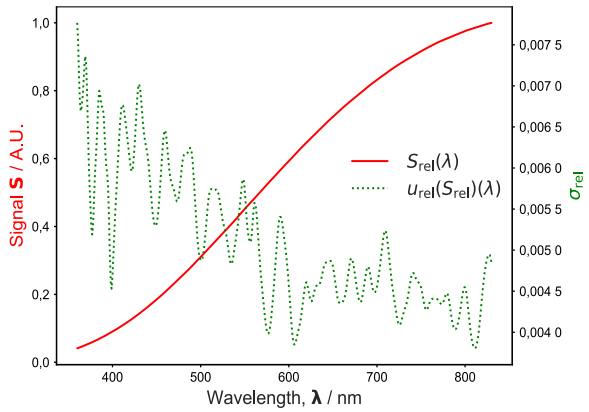

Figure 3 – Spectral distribution of a FEL lamp measurement

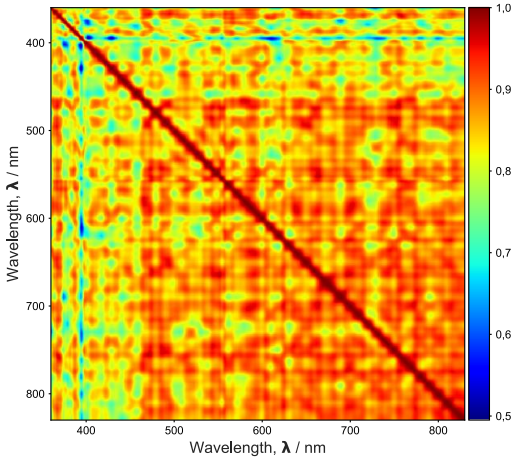

Figure 4 – Correlation matrix of the spectral distribution measurement of the FEL, shown in Figure 3

2.1.6 Output quantities

To show the effect of the different measurement uncertainties, several integrated quantities are calculated from the simulated spectral distribution representing the spectral irradiance / radiance of an FEL lamp. According to (CIE15:2018, 2018, chap. 7) the tristimulus values should be calculated with the following equations (using the notation of this paper and the nominal wavelength scale and $\bar{\boldsymbol{y}}(\lambda) = V(\lambda)$):

$$X = \sum_{i=0}^{N_\lambda - 2} \bar{\boldsymbol{x}}(\lambda_i) \cdot \boldsymbol{S}(\lambda_i) \cdot \Delta\lambda \,;\; Y = \sum_{i=0}^{N_\lambda - 2} \bar{\boldsymbol{y}}(\lambda_i) \cdot \boldsymbol{S}(\lambda_i) \cdot \Delta\lambda \,;\; Z = \sum_{i=0}^{N_\lambda - 2} \bar{\boldsymbol{z}}(\lambda_i) \cdot \boldsymbol{S}(\lambda_i) \cdot \Delta\lambda \qquad (5)$$

Using the random numbers generated with the basic approach presented here, the tristimulus values can be calculated as follows:

$$X = \sum_{i=0}^{N_\lambda - 2} \overline{x}(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{X}_i^r \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r) \tag{6}$$

$$Y = \sum_{i=0}^{N_\lambda - 2} V(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{X}_i^r \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r) \tag{7}$$

$$Z = \sum_{i=0}^{N_\lambda - 2} \overline{z}(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{X}_i^r \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r) \tag{8}$$

$$x = \frac{X}{X+Y+Z};\ y = \frac{Y}{X+Y+Z} \tag{9}$$

The correlated colour temperature (CCT) is calculated from the chromaticity coordinates (x,y) values based on standard algorithms according to (Robertson, 1968).

The spectral mismatch correction factor F (according to (ISO/CIE, 2014)) is defined by the following integral ratio, with the notation of this paper:

$$F = \frac{\int_{\lambda_{\min}}^{\lambda_{\max}} s_{\mathrm{rel}}(\lambda) S_{\mathrm{A}}(\lambda) d\lambda \int_{360\ \mathrm{nm}}^{830\ \mathrm{nm}} V(\lambda) S(\lambda) d\lambda}{\int_{360\ \mathrm{nm}}^{830\ \mathrm{nm}} V(\lambda) S_{\mathrm{A}}(\lambda) d\lambda \int_{\lambda_{\min}}^{\lambda_{\max}} s_{\mathrm{rel}}(\lambda) S(\lambda) d\lambda} \tag{10}$$

The spectral mismatch correction factor F for the measurement of the modelled spectral distribution $\boldsymbol{X}^r$ is calculated using the spectral responsivity of the photometer ($s_{\mathrm{rel}}(\lambda)$, Figure 1) without any uncertainty contribution, calibrated at CIE standard illuminant A, $S_{\mathrm{A}}(\lambda)$**.**

$$F = \frac{\sum_{i=0}^{N_\lambda - 2} s_{\mathrm{rel}}(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{S}_{\boldsymbol{A}}(\boldsymbol{\lambda}_i^r) \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r)}{\sum_{i=0}^{N_\lambda - 2} V(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{S}_{\boldsymbol{A}}(\boldsymbol{\lambda}_i^r) \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r)} \cdot \frac{\sum_{i=0}^{N_\lambda - 2} V(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{X}_i^r \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r)}{\sum_{i=0}^{N_\lambda - 2} s_{\mathrm{rel}}(\boldsymbol{\lambda}_i^r) \cdot \boldsymbol{X}_i^r \cdot (\boldsymbol{\lambda}_{i+1}^r - \boldsymbol{\lambda}_i^r)} \tag{11}$$

For simplicity, only one wavelength scale is used here. In practice, one would have to use separate wavelength scales for the spectral responsivity of the photometer, for the measurement of the spectral distribution of the DUT and for the spectral distribution of the calibration light source for which random numbers are generated independently.

