# Which diameter matters? 

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#### Abstract

Pin gauges are widely used in various applications, i.e. for gauging, as setting standards for mechanical and optical diameter measuring instruments, or for screw gauge calibration. Form deviations and surface roughness have a different influence dependent on the subsequent application of these gauges. In this paper we discuss how pin gauges are preferably calibrated and which measurands are to be determined for which application. In particular, the elastic deformation due to the measurement force and the determination of the form deviation are discussed. For the latter, an instrument based on three-point centerless roundness measurement is presented. The measurement methods and the best measurement capabilities are validated by internal comparisons between different instruments.


## 1 Introduction

Small diameter standards - often called pin gauges - are widely used in various applications, i.e. for limit gauging, as setting standards for diameter measuring instruments, both mechanical and optical (laser micrometers), or for screw gauge calibration "over the wires". In practice pin gauges are never perfect, but their form deviations and rough surface have a different influence dependent on the subsequent application of these gauges.

In this paper we discuss how pin gauges are preferably calibrated and which measurands are to be determined. We focus on cylindrical diameter standards in the range of typically 0.05 mm to 2 mm . First, the state of the art of pin gauge calibration, based on results of international comparisons, published CMCs and accreditation scopes is shortly investigated. The CMCs are compared with typical manufacturer specifications for the pin gauge tolerances. From the latter we conclude, that uncertainties in the order of $0.1 \mu \mathrm{~m}$ are required, which is often not in accordance with typically observed form deviations.

After an overview on the existing measurement principles for diameter and form, the different calibration methods and instruments available at METAS are presented. A recently developed instrument for three-point centerless roundness measurement according to ISO 4292 allows for the determination of form deviations in a very efficient way and for a diameter range which is hardly accessible with form testers. Among the influence factors which are investigated in more detail is the elastic deformation of the pin gauge due to the measurement force. The methods are validated by internal comparisons on several pin gauges of different quality.

To conclude, recommendations are given for the measurand and the extent of calibration in view of the functional application of the pin gauge.

## 2 Measurand versus application of pin gauges

The uses of pin gauges are manifold. They are used in large quantities as go / no go limit gauges for rapid gauging the diameter of small holes, particularly in watch industry, as setting gauges or calibration standards for setting mechanical, pneumatic or optical diameter measuring instruments to a known reference value, or finally for screw gauge calibration "over the wires". Dependent on the application, the right measurand is either an envelope (circumscribed) diameter, an average diameter, a
two-point diameter in a well defined direction, or a three-point diameter (Fig.1). Obviously, for "perfect" gauges, i.e. with no form deviation (roundness and cylindricity) and negligible surface roughness, these different measurands are all the same. In practice, however, actually encountered form deviations even on high quality pin gauges are often larger than expected and the differences between the different measurands may be considerably larger than the required tolerances.


Figure 1: Four different measurands for different applications of pin gauges: (a) Circumscribed diameter for limit gauging, (b) two-point diameter for the use as a setting standard for mechanical comparators, (c) two-point diameter for the use as a setting standard for optical comparators (laser scanners), or (d), three-point diameter for wires for screw gauge calibration.

Often, both the calibration service providers and the users are not sufficiently aware of these issues and rather consider the price and the measurement uncertainty for selecting the calibration service, instead of taking into account the subsequent use of the pin gauges for making an optimal choice. There are documents available that indicate the minimum extent of measurements a calibration of diameter standards should include [1, 2], but these do neither address the most suitable measurand and method nor include any discussion on measurement uncertainty.

## 3 Pin gauge calibration

### 3.1 Manufacturer specifications and measurement capabilities

A short market survey on the three major pin gauge manufacturers and providers showed, that there are no specifications on any form errors, but only an indication of the "precision" or "accuracy" of the pin gauge, which is probably to be understood as the largest possible deviation from the indicated diameter, i.e. as a tolerance limit. For two of the manufacturers, their best quality is specified to be $\pm 0.4 \mu \mathrm{~m}$ and $\pm 0.3 \mu \mathrm{~m}$, respectively, which is equivalent to the claimed best measurement capability of their own accredited calibration laboratory. The third manufacturer specifies an "accuracy" of $\pm 0.15 \mu \mathrm{~m}$, which is four times smaller than its accredited CMC, but probably corresponds to its claimed measurement capability outside the accreditation scope.

