

Dissertation

Jan Michael Spichtinger

Stitching interferometry for traceable form measurement of large optical freeform surfaces

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Dissertation
Mechanical Engineering

Stitching interferometry for traceable form measurement of large optical freeform surfaces

Jan Michael Spichtinger

Stitching interferometry for traceable form measurement of large
optical freeform surfaces

Von der Fakultät für Maschinenbau
der Technischen Universität Braunschweig
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Abstract

A novel stitching interferometry method for the form measurement of large and moderately curved optical surfaces is presented. It is implemented on a measurement system for specimen dimensions of up to 1000 mm x 200 mm in area and with radii of curvature down to 2 m. The system has a 5-axis positioning system which can align the employed Fizeau interferometer to any surface forms within the specifications for subaperture measurements. The presented method is based on wavelength-swept interferometry measuring the topography and the cavity length between interferometer and specimen. External angle measurements with two tiltmeters and – depending on the test specimen – with an autocollimator or rotary encoders provide the relative position of specimen and interferometer. For the stitching, only the offset of the subapertures has to be fitted. Specimens ranging from flat to freeform surfaces are measured to study different aspects of the measurement setup. Among others, the instrument transfer function is determined. An analytic uncertainty budget according for the measured topographies is derived. Furthermore, a method is introduced to estimate the radius uncertainty for spherical specimens based on the topography uncertainty and a simple Monte Carlo simulation. A novel concept of a three-dimensional autocollimator-like rotation sensor with a measuring range of up to 10° is presented. It can be used in the stitching interferometry setup in the future. Finally, Monte Carlo simulations of the measurement process and the evaluation are performed for the measured specimen. Various error contributions of the system components are taken into account. The measurements, the respective uncertainty budgets and simulations show a good agreement. The stitching interferometry method is transferable to a measuring system for specimens of up to 1.5 m in diameter and ≥ 10 m in radius of curvature which is currently being set up. For this purpose, further Monte Carlo simulations were performed to find the uncertainty contributions with the highest sensitivity for different applications. This is the accuracy of the angle measurement, and for strongly curved specimens the accuracy of the lateral positioning. At the end of this thesis, possibilities for the further development of the methods used are outlined and discussed.

Zusammenfassung

Ein neues Stitching-Interferometrie-Verfahren für die Formmessung an großen und moderat gekrümmten optischen Oberflächen wird vorgestellt. Es wurde an einem Messsystem für Prüflingsgrößen bis 1000 mm · 200 mm mit Krümmungsradien von 2 m oder mehr implementiert. Das System besitzt ein 5-Achs-Positioniersystem, das ein Fizeau-Interferometer zu jeder beliebigen Oberflächenform (innerhalb der Spezifikationen) für Subapertur-Messungen ausrichten kann. Die Methode basiert auf wellenlängen-

schiebender Interferometrie, die neben der Topographie auch die Kavitätslänge zwischen Prüfling und Interferometer misst. Zusätzlich werden mit zwei Neigungssensoren und - je nach Prüfling - einem Autokollimator oder Rotationsencodern die Winkel zwischen Interferometer und Prüfling gemessen. Beim Zusammenfügen der Subaperturen müssen dadurch nur noch die Höhenunterschiede zwischen den Subaperturen gefittet werden. Prüflinge mit ebenen Flächen bis hin zu Freiformflächen werden gemessen um verschiedene Aspekte des Messsystems zu untersuchen. Unter anderem wird die Instrumententransferfunktion für das Messsystem bestimmt. Für die gemessenen Topographien wird ein analytisches Unsicherheitsbudget aufgestellt. Außerdem wird eine Methode vorgestellt, die es erlaubt, die Unsicherheit des Radius für einen sphärischen Prüfling aus dessen Topographie-Unsicherheit anhand einer einfachen Monte-Carlo-Simulation zu schätzen. Ein neues Konzept für einen dreidimensional messenden Rotationssensor, der einem Autokollimator ähnelt, wird vorgestellt. Er hat einen Messbereich von bis zu 10° und kann in Zukunft in dem Messsystem verwendet werden. Schließlich werden Monte-Carlo-Simulationen des Messprozesses und der Auswertung für die gemessenen Prüflinge durchgeführt. Dabei werden verschiedene Fehlerbeiträge berücksichtigt. Die Messungen, die zugehörigen Unsicherheitsbudgets und die Simulationen zeigen eine gute Übereinstimmung. Das Stitching-Interferometrie-Verfahren kann auch auf ein Messsystem für Prüflinge bis zu 1.5 m im Durchmesser und Krümmungsradien von ≥ 10 m übertragen werden. Dieses System wird momentan aufgebaut. Dazu wurden Monte-Carlo-Simulationen durchgeführt um die Unsicherheitsbeiträge mit der größten Sensitivität zu finden. Der größte Beitrag ist die Genauigkeit der Winkelmessung, und für stark gekrümmte Prüflinge die Genauigkeit der lateralen Positionierung. Zum Schluss werden Möglichkeiten für die Weiterentwicklung der verwendeten Methoden vorgestellt.

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Previous publications

Parts of this work were published in advance. The publications to date are listed here. Many of the main results of the thesis were published in the following peer-reviewed paper:

- [1] Jan Spichtinger, Michael Schulz, Gerd Ehret, and Rainer Tutsch. “Traceable stitching interferometry for form measurement of moderately curved freeform surfaces”. In: *Optical Engineering* 61.09 (Sept. 2022). DOI: 10.1117/1.oe.61.9.094102

Interim results were published in the following conference proceedings:

- [2] Gerd Ehret, Jan Spichtinger, and Michael Schulz. “Status of highly accurate flatness metrology at PTB for optics up to 1.5 meters in diameter”. In: *Optical Metrology and Inspection for Industrial Applications VIII*. ed. by Gerd Ehret, Benyong Chen, and Sen Han. SPIE, Oct. 2021. DOI: 10.1117/12.2602680
- [3] Jan Spichtinger, Michael Schulz, and Gerd Ehret. “Absolute distance interferometry for subaperture stitching of large freeform optics”. In: *EPJ Web of Conferences* 238 (2020). Ed. by H. Michinel, M.F. Costa, and O. Frazao, p. 06011. DOI: 10.1051/epjconf/202023806011
- [4] Jan Spichtinger, Michael Schulz, and Gerd Ehret. “Optomechanical simulations of an advanced stitching interferometer set-up for large freeform optics”. In: *Optical Metrology and Inspection for Industrial Applications VII*. ed. by Gerd Ehret, Benyong Chen, and Sen Han. SPIE, Oct. 2020. DOI: 10.1117/12.2573704
- [5] Gerd Ehret, Jan Spichtinger, Manuel Stavridis, and Michael Schulz. “Setup of a new form measurement system for flat and slightly curved optics with diameters up to 1.5 metres”. In: *Seventh European Seminar on Precision Optics Manufacturing*. Ed. by Alexander Haberl, Gerald Fütterer, Oliver W. Fähnle, and Rolf Rascher. SPIE, July 2020. DOI: 10.1117/12.2564912
- [6] Jan Spichtinger, Gerd Ehret, Michael Schulz, Manuel Stavridis, and Clemens Elster. “Konzept für eine hochgenaue und rückgeführte optische Formmessung an großen Optiken bis 1,5 Meter”. In: *DGaO-Proceedings 2019 of the Deutsche Gesellschaft für angewandte Optik e.V.*. 2019. URL: https://www.dgao-proceedings.de/download/120/120_p1.pdf

Two patents have been filed:

- [7] Jan Spichtinger. “Autokollimator und Oberflächen-Messsystem”. DE10 2020 113 675.7. 2022. URL: <https://register.dpma.de/DPMAreger/pat/register?AKZ=1020201136757>
- [8] Jan Spichtinger. “Winkelmessgerät”. DE10 2021 109 392.9. 2023. URL: <https://register.dpma.de/DPMAreger/pat/register?AKZ=1020211093929>

During this work, the following peer-reviewed paper was also published without being a part of the dissertation:

- [9] Joachim Frühauf, Eva Gärtner, Zhi Li, Lutz Doering, Jan Spichtinger, and Gerd Ehret. “Silicon Cantilever for Micro/Nanoforce and Stiffness Calibration”. In: *Sensors* 22.16 (Aug. 2022), p. 6253. DOI: 10.3390/s22166253

1 Introduction

The properties of optical components and systems cannot be reliably optimized beyond the uncertainty with which its properties can be measured. Form measurements of optical elements are a key component to ensure their required performance whether for imaging or non-imaging applications. The performance of an imaging optic can be measured directly by measuring the point-spread function (PSF) of the system, or by measuring the surface forms and thus the resulting wavefront error. High-accuracy optics and optical measurement systems are needed in science and technology to push the boundaries of the measurable and to continuously improve high-tech products. Examples include but are not limited to telescopes optics, synchrotron optics, head-up display (HUD) systems and optical systems for computer vision.