Note:

> Using a non-nominal wavelength scale usually requires interpolation for further calculations. It is essential not to interpolate the values $\mathbf{S}^{\mathrm{r}}$ to the nominal wavelength scale $\boldsymbol{\lambda}$. The standard functions (e.g., colour matching functions, luminous efficiency functions, illuminants) used in the calculations should be interpolated to the usually non-equidistant random wavelength scale (CIE15:2018, 2018, chap. 7.2.3).
>
> Only in case of passing on to another user the value vector should be interpolated to a usually equidistant wavelength vector. Thereby the measurement uncertainties of the wavelength vector are converted to the amplitude values and their measurement uncertainties and correlations. This is comparable with the luminous flux measurement at a standard lamp. Here, the operating current is also specified as a nominal value (i.e. without measurement uncertainty) and the uncertainty in the setting/measurement of the operating current is included in the measurement uncertainty of the luminous flux value.

2.2 Parameters for random numbers

For simplicity, the random numbers used for simulating the uncorrelated and correlated contributions introduced above are drawn from normal distributions:

- Additive components (mean value $= 0$): $X^{\mathrm{r}} \sim \mathcal{N}(0;\sigma)$
- Multiplicative components (mean value $= 1$): $X^{\mathrm{r}} \sim \mathcal{N}(1;\sigma)$

The standard deviation parameter is selected in a way that one can adapt the result of the simulation conveniently to the situation in a concrete measurement setup:

- Uncertainty of the wavelength scale parameters is 1 nm.
- Uncertainty of the amplitude scale parameters is 1 %. (Usually, normalised data.)
- The uncertainty for the wavelength scale factor $k_{\lambda\mathrm{m\text{-}c}}$ is modelled with $k_{\lambda\mathrm{m-c}}^{r} \sim \mathcal{N}(1;0{,}001)$ due to the high sensitivity of this contribution.

2.3 Connection to physical models

The basic approach proposed here can be linked to the physical properties of measurement systems or at least to some properties usually used in "Black Box" models. A very first overview will be given here:

Table 1 – Connection to physical models

Symbol	Description	Origin in other (physical) models
$k_{\lambda\mathrm{m\text{-}c}}$	Wavelength scale factor	The measurement result of several narrowband spectral lamps for the wavelength calibration is the scale factor in nm/pixel or nm/° of the spectroradiometric system. The uncertainty of this scale factor can be used to model $k_{\lambda\mathrm{m\text{-}c}}$.
$\lambda_{\mathrm{a\text{-}c}}$	Wavelength scale shift of the whole scale	The stability (reproducibility or repeatability) of the homing/initialisation of a monochromator can be used to model $\lambda_{\mathrm{a\text{-}c}}$.
$\boldsymbol{\lambda}_{\mathrm{a\text{-}uc}}$	Wavelength scale shift of single measurement points	Reproducibility or repeatability of the wavelength setting at a single wavelength position.
$\boldsymbol{\lambda}_{\mathrm{a\text{-}b}}$	General uncertainty of the wavelength scale	A complicated relationship between different wavelength settings can be modelled by $\boldsymbol{\lambda}_{\mathrm{a\text{-}b}}$. An example of a physical-based modelling for such a parameter can be found in (White et al., 2012). The basis function approach can model the described behaviour if the correction is not possible.
$\boldsymbol{k}_{\mathrm{Sm\text{-}b}}$	Uncertainty of the absolute/relative amplitude calibration factor.	A sophisticated modelling of the calibration factor can be introduced, e.g., aging causing correlations between the calibration factors of different wavelength regions.
$S_{\mathrm{a\text{-}c}}$	Correlated uncertainty of a global offset	Modelling the global dark signal (e.g., generated by clamping).
$\boldsymbol{S}_{\mathrm{a\text{-}uc}}$	Uncorrelated uncertainty of the offset signals	Modelling the individual dark signal at every measurement position.
$\boldsymbol{S}_{\mathrm{a\text{-}b}}$	Uncertainty modelling for the offset.	A time-depended offset of a reference voltage can cause wavelength dependent offset values in the amplitude scale.

2.4 Basis function approach

The basis function approach is explained in (Kärhä et al., 2017) for Fourier basis functions and in (Vaskuri et al., 2018) for Chebyshev basis functions in detail. The implementation can be found in (19nrm02, 2023) in the MC Toolbox of the package in the file `FourierNoise.py`. A short summary is given below.