An analysis of the CMCs published by the national metrology institutes shows, that by the end of 2014 29 NMIs declare pin gauge measurement capabilities for a range $\leq 1 \mathrm{~mm}$ and 13 NMIs have CMCs with a range $\leq 0.1 \mathrm{~mm}$. Form these laboratories, 9 claim an expanded uncertainty $U \leq 0.1 \mu \mathrm{~m}$, whereas 3 indicate even $U \leq 0.05 \mu \mathrm{~m}$. Only a few of these CMCs have been validated by documented international comparisons, the measurements of which were accomplished 20 years ago [3]. A similar analysis was made on a national level among accredited laboratories of the Swiss Calibration Service SCS. 13 laboratories have pin gauge calibration with a diameter range $\leq 1 \mathrm{~mm}$ within their accreditation scope, with expanded uncertainties ranging between $0.3 \mu \mathrm{~m}$ and $0.6 \mu \mathrm{~m}$.

In conclusion, the claimed manufacturer tolerances and the calibration uncertainties are small enough, that a discussion on the most suitable measurand and on the influence factors compromising the application of the calibration result is more than adequate.

### 3.2 Traceable diameter calibration methods

We believe, that only tactile pin gauge measurements can provide more or less direct traceability, or being considered as primary methods, whereas optical (laser scanner) or pneumatic measurement methods do rely on independently calibrated reference pin gauges, at least at a level of sufficiently small uncertainty. We distinguish three different tactile diameter measurement principles (Figure 2):
a) Length comparator with parallel flat measurement anvils, exhibiting a line contact between the pin gauge cylinder and the flat probes. Special attention needs to be paid to the geometry of the anvils - any flatness or parallelism error directly influences the result. The probes might be zeroed in direct contact against each other, or with a gauge block or pin gauge of known length. Whereas the latter cases depend on calibrated reference standards, in the former cases the probe geometry is more critical.
b) Length comparator with a spherical measurement probe, exhibiting a point contact between the pin gauge cylinder and the probe. This method requires independent traceability for the probe sphere diameter. The comparator needs to provide the possibility to go around the pin gauge with the probe without loosing or compromising its position information and to find for the measurement line with the largest diameter.
c) 2 D or 3 D measuring instrument with a spherical measurement probe, exhibiting a point contact between the pin gauge cylinder and the probe. This method requires independent traceability for the probe sphere diameter. This is potentially the most accurate and versatile method, provided a well characterized probe sphere is available with an isotropic and highly repeatable probe head and a good position metrology for the multi-axis measurement stage.


Figure 2: Three different tactile pin gauge calibration methods: (a) Length comparator with two flat probes, (b) ID displacement comparator with spherical probe, (c) 2D or 3D measuring instrument with spherical probe.

METAS provides calibration services using all three above mentioned methods:
a) The Movomatic Movotelit instrument (Figure 3) is an Abbe type comparator with an incremental length measuring system, 30 mm range, variable measurement force and flat tungsten carbide anvils of 1 mm width. The instrument is well suited for the efficient calibration of pin gauges with diameters down to 0.05 mm and provides a best measurement capability of $U=0.08 \mu \mathrm{~m}$.
b) The length measurement machine LMM5, which is essentially based on a SIP coordinate measuring machine, reduced to one principle axis of measurement with a laser interferometer and a high resolution, low force probe head, was specially designed for the calibration of ring and plug gauges of larger diameter [4]. It provides a standard measurement capability down to 2 mm diameter for pin gauges with a CMC of $U=0.07 \mu \mathrm{~m}$. Smaller diameters are possible but not practical.
c) The METAS $\mu$ CMM [5] provides a best measurement capability of $U=0.05 \mu \mathrm{~m}$ for pin gauge diameters down to 0.2 mm . Smaller pin diameters are possible, but practical problems arise from holding and bending the pins. The $\mu \mathrm{CMM}$ gives not only the smallest uncertainty, but
also the best "picture" in terms of form deviation, on the other hand the measurement cost is relatively high and accessibility of pin diameter below a few millimetres from the front face is limited due to the available stylus lengths.