As the angular resolution of an imaging system depends on its aperture, modern scientific optical instruments like telescopes tend to get larger with each generation. The size of the optics requires an accurate manufacturing and a quality control to ensure the desired imaging qualities. The metrological traceability (see *International Vocabulary of Metrology* [10]) of the form measurements is the basis to ensure high standards for high precision optics and for their further improvement. During the production process, it is convenient to measure the form deviation in the production line, so that it can be directly corrected (for example during grinding, polishing and figuring of the optics [11, 12]). Reference standards whose form was traceably measured can ensure the accurateness of in-line measurement systems.

Freeform optics for imaging [13, 14] and other non-imaging applications play an increasingly important role because they allow for a more efficient and compact design of optical system with few elements. For example, off-axis imaging optics have wavefront errors that can be easily corrected with freeform optics instead of using several aspherical optical elements (e.g. Ref. [15]). Components that are parts of other systems such as car wind shields can have areas that are used for imaging, e.g. for head-up displays. Since they typically have freeform surfaces in the context of optics, a low uncertainty for the measurement is important to ensure a high quality of optical imaging.

In order to support this development towards larger optics with freeform components, Physikalisch-Technische Bundesanstalt (PTB), the German National Metrology Institute, wants to set up a versatile form measurement system for large optical components, that is capable of measuring large freeform optical surfaces [6, 5]. It will be mainly based on optical interferometry [16, 17], and to be more precise on stitching interferometry [18]. The system will be able to employ interferometric sensors, but also others as chromatic confocal sensors [19]. The desired specimen specifications that shall be measurable are listed in Table 1.1.

In this work, a smaller version of this form measurement system has been developed

Table 1.1: Specimen specifications for the proposed measuring system for large freeform optics at PTB.

Quantity	Specification
Specimen form	Flat to arbitrary freeform
Local radius of curvature	≥ 10 m
Type of surface	Optical surface
Diameter	up to 1.5 m

Table 1.2: Targeted specimen specifications for the form measurement system developed in this work.

Quantity	Specification
Specimen form	Flat to arbitrary freeform
Local radius of curvature	≥ 2 m
Type of surface	Optical surface
Specimen size	up to 1000 mm · 200 mm

whose methods are transferable to the proposed large system. The possible specimens are described in Table 1.2. Another possible future application is the form measurement of synchrotron mirrors, for example.

In Chapter 2, the state of the art of modern optics for different applications is discussed. Subsequently, an overview over different form measurement methods with examples is given. In chapter 3, specific objectives for the form measurement system are derived. A brief and comprehensive summary of the necessary theory is given in Chap. 4. In Chapter 5, the experimental setup of the stitching interferometry form measurement system and the applied methods are presented and analysed. Chapter 6 includes measurements performed with the form measurement system on different specimens, this is a flat, a spherical and a freeform specimen as well as a specimen with a chirp structure. In the following chapter 7, the traceability of the measurement results is explained. An uncertainty budget for the topography measurements on the basis of a model equation is given according to the *Guide to the Expression of Uncertainty in Measurement* [20] (GUM). Also, a method for the determination of the uncertainty of the radius based on the topography uncertainty is presented. The next chapter 8 comprises simulations of the measurement system and measurement and evaluation process to verify the method and to determine the uncertainty according to the *Supplement 1 to the “Guide to the expression of uncertainty in measurement” - Propagation of distributions using a Monte Carlo method* [21]. The simulations also serve for a comparison with the real measurements. In the end of this chapter, simulations for the proposed large form measurement system are performed to identify critical system parameters. In Chapter 9, the results are discussed and in Chapter 10 an outlook is given.

2 State of the art

First, several optical systems are presented. These are examples for what the measuring system shall be able to measure. Then, several up-to-date form measurement methods are described with examples of existing form measurement systems. Some of these methods are the basis for the envisioned measurement system.

2.1 Large moderately curved optics

Here, an overview is given over optical components that are possible specimens within the measurement capabilities of the envisioned form measurement system for large freeform optical surfaces presented later. The examples show different requirements for the form measurement systems regarding different specimens.

2.1.1 Telescope mirrors

Modern large telescopes are reflecting telescopes and have segmented primary mirrors. Examples of the largest ground-based telescopes under construction or in the planning stage are the *Extremely Large Telescope* [22] (see Fig. 2.1), the *Thirty Meter Telescope* [23] (see Fig. 2.2) and the *Giant Magellan Telescope* [24]. The first two telescopes have M1 mirror segments with diameters of approximately 1.4 m. The *Giant Magellan Telescope* has 8.4 m diameter M1 segments. The radius of curvature of the primary mirrors of the three telescopes is in between 36 m and 68.7 m and the total M1 mirror diameters are in the range of 24.5 m to 39.3 m.

The focal length f of a paraboloidal mirror with the surface form $h(x, y) = \frac{x^2 + y^2}{2R_c}$ ¹ is proportional to the curvature radius R_c at the origin of the paraboloid:

$$f = \frac{R_c}{2} \quad (2.1)$$

For a spherical mirror, this is valid under the condition that the diameter fulfils the condition $D \ll R_c$. The angular resolution according to the Rayleigh criterion is given by: [25]

$$\theta = \arcsin\left(1.22 \frac{\lambda}{D}\right) \quad (2.2)$$

where θ is the angular radius of the central maximum of the airy disk of the aperture diffraction pattern. D is the pupil diameter and λ the wavelength.

The segments of segmented mirrors can be in principal aligned in two steps [26]. First, their slopes, that is the tip and tilt angle, has to be aligned to let their optical axes intersect in one point. Then, the segment pistons, that is their position along

¹ x and y are the lateral coordinates of the mirror.

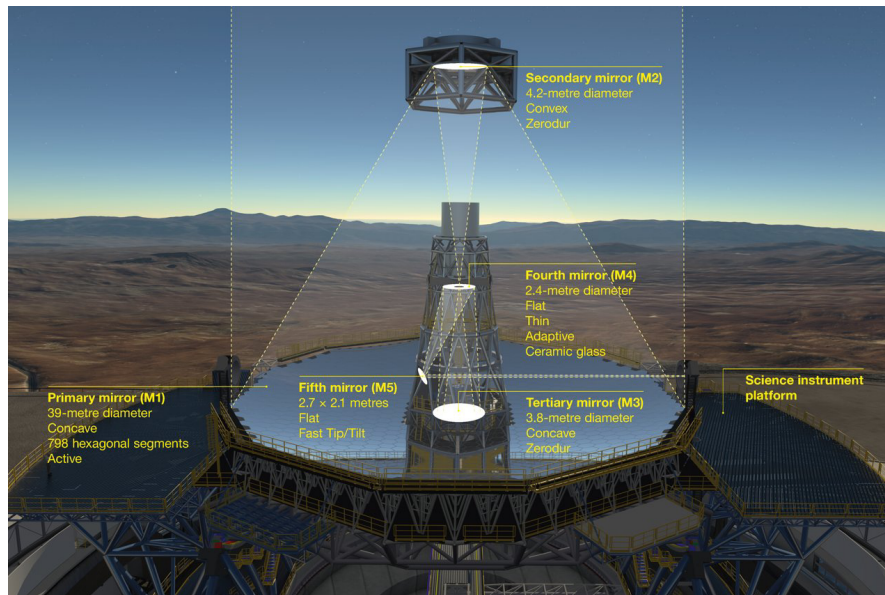


Image by ESO (https://commons.wikimedia.org/wiki/File:The_optical_system_of_the_ELT_showing_the_location_of_the_mirrors.jpg),
<https://creativecommons.org/licenses/by/4.0/legalcode>

Figure 2.1: Model of the Extremely Large Telescope (ELT) optics.



Courtesy TMT Observatory Corporation
(https://commons.wikimedia.org/wiki/File:Top_view_of_tmt_complex.jpg),
<https://www.tmt.org/page/image-use-policy>

Figure 2.2: Conception of the Thirty Meter Telescope with the segmented primary mirror.

their optical axis, have to be matched, so that the focal points fall in one point. An accuracy significantly smaller than the wavelength must be achieved. In practice, this is an iterative process. With matched phases, the segments act together as one optic which is diffraction limited and achieves a higher resolution than the single segment. If they are not properly aligned, the resolution is not higher than for a single segment.

Segmented telescope mirrors typically have active bearings to correct the position and the surface form which changes due to environmental conditions. As an example, the M1 mirror position actuators of the *Extremely Large Telescope (ELT)* of the *European Southern Observatory* have motion ranges of 15 mm for piston and tip/tilt correction [27]. The segments also have further actuators to correct form distortions. The number of actuators determines which spatial frequencies can be corrected. In general, only mid- to low spatial frequencies can be corrected. There is a different requirement for the different spatial frequency components of the form. The surface roughness of the optics has to be very small compared to the wavelength to avoid high frequency wavefront errors. The radius of curvature of the segments also has to be sufficiently accurate to avoid defocus errors. For single optics, the defocus error can be corrected by changing the position of the optical element. For segmented optics with deviating focal lengths, this is not possible [28].