Table 2 – Basis function approach

	Wavelength scale	Value scale
Random numbers	$\boldsymbol{\lambda}^{\mathrm{r}}_{\mathrm{a\text{-}b}} = \lambda + u_\lambda \cdot \boldsymbol{\delta}$	$\boldsymbol{S}^{\mathrm{r}}_{\mathrm{m\text{-}b}} = \boldsymbol{S}(1 + u_S \cdot \boldsymbol{\delta})$
Deviation function	$\boldsymbol{\delta} = \sum_{k=0}^{N_\mathrm{B}} \gamma_k \boldsymbol{f}_k$ or $\boldsymbol{\delta} = \sum_{k=0}^{N_\mathrm{B}} \gamma_k \boldsymbol{c}_k$	
Deviation function (single function with $\mathrm{k} = \mathrm{N_B}$)	$\boldsymbol{\delta}_s = \gamma_k \boldsymbol{f}_k$ or $\boldsymbol{\delta}_s = \gamma_k \boldsymbol{c}_k$	

With the generated standard normal distributed random numbers $Y_k \sim \mathcal{N}(0;1)$ one calculates the normalised weighting factors γ_k with $\gamma_k = Y_k / \sqrt{\sum_{j=0}^{k} Y_j^2}$

Fourier basis functions	Chebyshev basis functions
$\boldsymbol{f}_0(\lambda) = 1$	$g_0(\lambda) = \boldsymbol{T}_0(\lambda) = 1$ $\boldsymbol{T}_k(\lambda) = \cos\left(k \arccos\left(\frac{2\lambda - \lambda_1 - \lambda_2}{\lambda_2 - \lambda_1}\right)\right)$ $\boldsymbol{g}_k(\lambda) = \boldsymbol{T}_k(\lambda)/\sigma_k$
$f_k(\lambda) = \sqrt{2}\sin\left(2\pi k \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} + \phi_k\right)$	$\boldsymbol{c}_k(\lambda) = \cos(\phi_k) g_{2k-1} + \sin(\phi_k) g_{2k}$

ϕ_k is a uniformly distributed random number in the range of $[0,2\pi]$ and the wavelength range is described with $[\lambda_1;\lambda_2]$. σ_k is the standard deviation of $\boldsymbol{T}_k(\lambda)$.

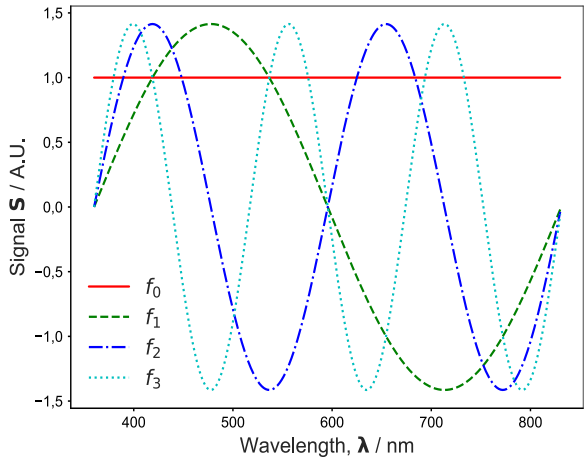

Figure 5 – Sample Fourier basis functions $\boldsymbol{f}_k$ with $\boldsymbol{\phi_k = 0}$

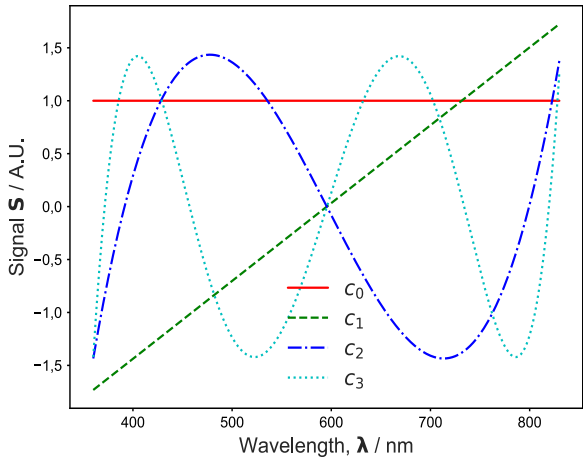

Figure 6 – Sample Chebyshev basis functions $\boldsymbol{c_k}$with $\boldsymbol{\phi_k = 0}$