Figure 3: Movotelit Abbe type comparator with an incremental length measuring system and flat anvils for pin gauge calibration.

### 3.3 Elastic compression correction

The measurement force leads to an elastic compression of the probing element and the pin cylinder at contact, which must be corrected for and might lead to a substantial uncertainty contribution. For point contact as for methods b) and c), the geometrical parameters are well defined and the application of the well known Hertz formula [6] for a sphere to flat contact leads reliable approximation of the correction. Also, on both of our instruments [4,5] we extrapolate the measurement force to zero resulting therefore in a virtually zero compression. In the case of the length comparator a) with flat anvils, we have to consider the line contact between a cylinder and a flat, which is described by the following equation [7]

$$
\begin{equation*}
\propto=2 \bar{P} \cdot\left(V_{1}+V_{2}\right) \cdot\left[1+\ln \left\{\frac{8 \alpha^{2}}{\left(V_{1}+V_{1}\right) \cdot \bar{P} \cdot D}\right\}\right] \tag{1}
\end{equation*}
$$

where $\bar{P}=\frac{P}{2 a}$ is the force per unit length, $D$ the cylinder diameter, $2 a$ the length of contact and

$$
\begin{equation*}
V=\left(1-\sigma^{2}\right) / \pi E \tag{2}
\end{equation*}
$$

with $E$ the Young's modulus and $\sigma$ the Poisson's ratio. Eq.(1) is essentially linear in the measurement force and slightly depends on the cylinder diameter. For a typical measurement force of 0.2 N and a steel cylinder between flat hard metal probes, the calculated compression amounts to $0.015 \mu \mathrm{~m}$ and $0.012 \mu \mathrm{~m}$ for a cylinder diameter of 0.05 mm and 2 mm , respectively.
We have tried to experimentally verify this elastic compression model [7]. For this we measured six steel pin gauges with the Movotelit length comparator under different measurement forces varying between 0.2 N and 1.0 N in steps of 0.1 N . We used tungsten carbide flat measurement anvils of 1 mm width. Each measurement value was taken as the mean of several measurements with different orientation of the pin gauge in order to reduce the influence of a roundness deviation of the pin. The measurements had to be corrected for the elastic deformation of the length comparator itself, which was determined from a measurement curve anvil against anvil, i.e. without pin gauge, again for the range of 0.2 N to 1.0 N . This measurement resulted in an elasticity or stiffness of the comparator of $0.015 \mu \mathrm{~m} / \mathrm{N}$. Figure 4 shows the results of the elastic compression measurements for six pin gauges from steel with diameters between 0.05 mm and 5 mm .


Figure 4: Elastic compression measurements for six pin gauges from steel with diameters between 0.05 mm and 5 mm on a length comparator with flat parallel tungsten carbide anvils of 1 mm width.

The experimental results show a fairly good agreement with the theoretical model for the diameters 5 mm and 3 mm , whereas for the smaller pins significant deviations are observed. The reason for this is most probably the surface roughness of the pin gauges. The peak roughness value $R \mathrm{p}$ was measured for the 3 mm and the two 1 mm gauges and resulted in $14 \mathrm{~nm}, 29 \mathrm{~nm}$ and 144 nm , respectively. The smaller roughness values are for pins with a lapped surface whereas the surface of the grinded 1 mm pin has a peak surface roughness a factor of 10 larger than the 3 mm lapped pin. The measurement force therefore seems to deform in a reversible and thus elastic way the roughness peaks and leads to a higher compression value than expected for a theoretically ideally smooth cylinder surface. The same probably applies for the two smallest pin gauges, which have also a grinded surface, but where roughness measurements were practically not possible.

Another factor which may lead to higher compression values is the imperfect geometry of the anvils. These are assumed to be perfectly flat and parallel. Any deviation in parallelism will lead to at least one of the two contacts to be not a line contact and thus result in a higher compression as well. The parallelism of our anvils was checked by a small sphere measured at different locations across the measurement faces and found to be better than $0.03 \mu \mathrm{~m}$.
The good agreement between theory and experiment, as shown in Figure 4 at least for the larger pin gauges with good surface roughness supports the formulae of Puttock and Thwaite [7]. In an earlier publication [3] large differences were shown between these formulae and another theoretical model for the elastic compression on a line contact between cylinder and flat.