The form of the mirrors is typically paraboloidal, off-axis paraboloidal or ellipsoidal. Freeform components of the mirrors (as they are also contained in an off-axis paraboloid) improve the imaging properties and allow more flexibility in the design [29, 30]. Power and astigmatism of paraboloids can be corrected to a certain degree by changing the position along its z -axis or by tilting the mirror or moving it laterally to the optical axis.

To take a closer look at an example: The Extremely Large Telescope primary mirror has 798 off-axis segments with 1440 mm in diameter [31]. It is supposed to work with optical instruments in the visible, near-infrared and mid-infrared region. In serial production, the root mean square (RMS) wavefront error (WFE) in Table 2.1 were achieved for the M1 segments. The segments are measured on an interferometric test bench using a spherical convex reference wavefront and CGHs.

Table 2.1: Achieved specifications of the ELT primary mirror segments in serial production [31].

Wavefront error	25 nm RMS
WFE with focus, astigmatism and trefoil term of the Zernike polynomials [32, 33]) removed	10 nm RMS
Surface roughness	1 nm RMS

An example for an operational telescope is the James Webb Space Telescope (JWST). It is a space-based telescope which has a primary mirror consisting of 18 segments with 1.3 m in diameter (see Fig. 2.3). It works in wavelength regions from the visible to the mid-infrared [34]. In space, there is no infrared noise as in the earth's atmosphere. A shield blocks infrared radiation from the sun.

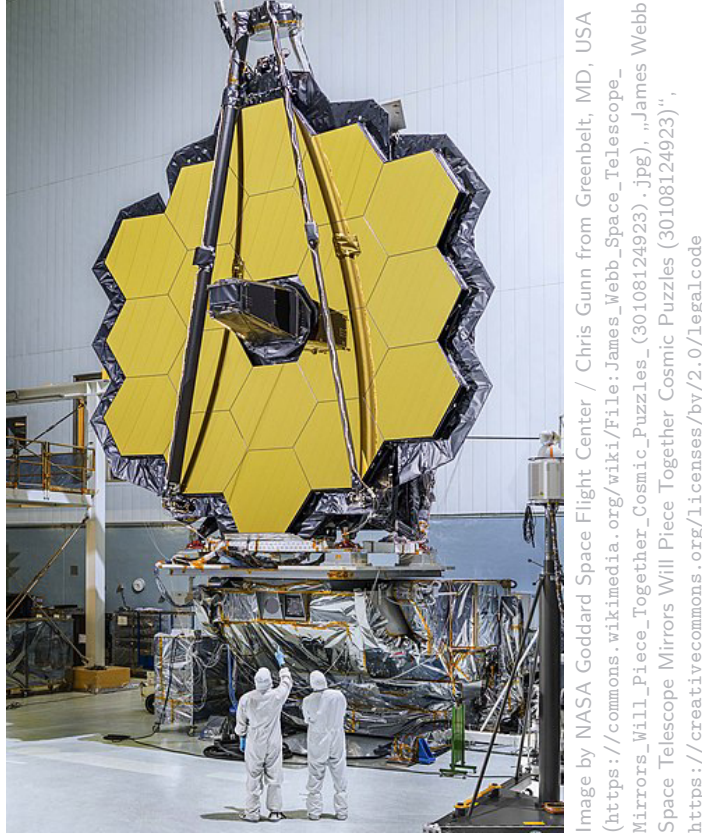


Figure 2.3: The JWST primary mirror.

For all scientific instruments of the JWST, a Strehl ratio larger than 0.8 was required [34]. The Strehl ratio S is the ratio of the theoretically maximum possible intensity of a peak and the achieved intensity in dependence of the wavefront error [35, 36, 37]. It can be calculated by

$$S = |\langle e^{i2\pi \frac{\delta}{\lambda}} \rangle|^2 = e^{-\sigma^2} \quad (2.3)$$

where λ is the wavelength and δ the RMS WFE. $\sigma^2 = \langle (\phi - \bar{\phi})^2 \rangle$ is the RMS deviation of the wavefront phase ϕ from its mean value $\bar{\phi}$. As the Strehl ratio depends on the wavelength, the requirements for the RMS WFE depend on the wavelengths to be observed with the telescope (or optical system in general). For the JWST, the criterion $S > 0.8$ was fulfilled for all its optical systems [34].

To conclude, a comparison of conventional and freeform optics in the context of astronomical telescopes and their manufacturing can be found in Reference [38].

2.1.2 Synchrotron mirrors

Another example for scientific optics with demanding form requirements are X-ray mirrors. They are used in the beamlines of synchrotron and free electron laser facilities [39, 40] to guide the generated X-ray radiation to an experimental setup. Synchrotron

radiation in general ranges from THz, IR, UV, EUV, VUV to soft and hard X-ray radiation. [39]. It is generated by the deflection of high energy electron beams with static or alternating magnetic fields. The mirrors for X-ray radiation are elongated because the incident beam has to arrive under a grazing angle to fulfil the criterion for total external reflection. For X-rays, the refractive index of materials is slightly smaller than 1, leading to total external reflection at grazing incidence. Typical lengths of these mirrors are a few 100 mm to 1 m. The radiations is, for example, used in scattering and absorption experiments in atomic, molecular and condensed matter physics and structural biology. An overview of many different kinds of applications in different wavelength regions can for example be found in Ref. [39].

The X-ray mirror forms range from flat, spherical, aspherical and cylindrical to freeform [41]. Flat optics redirect the beam, while curved optics focus the beam in one or two dimensions. The optics have demanding specifications with regard to a minimal surface roughness and a minimal form deviation from the design. An ideal surface form is important for the brilliance of the synchrotron radiation beam. The brilliance (or sometimes brightness) describes the photon flux $\dot{N}_{\text{photon}}$ per beam cross section (RMS beam sizes σ_x and σ_y), collimation of the beam (RMS beam angles σ'_x and σ'_y) and spectral bandwidth ($\frac{d\omega}{\omega}$) [42]:

$$B = \frac{\dot{N}_{\text{photon}}}{\sigma_x \sigma_y \sigma'_x \sigma'_y \frac{d\omega}{\omega}} \quad (2.4)$$

Exemplary requirements are slope errors of 1'' (RMS) and below, and roughnesses well below 1 nm (RMS) [41]. The specifications achieved typically depend on the form. There are also adaptive and active synchrotron mirrors as in Ref. [43].

2.1.3 Head-up display optics

HUD systems project a virtual image through a transparent screen that is in the line of sight for the given task of the user. Examples are the wind shield of a car or an aeroplane cockpit window (see Fig. 2.4). In the illumination path, freeform optics can be applied to achieve a compact size of the system [44]. Freeform optics can correct aberrations that may occur due to off-axis optical elements. An accurate measurement of the components and the screen guarantees the desired properties of the system. The reflecting screen can also have freeform properties. Many head-up display systems are head-mounted display (HMD) [45] and fall into the category of small optics. Here, freeform optics offer a compact design as well [15].

2.2 Form measurement systems

Next, an overview of different types of form measurement systems and their applications is given. This is a selection from a multitude of different system.

Commercial examples include the *LuphoScan* by *Taylor Hobson* [52] using a scanning point multi-wavelength interferometer, and the NANOMEFOS [53] (by *Netherlands Organization for Applied Scientific Research and Dutch United Instruments*) using a differential confocal sensor consisting of two focus sensors (with different focal lengths) [54].

The so-called nanopositioning and nanomeasuring machine NPMM-200 [55] is a highly accurate positioning system measuring all 6 degrees of freedom according to the Abbe comparator principle. It has a measurement volume $200\text{ mm} \cdot 200\text{ mm} \cdot 25\text{ mm}$ and is suitable for measurements with optical and tactile sensors. The positioning accuracy is at the nanometre level. The whole system is in a thermally controlled chamber to minimize environmental influences.

Tactile profilometry

Instead of an optical sensor, a tactile probe is scanned over the specimen. Typically, the force that occurs between the probe and the surface is kept constant to avoid large forces on the surfaces. Forces can be well below 1 mN. The probe tips consist, for example, of diamond spheres with different diameters. The direct measurement data is the convolution of the surface profile and the form of the sphere. The convolution is one limit to the achievable instrument transfer function. Measurements can be continuous or pointwise. The accuracy strongly depends on the accuracy of the positioning system for specimen and probe tip. It is often improved with interferometers. One commercial example is the UA3P (Ultrahigh Accurate 3-D Profilometer) by *Panasonic* [56, 57].

2.3 Interferometry

2.3.1 Subaperture stitching interferometry

Stitching interferometry [18] is a well-established solution for specimens that are too large or too curved for a single aperture measurement with an interferometer. There is a vast amount of different stitching interferometry techniques. They differ in the type of interferometer, the reference wavefront used, the positioning system, the stitching algorithm (especially the parameters to be fitted) and how positions and angular positions are measured (built-in encoders and/or additional measuring devices).

A selection will be presented that is relevant to the method developed in this work.