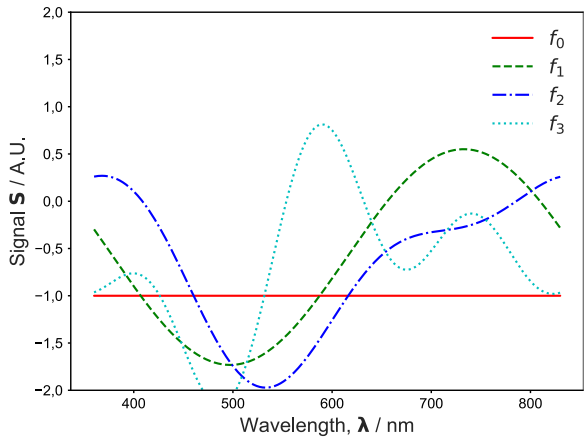

Figure 7 –Combined Fourier basis functions δ_k

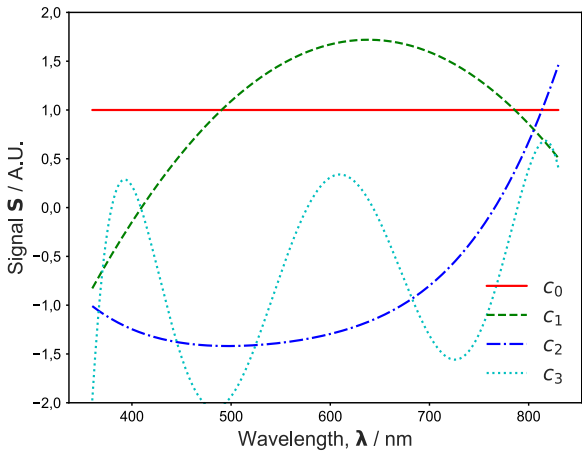

Figure 8 – Combined Chebyshev basis functions δ_k

The partially correlated realisation of the random number $\boldsymbol{\lambda}^{r}_{\mathrm{a-b}}$ with N_B orthogonal basis functions $f_k(\lambda)$ (based on Fourier or Chebyshev basis functions), the weighting factors γ_k and the standard uncertainty u_λ=1 nm (here, u_λ is a parameter of the simulation, representing the standard deviation of the noise to be generated) are expressed as shown in Table 2.

Using the deviation function with a single basis function only, as shown in the last line of Table 2 and Figure 5 and Figure 6, checks the direct influence of a specific basis function order. An example of the combination of basis functions is shown in Figure 7 for Fourier basis functions and Figure 8 for Chebyshev basis functions.

Iterating the MCS over several basis functions $N_\mathrm{B} = 0 \ldots N_\lambda/2 - 1$ will result in a lot of information. $N_\lambda/2 - 1$ is the maximum number of basis functions possible due to the Nyquist rule. Besides the basis function number for the maximum impact on the final uncertainty, three specific points are usually of particular interest:

- The evaluations based on the FEL measurement results from the PTB are shown as an additional point (and a corresponding horizontal line) in the graphs (description 'PTB').
- The fully-correlated case is also shown in the graphs as a separate point (description 'corr').
- $N_\mathrm{B} = N_\lambda/2 - 1$: The fully-uncorrelated case is separately marked (description 'un-corr').

Therefore, modelling using the additional scalar values $k_{\mathrm{Sm\text{-}c}}$ and $k_{\mathrm{Sm\text{-}uc}}$ was not necessary in (*1*) because both special cases are included in the basis function approach.

2.5 Model application

The subsequent application of these types of contributions (and all together at the end) estimates the possible sensitivity coefficients (the variation of the output quantity concerning the variation of the input quantity). This kind of sensitivity analysis proposed by (JCGM 101:2011, 2011) in Annex B is called one factor at a time (Razavi and Gupta, 2015).

3 Results

A blackbody spectral distribution is used as input quantity to make the simulation as simple as possible. Based on the theoretical blackbody radiator, the amplitude and wavelength values are modified during the MCS as described above. With the modified spectral distributions, one can calculate output integral quantities (e.g., tristimulus value Y, chromaticity coordinates x and y, the correlated colour temperature CCT and the spectral mismatch correction factor F) and study their behaviour during the simulation. These data estimate the probability distribution functions and statistical parameters for all output quantities, including mean and standard deviation. As a first step, their relations are described with linear correlation coefficients and correlation plots.

The implementation of this simulation can be found in (19nrm02, 2023). All diagrams, tables and calculations based on this Python package are made in the Jupyter Notebook `MCSim_PM.ipynb`. Calculations in the MCS were performed with 20 000 trials.

3.1 Overview

For the first setting, the basis functions are modelled with cumulative Chebyshev basis functions of order $N_\mathrm{B} = 7$ (description: "basis c 7"). A separate analysis of the influence of the kind of basis function, the order, and the difference between a single basis function and the cumulative approach will be shown in section 3.2.

Table 3 shows the output table of a measurement uncertainty calculation with the uncertainty contributions of every input quantity of the model to selected output quantities.

Figure 9 and Figure 10 show the probability distribution function for the relative tristimulus value Y the variation of the different model parameters.

Figure 11 and Figure 12 show the influence of the different model parameters on the evaluated chromaticity coordinates.

Figure 13 to Figure 15 show selected correlation matrixes of the generated SD inside the MSC. Figure 13 shows the correlation matrix for the SD based on the $\boldsymbol{k}_\mathrm{Sm\text{-}b}$ modelling with $N_\mathrm{B} = 7$ and Chebyshev basis functions. Figure 14 shows the same calculation with Fourier basis functions. Figure 15 shows a correlation matrix for the configuration of Figure 14 but with a single basis function of order 7 only.

Figure 16 shows the PDF of all selected output quantities and their correlation.

3.2 Evaluation of different basis functions

Observing the influence of the model parameter $\boldsymbol{k}_\mathrm{Sm\text{-}b}$ only, we can vary the kind of basis function, the number of basis functions N_B and how we use the basis functions (all together up to a specific order as called cumulative or as a single basis function ('s') only).

Figure 17 to Figure 21 show the effect of the number of basis functions and the kind of basis function modelling on the measurement uncertainty of the different selected output quantities.

4 Discussion

The simulations show the expected behaviour, i.e., uncorrelated and fully-correlated contributions have no significant influence on chromaticity coordinates and the other evaluated quantities. The exception is the contribution from the additive, fully-correlated errors in the values of the wavelengths (e.g., caused by the homing/initialising procedure of a monochromator or by the wavelength adjustment of an array spectroradiometer with a few spectral lines only), which makes significant contributions to nearly all investigated output quantities.