## 4 Centreless roundness measurement

Dependent on the application, the roundness deviations of pin gauges may become crucial. As introduced in chapter 1, for limit gauging the functional envelope diameter is larger than the mean diameter by about the peak-valley roundness deviation, and the same can apply for the three-point diameter as it is used for thread measurement over the wires. From experience we know, that
centerless grinded or lapped pin gauges often show large roundness deviations, typically with forms of constant diameter with three, five or even higher harmonic lobes. Two point diameter measurements do not reveal this kind of constant diameter form deviations. Figure 5 shows three examples of centerless machined pin gauges, which all show large roundness deviations up to $12 \mu \mathrm{~m}$.


Figure 5: Examples of centerless machined pin gauges with large constant diameter form deviations with 3, 5 or 11 lobes. The roundness measurements were made with a Talyrond 300 form tester.

It is obvious that a simple two point diameter measurement is not sufficient, but a calibration of the form deviation is necessary. A roundness measurement with a rotating spindle form tester, as the one used for the measurements shown in Figure 5, is time consuming, thus expensive, and gets difficult for small diameters below 1 mm . A good alternative is the centerless measurement of three point diameter variation according to ISO 4292 [8].

We have developed a simple three-point measurement instrument specially designed for small diameter pin gauges (Figure 6). It is based on a commercial comparator, where the anvils were replaced by a small tungsten carbide cylinder of diameter 0.5 mm and lapped front face on the one side and a V-shaped anvil on the other side. The V-shaped anvil is built of two cylinders from Easium® [9] lapped on the long side to a flat and glued on a V support. Easium® is an extra hard material with excellent tribological characteristics. We have two instruments, one with a $72^{\circ} \mathrm{V}$ exhibiting good sensitivity for $3-, 9$ - and 11 -lobed form and one with a $108^{\circ} \mathrm{V}$ ideal for 5 - and 7 -lobed forms. The roundness deviation is obtained from the three-point diameter variation multiplied by the so-called F-factor, which depends on the V angle and the number of undulations. These F-factors are tabled in the standard ISO 4292.


Figure 6: Three-point measuring instrument with a V-shaped anvil for the measurement of roundness deviation of pin gauges with diameter down to 0.2 mm .

Roundness measurements are carried out by manually rotating the pin gauge to about 15 to 20 positions within one full revolution and releasing a diameter measurement in each position. During these measurements the diameter variation is recorded and displayed on a graph (Figure 7). After a first cycle with the $72^{\circ} \mathrm{V}$ the number of lobes is counted and decided, whether a second cycle with the $108^{\circ} \mathrm{V}$ is necessary to get better sensitivity. Finally the larger of the obtained peak-to-valley diameter variations is multiplied by the F-factor appropriate for the observed number of undulations. Figure 7 shows the result of a 3-lobed 1.5 mm diameter pin gauge. The measurement is compared with the result obtained with a Talyrond 300 form tester. The two measured $R O N t$ values are $0.65 \mu \mathrm{~m}$ and $0.64 \mu \mathrm{~m}$, respectively, and thus in almost perfect agreement. The expanded uncertainty of the threepoint centerless roundness measurement is estimated to about $0.2 \mu \mathrm{~m}$, whereas the uncertainty of the form measurement was given by $0.1 \mu \mathrm{~m}$.


Figure 7: Three-point diameter variation (left) of a 1.5 mm pin gauge as recorded by the centerless roundness measuring instrument, polar representation of this measurement (middle) and roundness measurement with a rotating spindle instrument (right) for comparison.

## 5 Validation

The different measurement methods presented above were validated by internal comparisons on a series of six pin gauges with diameters ranging from 0.5 mm to 2.29 mm .

### 5.1 Diameter

The diameter of the six gauges was measured by the Movotelit instrument (method a) between flat anvils of 1 mm width and by the $\mu \mathrm{CMM}$ (method c) with a 1 mm diameter spherical probe. In the former case an average two-point diameter in different directions was determined, in the latter case the circumference was scanned with a point density of 300 points $/ \mathrm{mm}$ and the diameter of the LS circle was determined. Figure 8 shows the results for the six pin gauges, each measured at three heights. The agreement is fairly good, although the measurand is not exactly the same. The $\mu \mathrm{CMM}$ results are in the average about $0.04 \mu \mathrm{~m}$ larger, but there is no explanation for this systematic difference.