In Ref. [58], a patent assigned to *QED international Technologies Inc.*, a stitching interferometry setup for spherical specimens is presented. Using a multi-axis positioning system, a specimen is measured in many subapertures. The stitching is formulated as a linear problem that includes local errors like positional and angular misplacement and global errors for reference wave, pixel scale and distortion as parameters to be determined.

QED also presented a stitching interferometry for the form measurement of optics with freeform components [59]. The stitching is based on a six degree-of-freedom rigid-body fit. The measured optics have up to 1300 μm deviation from a spherical form. Specimens up to 200 mm in diameter were measured.

The traceable multiple sensor (TMS) [60] is a self-referencing type of stitching interferometry. A profile is measured in many steps with a multiple sensor which can be a Fizeau interferometer. A point of the topography is redundantly measured by different pixels. The angle between interferometer and specimen is measured with an autocollimator. From this information, a set of linear equations can be set up yielding the most probable topography and the pixelwise systematic topography error of the sensor. This method is limited to almost flat specimens. The error of the angle is assumed to be purely statistical. Systematic errors of the autocollimator cannot be corrected.

The *two-dimensional tilt-measurement-based stitching interferometry* (2D-TSI) [61] is a method similar to the TMS. A specimen on a linear stage is scanned with a Fizeau interferometer. The tilt errors of the linear stage relative to the measurement table are measured with two tiltmeters. During stitching, the angles are fixed and not fitted. Only the height difference of overlapping subapertures is minimized. This method was applied for a strip of merged subapertures, i.e. $1 \times n$ subapertures, but it can be transferred to a raster of subapertures. The accuracy of the tiltmeters is an important contribution to the topography accuracy as it is integrated over the specimen size.

In Reference [62], a stitching interferometry setup with a white light interferometer and a 3-axis positioning system is presented. The subapertures are stitched with a simulated annealing [63] algorithm iteratively searching for their optimal arrangement in height. The subaperture tilts are fixed. For higher surface slopes, a higher magnification is used to resolve the interference pattern. Ref. [64] presents a stitching interferometry setup for small complex-shaped optics with diameters in the order of 12.7 mm and strong curvature. The specimen is placed on a hexapod so that it can be measured in subaperture measurements. The interferometer is a Michelson type with wavelength sweeping.

An example of the measurement of synchrotron mirrors with stitching interferometry is presented in Ref. [65].

2.3.2 Interferometry with annular stitching for aspheres

With a spherical reference wavefront, spherical topographies can be measured in null-configuration. Aspheres are rotationally symmetric but contain non-spherical surface components. By changing the distance between interferometer and specimen, the null-configuration can be achieved for different annular sections of the asphere and then be stitched together. One example is Ref. [66].

2.3.3 Interferometry using computer generated holograms

Computer-generated holograms (CGHs) are diffractive optical elements that change the phase of a wavefront. For an interferometer, a reference wavefront can be designed with a CGH matching the nominal topography of a specimen to be measured. The interferogram then gives the deviation of a specimen from the design. An overview can amongst others be found in Ref. [67].

2.3.4 Non-null interferometry with inverse ray tracing

Non-null interferometry is a kind of interferometry, in which the reference wavefront and the specimen wavefront have an optical path difference that does not vanish by design for an ideal reference and specimen. The tilted-wave interferometer (TWI) [68, 69], based on a Twynman-Green interferometer, has an illumination consisting of several single light sources, which illuminate the specimen under different angles. Thereby, high slopes on aspheres can be measured. The raw measurement data consists of overlapping interferograms for the different light sources. The interferograms are far from the null-configuration. Also, a virtual model (but not necessary a full physical model) of the interferometer is set up. Using inverse ray tracing with the model, the wavefront errors caused by the interferometer optics are reconstructed by recreating the observed interference patterns with the model. The model is often called Virtual Twin. The TWI typically measures small aspherical optics.

2.4 Interferometry for large specimens without stitching

In this section, some examples of interferometry for large specimens with a single sub-aperture measurement are presented.

2.4.1 Large flat surfaces

There are Fizeau interferometers with beam expanders for very large optical flats, e. g. a transmission flat with 32 inch in diameter [70]. At larger sizes, production becomes more elaborate and a sufficient stability of the interferometry setup must be ensured. With increasing size of the reference surface, the bearing also has an increasing influence on deformations due to gravity [71]. With a three-flat test, deformations due to mounting can be compensated [72]. Large optical flats can be used for redirecting beams or as references for testing other components.

2.4.2 Tower-mounted interferometer for large concave mirrors

Large concave mirrors, for example in telescopes, can be measured with an interferometer (or additional deflection optics) mounted on a tower in the centre of curvature of the mirror. The advantage is that only a single aperture has to be measured. However, a large setup with an optical path length in the range of the specimen focal length is required. The lateral resolution is limited by the image sensor of the interferometer.

Due to the large distance between the interferometer and the test specimen, the system is susceptible to environmental influences (changes in the refractive index, vibrations, et cetera). The errors due to disturbances can be minimized to some extent, for example by single measurements with simultaneous acquisition of several phase shifts.

Mirrors with a primary spherical form and additional other form components (e.g. ellipsoid, freeform) can be measured with CGHs.

An example is the interferometry setup at the University of Arizona, see Ref. [73].

2.5 Radius measurement based on cavity length measurement

This method is based on a wavelength-sweeping Fizeau interferometer working with a spherical reference wavefront [74]. The setup is aligned so that the centre of curvature of the reference sphere and the specimen are in the same position (for either a convex or concave sphere). With an interferometric distance measurement, the radius of the spherical specimen can be directly calculated as the difference between cavity length and radius of curvature of the reference sphere.

The uncertainties of the reference surface and the uncertainty of the (compensated) position of the specimen is higher than the uncertainty of the cavity length measurement.

2.6 Wavefront measurement for large specimens

A Shack-Hartmann sensor (SHS) [75, 76] uses a microlens array to measure the wavefront angle reaching each microlens by measuring the lateral displacement of the focal spot on an image sensor (see Fig. 2.5). An example for an application measuring an X-ray mirror is found in Ref. [77].

2.7 Comment on accuracy, precision and uncertainty

It sometimes occurs that the distinction between precision, accuracy and uncertainty is not exact for some form measurement setups, and that the uncertainty is incompletely

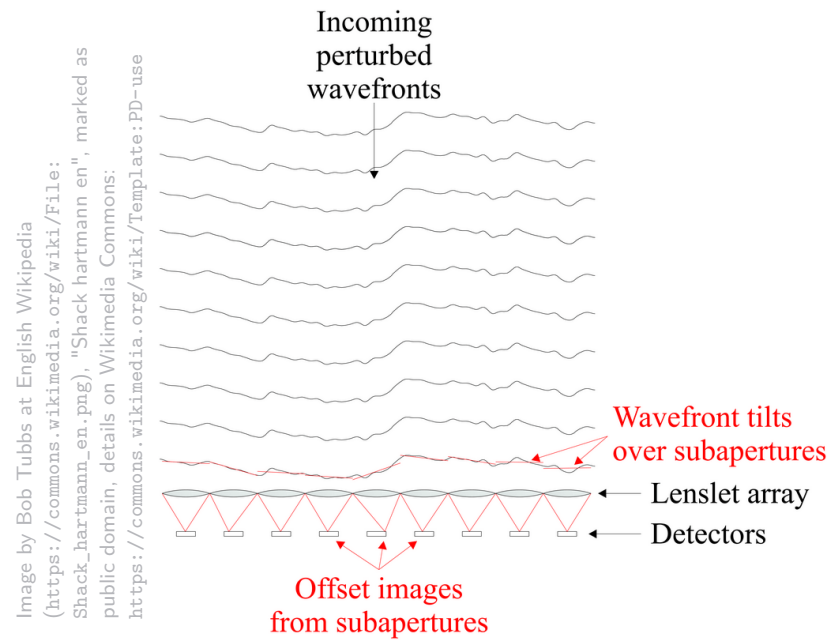


Figure 2.5: Principle of a Shack-Hartmann sensor. The lateral positions of the focal points correspond to the wavefront tilt of the subapertures.

specified as the precision only. Thereby, the accuracy (which contains unknown or uncorrected systematic errors) is neglected leading to an apparently smaller uncertainty.

3 Concept and objectives

The underlying goal of PTB is to develop a form measurement system for the test specimens presented in the state of the art section 2.1. In this context, the main aim of this work is to develop and evaluate methods for the highly accurate metrology for the optical form measurement of large and moderately curved conventional and freeform optics with diameters of up to 1.5 m and with local radii of curvature larger than 10 m.

For the form measurement system, stitching interferometry is chosen because interferometry offers sub-nm resolution. Furthermore, stitching interferometry measuring devices can be set up in large standard laboratories and promise flexibility in the specifications of the targeted specimens. By contrast, a tower-based interferometer for curved optics cannot measure flat specimens. Areal measurements, as performed in stitching interferometry, are also advantageous for determining the absolute form. Within the subapertures, a very accurate relationship exists between the topography points. The sensor does not have to be too close to the SUT. A further advantage of interferometry is that the distance between specimen and interferometer can be directly measured. The angle relationships between the measuring head and the test specimen (and thus between the subapertures) will be additionally measured in order to also obtain an accurate relationship.