However, it was shown by modelling with orthogonal basis functions that partial correlations contribute significantly to the measurement uncertainty. In the case of the slowly changing tristimulus functions, only the long-wave basis functions (N_B is small) produce effects that do not cancel each other out. Therefore, these reflect the possible maximum effects of correlations. This is a possible contribution to the measurement uncertainty that needs to be considered for physical models.

The approach based on using single basis functions generally provides information about the most sensitive frequency for the basis function approach. It is usually observed for intermediate and more specific spectral frequencies, those related to correlations whose disregard leads to a greater underestimation of the uncertainty.

The behaviour for the spectral mismatch correction factor F (Figure 21) is different from the other integral data evaluations. This is because the high frequency difference between the $V(\lambda)$ function and the photometer #73 (Figure 1) used here also contains higher frequencies. Therefore, higher orders of the basis function cause more deviation in F than lower orders.

Table 3 – Evaluation table for the MU calculation using the basic approach

Quantity	*Unit*	*Mean*	*StdDev.*	*Distribution*	Y	$u(Y)$	x	$u(x)$	y	$u(y)$	CCT / K	$u(CCT)$ / K	F	$u(F)$
T	K	3077,0	1,000	normal	1,000 0	0,002 7	0,431 55	0,000 07	0,402 16	0,000 03	3077,0	1,0	1,002 4	0,000 010
$\lambda_{\text{a-c}}$	nm	0,0	1,000	normal	1,000 0	0,005 8	0,431 55	0,000 56	0,402 15	0,000 25	3077,0	7,7	1,002 4	0,000 074
$\lambda_{\text{a-uc}}$	nm	0,0	1,000	normal	1,000 0	0,000 5	0,431 55	0,000 10	0,402 15	0,000 12	3077,0	1,7	1,002 4	0,000 023
$\lambda_{\text{a-b}}$	nm	0,0	1,000	basis c 7	1,000 1	0,006 5	0,431 54	0,000 85	0,402 15	0,000 78	3077,2	13,8	1,002 4	0,000 106
$k_{\lambda\text{m-c}}$	1	1,0	0,001	normal	1,000 0	0,003 2	0,431 56	0,000 21	0,402 16	0,000 08	3077,0	3,1	1,002 4	0,000 031
$S_{\text{a-c}}$		0,0	0,010	normal	1,000 1	0,010 1	0,431 55	0,000 00	0,402 16	0,000 00	3077,0	0,0	1,002 4	0,000 000
$S_{\text{a-uc}}$		0,0	0,010	normal	1,000 0	0,000 8	0,431 55	0,000 14	0,402 16	0,000 13	3077,0	3,0	1,002 4	0,000 038
$S_{\text{a-b}}$		0,0	0,010	basis c 7	1,000 1	0,011 3	0,431 56	0,000 97	0,402 14	0,000 88	3076,8	19,9	1,002 4	0,000 150
$k_{\text{Sm-b}}$		*0,0*	*0,010*	*basis c 7*	0,999 9	0,011 4	0,431 56	0,000 98	0,402 14	0,000 88	3076,8	20,1	1,002 4	0,000 151
All					**1,000 2**	**0,021 2**	**0,431 55**	**0,001 72**	**0,402 12**	**0,001 51**	**3076,9**	**32,5**	**1,002 4**	**0,000 255**

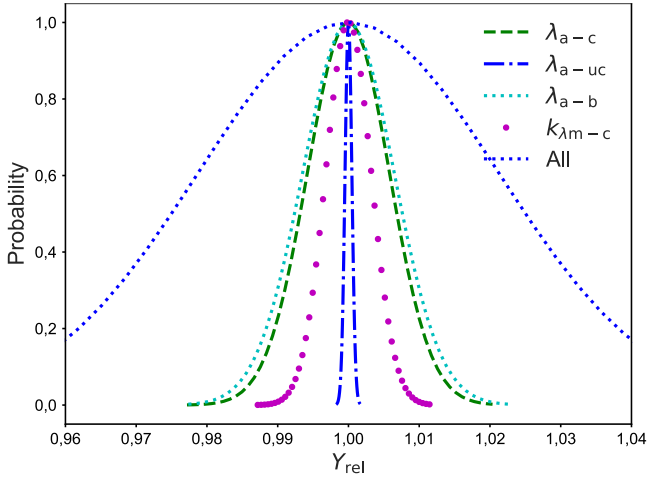

Figure 9 – Histogram of the relative tristimulus value Y_{rel} based on the wavelength uncertainty contributions

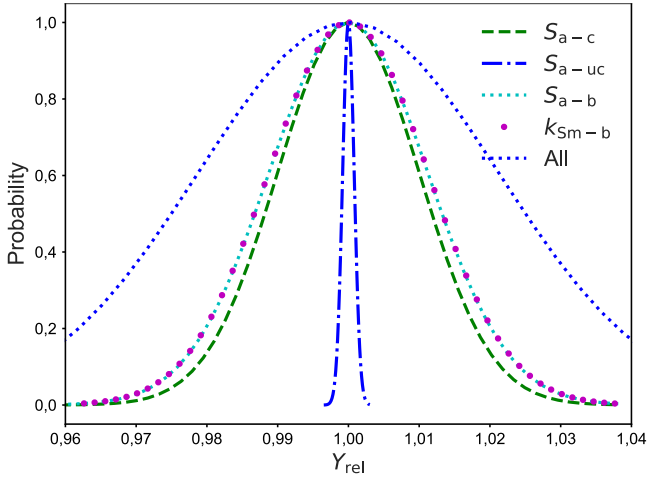

Figure 10 – Histogram of the relative tristimulus value Y_{rel} based on the value uncertainty contributions