### 5.2 Roundness

Roundness measurements on the same pin gauges were compared with three different instruments and methods, i.e. with the three-point centerless instrument, the $\mu \mathrm{CMM}$ in the scanning mode and the Talyrond 300 form tester. The results are shown in Figure 9, again for three heights of each pin gauge. The agreement between the instruments as roughly within $\pm 0.1 \mu \mathrm{~m}$, obviously best for the pin with the smallest form error ( $\emptyset 2.29 \mathrm{~mm}$ ), and worst for the $\emptyset 0.65 \mathrm{~mm}$ pin gauge, which had a roundness deviation of RONt $=1.8 \mu \mathrm{~m}$. This difference is explained by the fact, that the TR300 form measurements were filtered with 15 UPR, whereas the $\mu$ CMM measurements were filtered with 150 UPR.


Figure 8: Comparison of diameter measurements between the Movotelit length comparator and the $\mu C M M$ for six different pin gauges, at three different heights of each gauge. Error bars are expanded uncertainties with $k=2$.


Figure 9: Comparison of roundness measurements between the three-point centerless roundness instrument, the TR300 form tester and the $\mu C M M$ for six different pin gauges, at three different heights of each gauge. Error bars are expanded uncertainties with $k=2$.

### 5.3 Circumscribed diameter

As pointed out in the introduction, for limit gauging applications the minimum circumscribed or envelope diameter is relevant. This may be determined straightforward from the $\mu \mathrm{CMM}$ measured profile, however at high cost. For pin gauges with a sufficiently regular roundness deviation the circumscribed diameter may also be estimated by taking the sum of the average two-point diameter in different directions and the roundness deviation, typically resulting from a tree-point diameter variation. Although this method is only approximate and does not work for all imaginable form deviations, it is more efficient, does not require an expensive $\mu \mathrm{CMM}$ and turns out to work well in practice. For limit gauging it is certainly much closer to the functional use than an average or least squares diameter. Figure 10 shows comparison measurements for the same pin gauges as above. The agreement between the two methods is within $\pm 0.1 \mu \mathrm{~m}$ and thus remarkably good.


Figure 10: Comparison of circumscribed diameter values, measured by the $\mu C M M$ and determined as the sum of average two-point diameters measured on the Movotelit and three-point centerless roundness values, at three different heights of each gauge. Error bars are expanded uncertainties with $k=2$.

## 6 Conclusion

We have shown, that two-point measurements between flat probes can give reliable diameter results, if

- the average in different directions is taken;
- the measurement anvils have a good geometry in terms of flatness and parallelism;
- and if low measurement forces are applied together with the appropriate correction for the elastic deformation.
Expanded measurement uncertainties below $0.1 \mu \mathrm{~m}$ can be reached.
Furthermore, the roundness deviation of small pin gauges can be determined efficiently and reliably by centerless measurement of the three-point diameter variation, at least for centerless grinded pins which usually have constant diameter form deviations with an odd number of lobes. The centerless roundness measurements show good agreement with spindle roundness measurements and allow for smaller diameters to be measured with an expanded measurement uncertainty of about $0.2 \mu \mathrm{~m}$.

In response to the two existing documents on the extent of calibration for cylindrical diameter standards [1, 2] we propose - depending on the application - to carry out the following measurements for routine calibrations of pin gauges, based on the above experience and the customer's functional need:

## 1. Limit gauges:

- circumscribed diameter determined as the sum of the average two-point diameter and the roundness deviation determined by a centerless three-point diameter variation or a form tester;
- at 1 measurement height close to the end of the pin.

2. Wires for screw gauge calibration:

- average two-point diameter;
- roundness deviation determined by a centerless three-point diameter variation or a form tester;
- at 1 or eventually 3 measurement heights around the centre of the wire.


## 3. Setting gauges:

- average two-point diameter or two-point diameter in a defined direction;
- roundness deviation determined by a form tester;
- at 3 measurement heights around the centre of the pin gauge.


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