As outlined in Chap. 2, stitching interferometry has been applied to a variety of specimens. The novelty of this work is the tailoring of stitching interferometry for specimens that are large and have freeform surfaces with moderate surface gradients. In comparison to Ref. [64], the edge length of the measurable specimens in this work is larger by more than one order of magnitude, while the surface slopes are smaller by more than one order of magnitude.

Since the 1.5 m positioning system is not yet available, a setup with a measurement area of 200 mm · 1000 mm will be implemented with a 10 mm Fizeau interferometer for subaperture stitching. The system uses a tunable laser and a 5-axis motion system. It is also a fully-fledged measuring system in itself. On the other hand, it can demonstrate the methods for the large measurement system. The specimens to be measured in this work range from 50 mm in diameter to a rectangular shape of 200 mm · 500 mm. The local radii of curvature are 2 m or larger.

A subaperture stitching interferometry method has been developed, that it is suitable for all kinds of reflective surfaces from flats to freeform surfaces. The stitching algorithm was formulated as a linear problem. This allows for a comprehensive understanding of the results of the stitching. A model based uncertainty budget according to the "*Guide to the Expression of Uncertainty in Measurement*" [20] has been established together with a traceability chain. The uncertainties of the input quantities of the measurement have been assessed, and it was analysed how they propagate through the data evaluation

and the stitching algorithm. In doing so, the sensitivity for the different uncertainty contributions was calculated. The goal is the calculation of uncertainty in a simple but accurate way using the uncertainty contributions. Furthermore, a method is presented for estimating the uncertainty of the radius of curvature using a simplified Monte Carlo simulation. In addition, ways to measure the angular position of the interferometer and sample are being investigated.

The measurement system will also be virtually implemented in a simulation based on a geometric model. Several Monte Carlo simulations will be performed for a sensitivity and error analysis according to Ref. [21]. The results for the existing measurement system will be compared with the analytic uncertainty budget and the results from the measurements.

All methods that are developed shall be transferable to the larger setup with a larger interferometer and positioning system. Therefore, a simulation of the future large measurement system is required and will be presented at the end of the thesis in Section 8.4. The simulation allows to investigate the theoretical limits of the measurement method for given component specifications of the 1.5 m measurement system.

4 Theory

In this chapter, the theoretical basics applied in this work are explained. First, the necessary general theory of classical optics is outlined. Then, the theory for wavelength-swept interferometry for topography and cavity length measurements is explained, including topics such as signal processing, the retrace error of a Fizeau interferometer and effects of defocus in an interferometer with imaging optics. Two basic stitching algorithms for subaperture stitching interferometry are presented.

Subsequently, the stitching algorithm developed in this work using topography, cavity length and tilt measurements is presented. An extension of the algorithm with an alignment parameter for an external angle sensor is also explained.

Afterwards, the theory for the angles sensors (rotary encoders, autocollimator and tiltmeter) is briefly presented, as they are used in the measurement setup. The geometry of positioning systems is considered to understand the occurring geometrical errors. This is one of the foundations of the model based uncertainty budget (Sec. 7.2) and the simulations (Chap. 8).

4.1 Overview over classical optics frameworks

The fundamental description of classical optics is given by the Maxwell equations [78]. They are discussed thoroughly in introductory texts like Ref. [79, 80] and many others. For the Fizeau interferometer, the plane wave and spherical wave solution of the Maxwell equations are of interest as they describe relevant effects in interferometry. A useful description of the propagation of light in the wave model is Fourier optics [81]. In its simplest form, it describes the propagation of electrical or magnetic fields from one plane to another, only considering of the field types. Optical elements can be modelled as changing the amplitudes and phases of an electrical field. This is most simply done under the assumption of thin optics, for example thin lenses. This is the assumption that an optical element does not change the lateral positions of electromagnetic (EM) fields during the propagation within an optical element. Polarization of the light can also be considered as described in Ref. [81]. Fourier transforming properties also occur when light travels an infinite distance, or at least much further than the wavefront's lateral structures. The Fourier transforming properties are also inherent to convex optical lenses. The electromagnetic field in the back focal plane is equivalent to the Fourier transform of the field in the front focal plane. An imaging system of two lenses can be understood as a double Fourier transform of the electrical field in the object plane.

A simplification of the model of electromagnetic fields is the model of ray optics (see for example Refs. [33, 12, 82, 83]). The ray optics model describes electromagnetic

waves through rays that travel in direction of the wave vector, that is, perpendicular to the wavefront. With ray optics, refraction of light can be modelled, but not diffraction. It is especially used for the design of optical systems. The refraction at an interface between two materials is described by Snell's law

$$n_1 \sin(\alpha_1) = n_2 \sin(\alpha_2) \quad (4.1)$$

where n_1 and n_2 are the refractive indices of the two materials and α_1 and α_2 the ray angles relative to the interface normal. For reflections, the angle of incidence and reflection onto a reflective surface are equal. The phase corresponding to the optical path length of a ray can also be calculated by integrating the index of refraction over the optical path, therefore ray optics can also describe the interference of two rays travelling a different path.

A further simplification is paraxial ray optics (see e.g. Refs. [33, 12, 82, 83]). Under the assumption that the rays only have small angles relative to the optical axis, the law of refraction can be linearised with the small angle approximation for the sine terms in Eq. 4.1 (compare Ref. [12]). Optical elements (including free space) can be expressed as matrices which are multiplied with a vector describing the optical rays. This allows for a linear description of a complex optical system. Optics can be modelled with propagation within the optics or, if neglected, in the thin lens approximation [12]. The ray optics formalism can also be used for Gaussian beams [12] by expanding the vector describing a ray, while the matrices representing optical elements remain unchanged. Another formalism, that describes coherent monochromatic light including polarization is the Jones calculus [33].

As long as the assumptions of the respective models are fulfilled, they are equivalent to each other. In this work, the models of wave optics and ray optics and the ideas of Fourier optics are used, since the occurring light sources are sufficiently monochromatic and coherent, and polarization effects are not used.

4.2 Wavelength swept interferometry with a Fizeau interferometer

In this section, the theory for wavelength swept interferometry using a Fizeau interferometer is explained. Starting from the optical setup of a Fizeau interferometer, the whole measurement process from the interferogram formation to the measurement and the data evaluation is outlined.

4.2.1 Fizeau interferometer

The Fizeau interferometer [16, 12, 17] is a widespread device to measure optical surface forms. A typical design is shown in Fig. 4.1. It uses coherent light from a laser source

that is reflected by the surface to be tested and by a reference surface. In a Fizeau interferometer, the reference surface is also the beam splitter. For ideally flat reference and test surfaces, the path through the interferometer optics is exactly the same for both reflected beams. Hence, any imperfections of the interferometer optics or other perturbations in the instrument causing changes in the optical path length are experienced by both rays equally. As the change of the optical path length applies for both rays, it does not change their optical path difference and the formed interference pattern. This principle is called "common path principle". The larger the difference between the gradients of the reference flat and the SUT are, the larger the measurement error due to of the optics imperfections becomes (see Sec. 4.2.9). This is also applying if the reference optic is not a flat but a sphere or a CGH for a matching SUT.

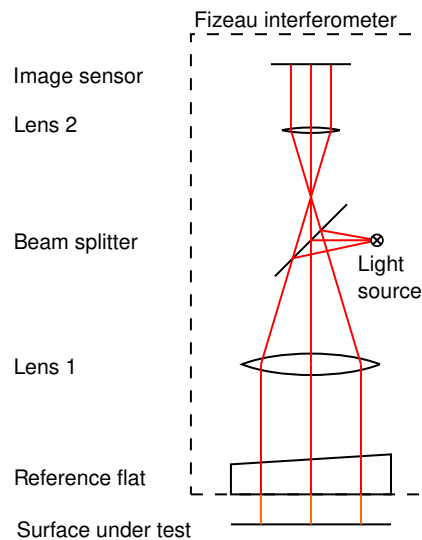


Figure 4.1: Typical design of a Fizeau interferometer. The light is emitted from a point source. From there, the rays travel to the reference flat and the SUT. From both surfaces, a fraction of the intensity is reflected and imaged onto the image sensor. In the drawing, the SUT and the reference flat are perfectly parallel, therefore the common path principle for the reflected rays is fulfilled. (The refraction at the inclined upper surface of the reference flat is not shown in the figure.)

4.2.2 Topography measurement

To determine the topography $h(x, y)$ of a SUT, the intensity of the obtained interference pattern formed by reflections from the SUT and the reference flat has to be evaluated. In the simplest form, this can be done with a single interference pattern with an ambiguity of the sign of the topography. To overcome this ambiguity, the reference flat can be shifted with a piezo stage. This is called *phase shifting interferometry*. Another possibility is a wavelength sweeping of the coherent light source. In the following paragraphs, the

wavelength-swept interferometry [84] is explained. It has the advantage of not requiring mechanical shifts of the reference flat and also being able to measure the cavity length (the distance between the SUT and the reference surface).