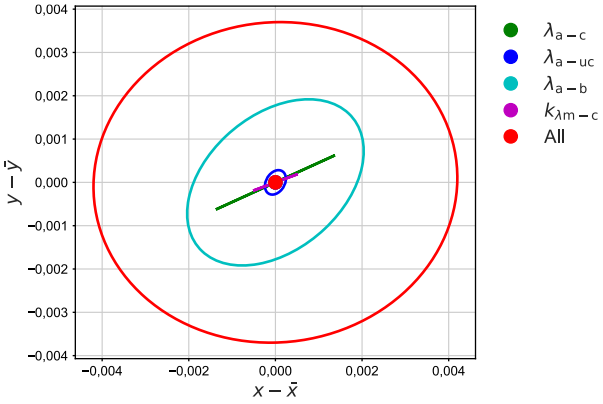

Figure 11 – Covariance for the chromaticity coordinates based on the wavelength uncertainty contributions

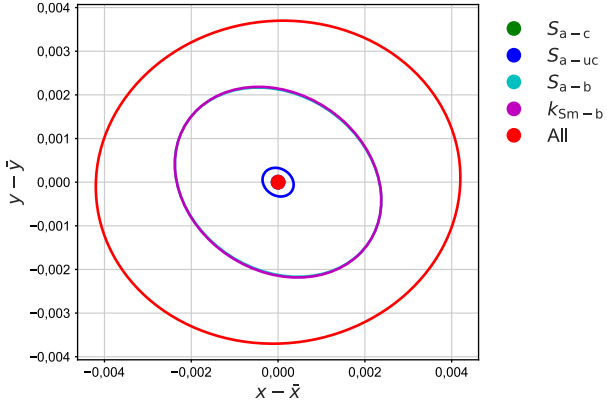

Figure 12 – Covariance for the chromaticity coordinates based on the value uncertainty contributions

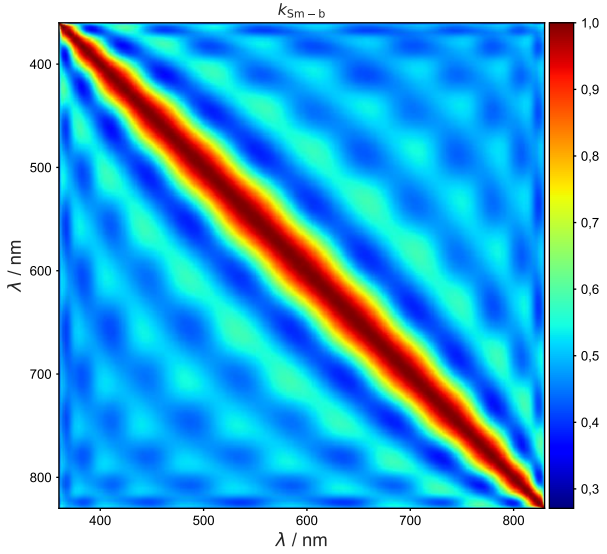

Figure 13 – Correlation matrix for the SD base on the $k_{\mathrm{Sm\text{-}b}}$ modelling with $N_{\mathrm{B}} = 7$ Chebyshev basis functions

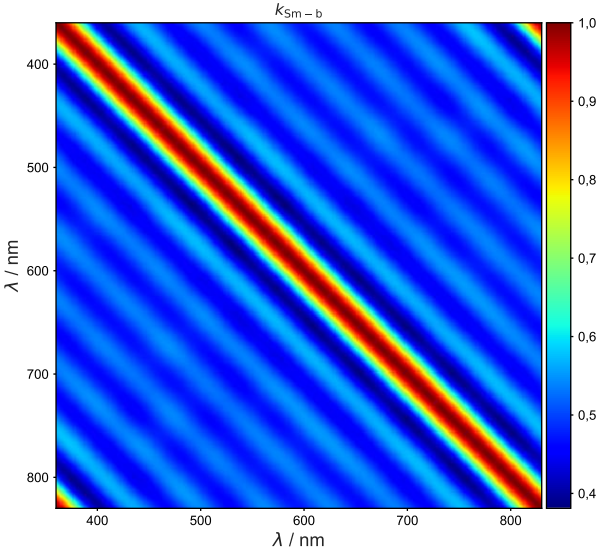

Figure 14 – Correlation matrix for the SD base on the $k_{\mathrm{Sm\text{-}b}}$ modelling with $N_{\mathrm{B}} = 7$ Fourier basis functions

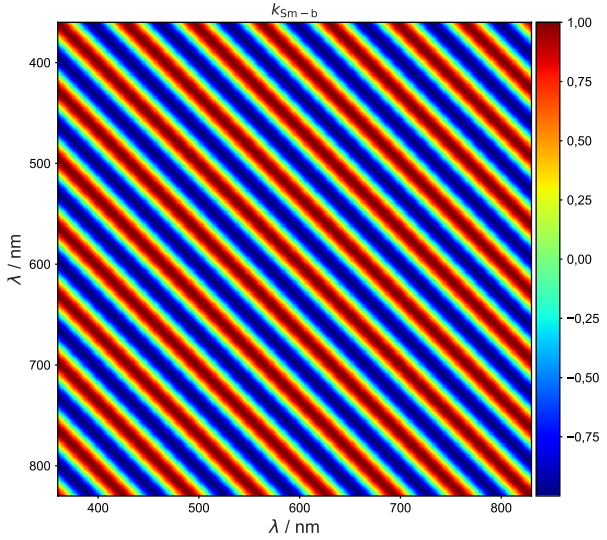

Figure 15 – Configuration like Figure 14 with a single basis function only

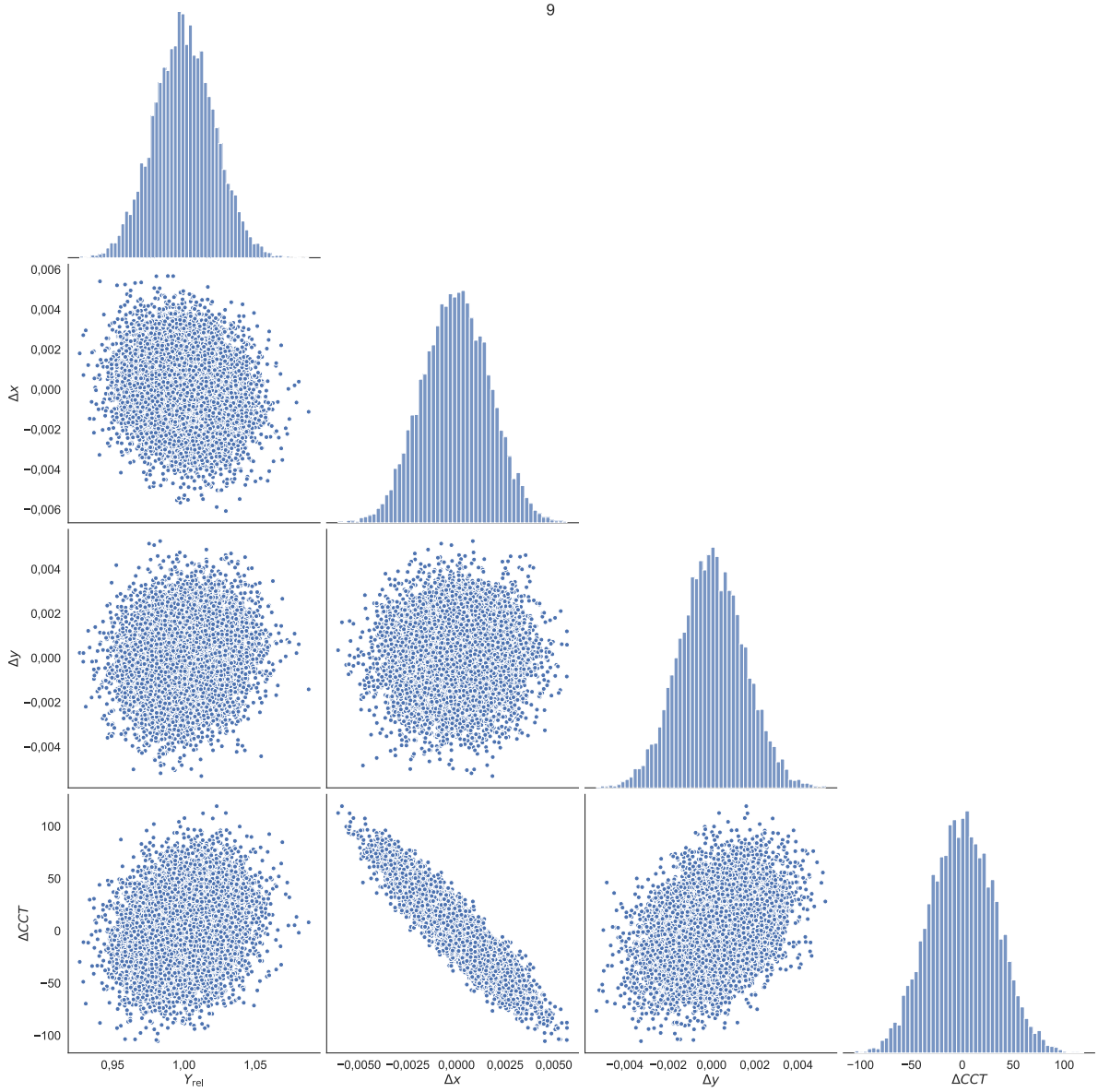

Figure 16 – Correlation of the integral output quantities for the $k_{\mathrm{Sm\text{-}b}}$ part of the model

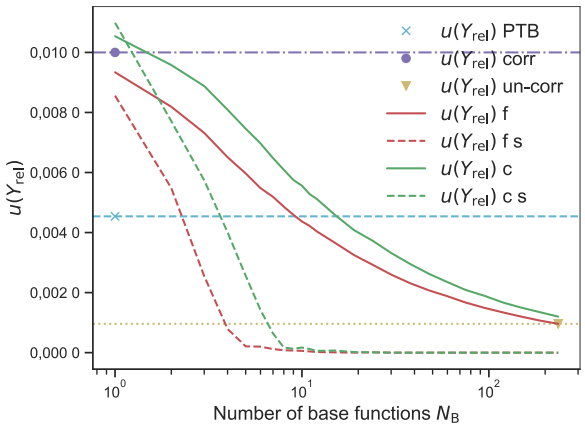

Figure 17 – Influence of the basis function setting on the tristimulus value Y evaluation

The graphs Figure 17 to Figure 21 show the number of basis functions on the horizontal axis (log scale) and the measurement uncertainty of different quantities (tristimulus value Y, chromaticity coordinates (x, y), CCT and spectral mismatch correction factor F) caused by the model parameter $\boldsymbol{k}_{\text{Sm-b}}$ only on the vertical axis.