4.2.3 Formation of the observed interference pattern

The phase of a ray reflected by the SUT is given by:

$$\phi(x, y) = 2\pi \frac{2d(x, y)}{\lambda} \quad (4.2)$$

where d is the distance between reference surface and SUT at the lateral position (x, y) and λ the wavelength in air. Expressing the wavelength with $\lambda = \frac{c_0}{nf}$, the phase can be rewritten as

$$\phi(x, y) = 2\pi \frac{2d(x, y)f}{c_0 n} \quad (4.3)$$

where f is the frequency of the light, c_0 the speed of light in vacuum and n the refractive index. To calculate the intensity of the interference pattern, it is assumed that the reference wavefront has the overall phase ϕ_{ref} . The intensity of the interference pattern is given by [16, 12]

$$I(x, y) = |E_1(x, y) + E_2(x, y)|^2 \quad (4.4)$$

where E_1 and E_2 are the electrical fields of the light reflected from the SUT and the reference flat. In dependence of the phases of the respective wavefront, the intensity is given by

$$I(x, y) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi(x, y) - \phi_{\text{ref}}) \quad (4.5)$$

The phase difference between two points is given by

$$\Delta\phi(x_1, y_1, x_2, y_2) = \frac{4\pi f}{c_0 n} (d(x_2, y_2) - d(x_1, y_1)) \quad (4.6)$$

An example for a spherical topography and the corresponding interference pattern is given in Fig. 4.2 and 4.3.

4.2.4 Retrieval of the phase information with wavelength sweeps

The image sensor of the Fizeau interferometer measures the intensity distribution described in Eq. 4.5. Now, it will be considered how the intensity of the interference pattern changes with the wavelength. Only the interference term of Eq. 4.5 is regarded. The phase of the reference is chosen as $\phi_{\text{ref}} = 0$ for simplicity. The intensity at a position (x, y) is given by

$$I(x, y) \propto \cos(\phi(x, y)) \quad (4.7)$$

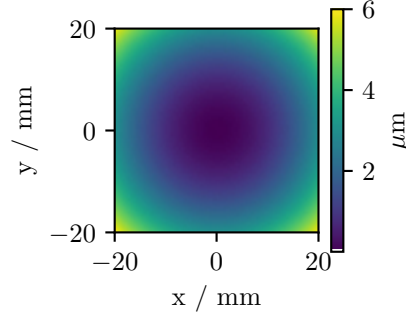


Figure 4.2: Example for a specimen topography (the SUT).

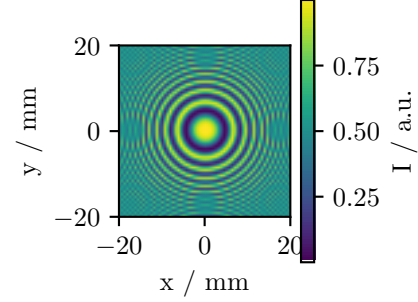


Figure 4.3: Intensity distribution of the interference pattern of a planar wavefront reflected by the topography in Fig. 4.2 with a plane reference wavefront.

To take the effect of small wavelength changes $\Delta\lambda$ into account, a first order Taylor expansion of Eq. 4.2 at $\lambda = \lambda_0$ is calculated and used in Eq. 4.7. The intensity is given by

$$I(x, y, \Delta\lambda) \propto \cos\left(2\pi\frac{2d(x, y)}{\lambda_0} - 2\pi\frac{2d(x, y)}{\lambda_0^2}\Delta\lambda\right) \quad (4.8)$$

The cavity length $d(x, y)$ can be split into $d(x, y) = h(x, y) + d_0$ where $h(x, y)$ is the topography relative to the mean cavity length d_0 . This gives

$$I(x, y, \Delta\lambda) \propto \cos\left(2\pi\frac{2[h(x, y) + d_0]}{\lambda_0} - 2\pi\frac{2[h(x, y) + d_0]}{\lambda_0^2}\Delta\lambda\right) \quad (4.9)$$

In the first fraction, d_0 can be neglected as it is a constant phase offset for all pixels. In the second term, $h(x, y)$ can be neglected because it is typically three orders of magnitude smaller than d_0 (for example for a topography with a $10\mu\text{m}$ height and a cavity length in the order of 10mm). This leads to

$$I(x, y, \Delta\lambda) \propto \cos\left(2\pi\frac{2h(x, y)}{\lambda_0} - 2\pi\frac{2d_0}{\lambda_0^2}\Delta\lambda\right) \quad (4.10)$$

The first term is a constant but position dependent phase caused by the topography. The second term is a position independent phase term changing with the wavelength sweep. The prefactor can be defined as the frequency

$$\nu = \frac{2d_0}{\lambda_0^2} \quad (4.11)$$

with which the phase changes during the wavelength sweep. During the wavelength sweep, the intensity signal is recorded at every pixel of the interferometer. The intensity

signal could look like the signal shown in Fig. 4.4.

A fast Fourier transform [85, 86] yields the frequency ν and the wrapped phase map $\psi(x, y)$ of the signal. The phase in this example is correctly determined as zero, and the amplitude as 0.5 (see Fig 4.5). The retrieved phase being wrapped means that is it only the relative phase between the pixels and not an absolute phase.

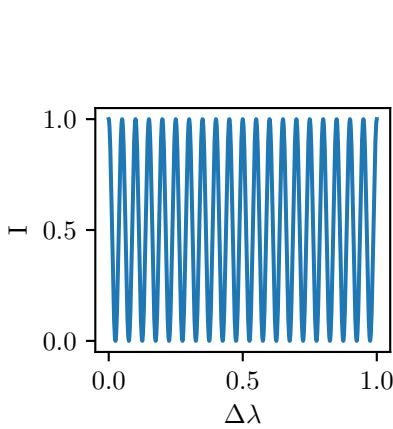


Figure 4.4: Normalized interference signal at one pixel during a wavelength sweep. The phase of the signal at $\Delta\lambda = 0$ is zero. All axes are in arbitrary units.

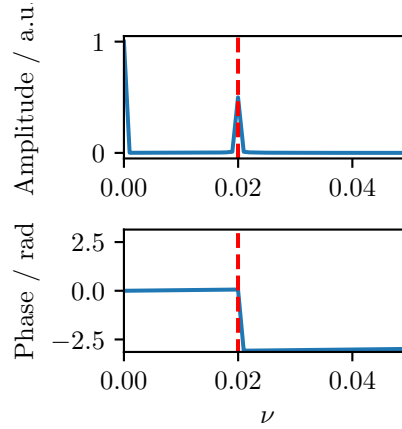


Figure 4.5: Amplitude and phase for the signal in Fig. 4.4 retrieved from a Fast Fourier transform.

An overview over the current state of the art can be found in Ref. [84].

4.2.5 Phase unwrapping and reconstruction of the topography

The retrieved wrapped phase ψ can be used to obtain the topography. Therefore it has to be unwrapped to yield the unwrapped phase $\phi(x, y)$. Under the assumption, that the phase differences at neighbouring positions (or pixels) are smaller than π (or that their height difference is smaller than $\lambda/2$) and that there is no noise present, this problem has a unique solution with exception of a constant offset term.

In the one-dimensional case, the phase unwrapping from the i -th pixel to the neighbouring $(i + 1)$ -th pixel is described by the following rule

$$\phi_{i+1} = \begin{cases} \phi_i + \psi_{i+1} & \text{if } |\psi_{i+1} - \psi_i| < \pi \\ \phi_i + \psi_{i+1} - 2\pi & \text{if } \psi_{i+1} - \psi_i \geq \pi \\ \phi_i + \psi_{i+1} + 2\pi & \text{if } \psi_{i+1} - \psi_i \leq -\pi \end{cases} \quad (4.12)$$

The first phase can be set to $\phi_0 \equiv 0$. With noisy phase data, artefacts can arise when the phase difference of two neighbouring pixels is close to π .

The two-dimensional phase unwrapping is unambiguous without noise and can be calculated in the same way as for the one-dimensional case by following a continuous path through the phase map while following Eq. 4.12. With presence of noise, there can be an ambiguity leading to different unwrapped phase maps when following different paths. In this case, an approach assigning weights to the phase steps as described in Ref. [87] can be applied. If contradictions during the unwrapping arise, the phase steps with higher weights determine how the phase is unwrapped. Of course, there are numerous other approaches as well.

The unwrapped phase can be converted to the topography by a multiplication with the central wavelength λ_0 according to

$$h(x, y) = \frac{\lambda_0}{2} \frac{\phi(x, y)}{2\pi} \quad (4.13)$$

An example for a wrapped phase and its corresponding unwrapped phase can be found in Fig. 4.6 and 4.7.