The four different versions show:

PTB	Calculated with PTB data
corr	Fully-correlated random numbers
un-corr	Fully-uncorrelated random numbers
f	Fourier basis functions (cumulative)
f s	Fourier basis function (single)
c	Chebyshev basis function (cumulative)
c s	Chebyshev basis function (single)

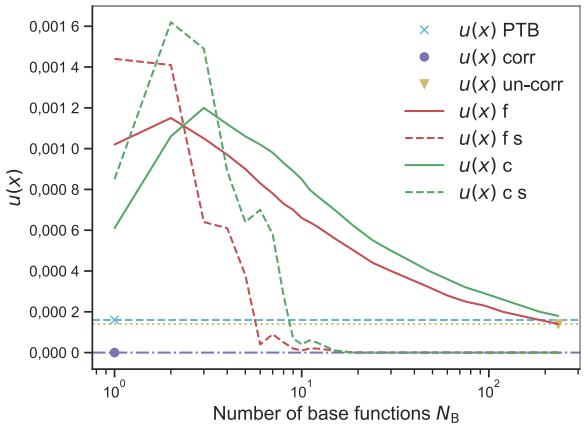

Figure 18 – Influence of the basis function setting on the x evaluation

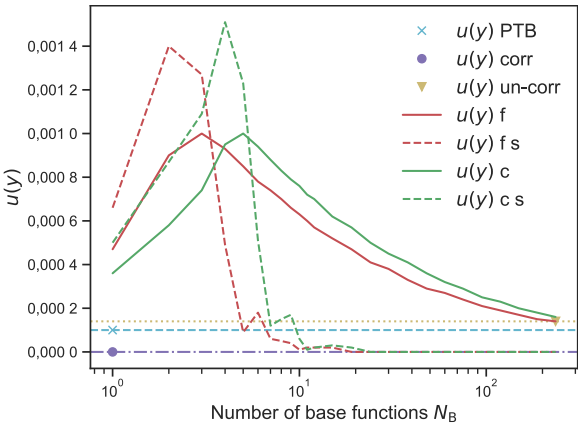

Figure 19 – Influence of the basis function setting on the y evaluation

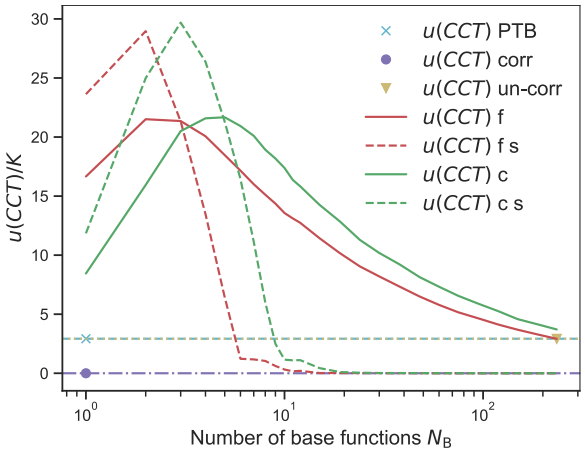

Figure 20 – Influence of the basis function setting on the CCT evaluation

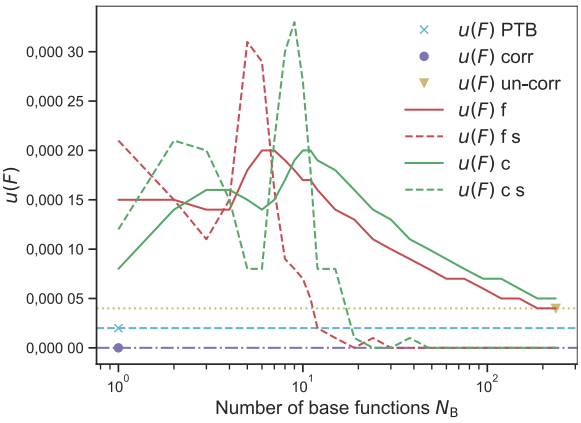

Figure 21 – Influence of the basis function setting on the spectral mismatch correction factor F evaluation

5 Conclusion

The proposed basic modelling approach is an easy tool to understand the origin of possible significant contributions to measurement uncertainty caused by correlation in the spectral data used as input quantities or by the evaluation process.

MCS has shown that when measuring quantities derived from spectral integrations, the full-correlated and uncorrelated errors contribute much less to the uncertainty than the partially correlated errors modelled by the basis functions. As expected, the basis functions with fewer terms (functions with low spectral frequencies representing errors with short autocorrelation length) provide a more considerable uncertainty in measuring these spectrally integrated quantities and thus provide a reasonable estimate of the maximum uncertainty.

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