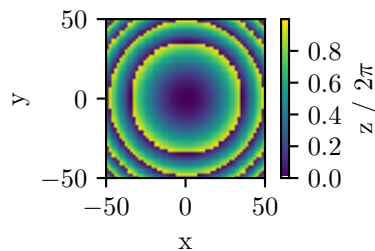


Figure 4.6: The figure shows an example of a two-dimensional wrapped phase map.

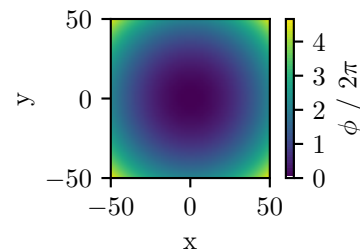


Figure 4.7: Unwrapped phase retrieved from the wrapped phase map in Fig. 4.6.

4.2.6 Determination of the cavity length

The cavity length d_0 , that is, the absolute distance from the specimen to the reference surface, can be calculated from the phase difference between the wavefronts reflected by both elements. From Eq. 4.10 it can be seen that the frequency $\nu = \frac{2d}{\lambda_0^2}$ of the intensity change during the wavelength sweep is proportional to the cavity length. From Eq. 4.11 follows that the cavity length is

$$d_0 = \frac{\nu \lambda_0^2}{2} \quad (4.14)$$

The frequency ν can be obtained from the Fourier transform of $I(x, y, \Delta\lambda)$. In the example from Fig. 4.5, ν can be directly read from the amplitude spectrum. The synthetic wavelength [88] Λ is often used in the context of wavelength swept interferometry. It is

the ratio

$$\Lambda = \frac{\lambda_0^2}{\Delta\lambda} \quad (4.15)$$

With the synthetic wavelength, the total phase change for a wavelength tuning range (or step) $\Delta\lambda$ for the cavity length d can be calculated:

$$\Delta\phi = 2\pi d/\Lambda \quad (4.16)$$

4.2.7 Signal processing

During the wavelength sweeping, an intensity signal is recorded for each pixel in dependence of the changing wavelength. The Nyquist frequency f_{Nyquist} , that is, the highest frequency of a signal that can be resolved, is given according to the Nyquist-Shannon-theorem [89] by

$$f_{\text{Nyquist}} = \frac{1}{2}f_{\text{sampling}} \quad (4.17)$$

where f_{sampling} is the sampling rate of the sensor. According to the Nyquist-Shannon sampling theorem, the frequency of the interference pattern oscillations during the wavelength sweep may not be higher than half the sampling rate of the image sensor. Above the Nyquist frequency, aliasing occurs [86, 85].

The frequency and phase of an interference signal can be determined with Fourier analysis as described in Section 4.2.4 by decomposing the signal in its frequency components. To increase the resolution of the frequency spectrum for the cavity length measurement, the signal $I(\Delta\lambda)$ can be zero padded. This allows to determine the peak position of a frequency component with higher accuracy. However, the sampling theorem still applies. Two very close peaks merged into one cannot be separated by zero padding. A general overview over zero padding for FFTs can for example be found in Ref. [85, 86]. An example for zero padding is shown for the signal in Fig. 4.8. The signal has only a few measurement points. To determine its main frequency, the absolute value of the Fourier spectrum is calculated by Fast Fourier Transform (FFT) and shown in Fig. 4.9. The spectrum of the raw signal (blue) has the same number of data points as the original signal and therefore a limited resolution. When reading the peak position directly from the graph, it would have a large error. To find a more accurate position of the maximum peak, padding the signal with $n = 2079$ zeros (as an arbitrary example) at the end of the signal increases the resolution of the spectrum to the number of data points including the zero values. The maximum can be found with a resolution according to the higher spectrum resolution. The zero padding acts equivalent to a rectangular window function in the time domain. This is equivalent to a convolution of the spectrum with a sinc function in the frequency domain [85].

The phase for each frequency can also be retrieved. The phase for the frequency components, that is the angles of the complex values from the Fourier transform, is obtained and shown in Fig. 4.10.

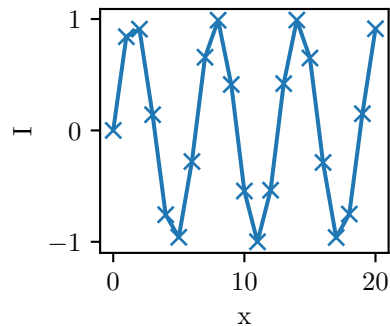


Figure 4.8: An exemplary signal with 21 data points as example.

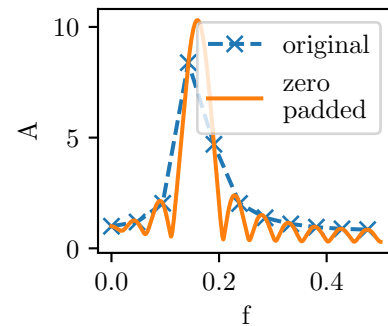


Figure 4.9: This shows the amplitude spectrum of the signal in Fig. 4.8. The blue signal is the real part of the Fourier transform of the original signal. The orange curve is the real part of the Fourier transform of the signal with $99 \cdot 21 = 2079$ zeros appended.

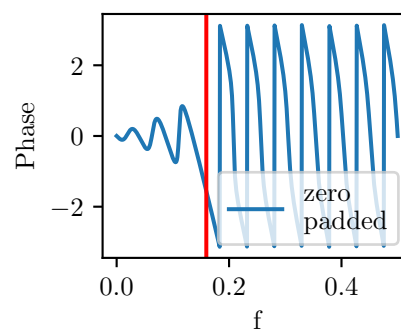


Figure 4.10: The imaginary part of the fast Fourier transform (FFT) spectrum. The phase can be directly read for each frequency. The red line is the peak position from Fig. 4.9.

4.2.8 The imaging optics of the Fizeau interferometer and effects of defocusing on the interferogram

Ideally, the SUT is in the object plane of the interferometer imaging optics, so that the wavefront is imaged directly after the reflection at the SUT. If this is not the case, the measured phase can be different from the wavefront right at the specimen surface. This effect is larger the more the reflected wavefront departs from the reference wavefront. In Figure 4.11, this effect is sketched using ray optics. The ray reflected from the surface

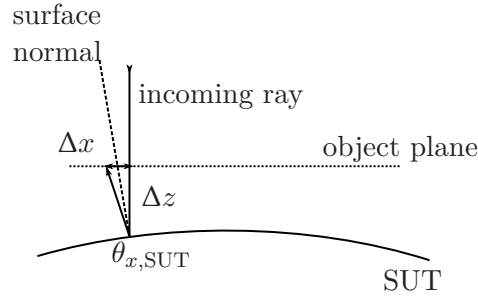


Figure 4.11: In this example, the SUT is at the distance Δz from the object plane of the interferometer imaging optics. The incoming ray parallel to the optical axis is reflected under a certain angle due to the slope $\theta_{x,\text{SUT}}$ of the SUT. Travelling back to the object plane from where it is imaged, it gets a lateral shift Δx .

has an angle of reflection of two times the gradient of the surface $\frac{\partial z}{\partial x}$. The displacement of the ray at the object plane is given by

$$\Delta x = \sin\left(2\frac{\partial z}{\partial x}\Delta z\right) \quad (4.18)$$

$$= \sin(2\theta_x\Delta z) \quad (4.19)$$

where Δz is the distance from the object plane to the SUT. Therefore, the defocusing distorts the wavefront. For the y -coordinate, the displacement of a ray caused by defocusing is calculated equivalently.

In addition, there is a change in optical path length depending on the specimen slope, caused by the propagation of the rays from the surface to the object plane under a non-zero angle. It can be calculated by

$$\Delta OPL = n_{\text{air}}\left(\frac{\Delta z}{\cos(2\theta_x)}\right) \quad (4.20)$$

For measurements in air, the refractive index is $n_{\text{air}} \approx 1$. When the defocus Δz as well as the topography gradients θ_x are small, the dependence of ΔOPL on $\cos(2\theta_x)$ is negligible. Therefore, ΔOPL approximately causes a position-independent phase offset for typical applications of the Fizeau interferometer not too far away from the null-configuration. Also, for an ideally flat SUT and a flat illumination wavefront, the defocusing has no effects.

Apart from the changes in optical path length, the lateral resolution of the imaging system also becomes worse due to defocusing. This effects mainly the higher frequency components of the imaged wavefront, and consequently of the phase map encoding the topography information. Therefore, these wavefronts might not only be distorted due to defocusing but also be missing higher spatial frequencies. This was, for example, investigated in Ref. [90].

The cavity length measurement is not significantly affected, because it does not rely on the resolution of high lateral frequencies of the phase map, but only on the frequency of the interferogram's oscillations during the frequency sweeping.

The effects of a defocused interferometer on the reconstructed wavefront can be corrected by propagating the wavefront with ray optics or Fourier optics to the object plane when the target cavity length for a perfectly focused imaging and the actual cavity length are known. High frequency spatial components of the wavefront lost by defocusing can not be recovered.

4.2.9 Fizeau interferometer in non-null configuration: The retrace error

In non-null configuration, the common path principle of the Fizeau interferometer is violated. This means that rays reflected from the SUT and the reference flat do not follow the same path through the optical system to the image sensor (see Fig. 4.12). In an ideal imaging system, there would be no optical path difference between different paths. However, real optics never have perfect surfaces. Surface form errors are typically in the range of tens of nanometre.

Assuming that the opto-mechanics of the Fizeau interferometer are sufficiently stable, the optical path length differences due imperfections of the optical surfaces are in principal deterministic. They depend on the incident ray vector, that is, the starting point and the angle of a ray in the object plane (that ideally coincides with the SUT). In other words, it depends on the topography and position of the SUT relative to the interferometer. Furthermore, the path through the optics depends on the wavelength.

There are several approaches to handle the retrace error. A first and simple approach is to measure the maximum retrace error as demonstrated in Ref. [71]. With maximum surface gradients of the topography, it can be taken into account in an uncertainty budget. A more complex approach is to measure the retrace error by measuring a known surface in different positions and tilts relative to the interferometer to obtain a function describing the retrace error in dependence of the relevant parameters.

In a further step, a simulation of the optical system of the interferometer with parameters for the form errors of the optics could be set up. Measuring a specimen in different positions and angles, it can be attempted to fit the parameters of the virtual interferometer to generate virtual measurement data that is in accordance with the real measurement data including the retrace error. This can be called a *virtual twin*. A virtual twin is also used in the Tilted-wave interferometer [91].

4.3 Stitching interferometry

Stitching interferometry [18] is a method in which the topography of an optical surface of a specimen is measured in several partial measurements, so-called subaperture mea-

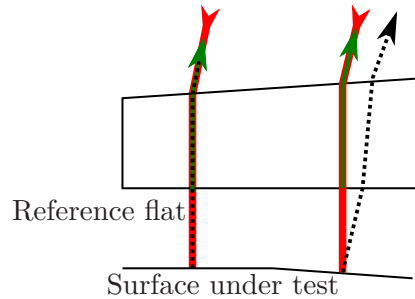


Figure 4.12: A sketch explaining the retrace error. The incoming rays (red) are reflected at the reference flat and the SUT. On the left side, the common path principle is fulfilled. On the right, the surface gradient leads to a different path for the black ray reflected from the SUT than for the green one reflected from the reference flat. The paths of these rays remain different throughout the imaging optics (not shown).

surements. This method is often applied when the specimen to be measured is too large for the aperture of the interferometer, or when the gradients of the specimen surface are too steep to measure the whole topography in one measurement. The stitching algorithm fitting together several or many subaperture measurements is a mathematical optimization. Various deviations between the subapertures can be minimized with different parameters to be optimized. Two approaches are presented in the following two subsections.

4.3.1 Stitching with height offset only

In the simplest case of stitching interferometry, an interferometer is scanned over a flat specimen with one linear stage. The approximately flat surface is oriented perpendicular to the optical axis of the interferometer and parallel to the linear stage. The topography is measured at several positions, so that the subapertures overlap laterally. The distance from the interferometer to the specimen does not have to be measured. The respective topography of a subaperture measurement will be called subtopography. To stitch the obtained subtopographies, each subtopography gets an individual height offset o_i to be determined. The lateral coordinates of the subtopographies are fixed as they are known from the linear stage and the interferometer. In the overlapping region of two subapertures i and j (i.e. that have an intersection in the x - y -plane), the mean height of each topography in the region of overlap is calculated as z_{ij} and z_{ji} . z_{ij} is the mean height of topography i in the region of overlap with the subaperture j . Figure 4.13 explains the variables in 2D.

The stitching can be expressed as a system of equations:

$$(z_{ij} + o_i) - (z_{ji} + o_j) = 0 \quad (4.21)$$

The height differences of all overlapping subapertures i and j are then minimized by

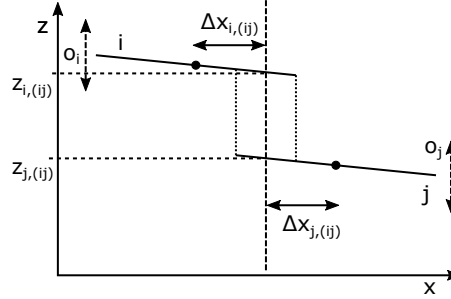


Figure 4.13: Figure illustrating the quantities describing the relative position of two subapertures in 2D.

fitting the offsets o_i in order to minimize the sum of the quadratic height differences of all pairs of overlapping subapertures in a linear least squares approach:

$$\text{Minimize } \sum_{i,j} \|(z_{ij} + o_i) - (z_{ji} + o_j)\|^2 \quad (4.22)$$

An overview of the numerical treatment of linear equation systems can be found in Ref. [92]. In Figure 4.14, it is shown how the subapertures are shifted by the stitching.

This basic approach can be extended by including many others parameters aside from the subaperture height offset. Examples are the lateral position and the tilt of the subtopographies, the magnification factors and systematic errors of the Fizeau interferometer. It can also be useful to express topography data points from the interferometric measurement in terms of polynomials, for example Zernike polynomials, instead of computing with single data points. This can reduce the computational time.

If the standard deviations of the parameters to be fitted are known, a ridge regression (using the Tikhonov regularization) can be applied [93]. Thereby, the standard deviations are used as weight factors in Eq. 4.22 (for example if they are known from the measurement). However, when unknown systematic errors are present, the regularization can produce unwanted results as purely statistical errors are assumed for this kind of regularization.

If the optimization problem is non-linear, it can be solved with non-linear least squares (for example when the lateral positions of the subapertures are also fitted). For high-dimensional non-linear problems, evolutionary algorithms like the genetic algorithms and evolution strategies can be efficient iterative approaches to find a solution. For an overview, see Ref. [94].

4.3.2 Stitching with offset and tilts

Additionally to the offset, the tilts of the subapertures can be fitted when it is assumed that some of the subapertures are erroneously tilted. The gradients of the subtopographies i and j in the region of overlap can be determined as m_{ij} and m_{ji} , following the

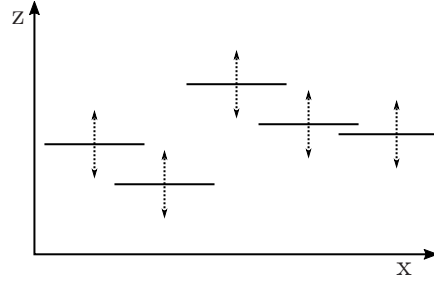


Figure 4.14: Five subtopographies seen from the side before stitching. The arrows indicate the way they can be shifted by adding offsets o_i to them. Here, the specimen was a flat surface and there is a unique solution, apart from an overall offset.

same nomenclature as z_{ij} in Eq. 4.22. For example, a plane can be fitted to the data points of the subtopographies. The system of equations in Eq. 4.21 is expanded to

$$(z_{ij} + o_i + x_{ij}t_{x,i} + y_{ij}t_{y,i}) - (z_{ji} + o_j + x_{ji}t_{x,j} + y_{ji}t_{y,j}) = 0 \quad (4.23)$$

$$(m_{x,ij} + t_{x,i}) - (m_{x,ji} + t_{y,j}) = 0 \quad (4.24)$$

$$(m_{y,ij} + t_{y,i}) - (m_{y,ji} + t_{y,j}) = 0 \quad (4.25)$$

where $m_{x,ij}$ is the gradient in x -direction of subtopography i in the region of overlap with the subaperture j . $t_{x,i}$ and $t_{x,j}$ are tilts for the x -direction added to subaperture i and j . These tilts are to be determined. x_{ij} and y_{ij} are the x - and y -distance of the centre of subaperture i to the centre of the region of overlap with subaperture j .

This kind of stitching works very well if the subtopographies have no form error. However, if the determined gradients m have a small systematic error (for example from a form error of the reference flat), a parabolic topography error will arise. This is qualitatively shown for a two-dimensional example in Fig. 4.15 and 4.16. The systematic gradient error is called $\epsilon_{m,\text{sys}}$. Δx is the distance between the centres of neighbouring subapertures. For each additionally stitched topography, the gradient error increases by $\epsilon_{m,\text{sys}}$. The height error ϵ_h of the N -th subaperture is given by

$$\epsilon_h(N) = \Delta x \epsilon_{m,\text{sys}} + 2 \cdot \Delta x \epsilon_{m,\text{sys}} + \dots + N \cdot \Delta x \epsilon_{m,\text{sys}} \quad (4.26)$$

$$= \Delta x \sum_{i=0}^N \epsilon_{m,\text{sys}} \cdot i \quad (4.27)$$

$$= \Delta x \frac{N^2 + N}{2} \cdot \epsilon_{m,\text{sys}} \cdot i \quad (4.28)$$

$$\approx \Delta x \frac{N^2}{2} \cdot \epsilon_{m,\text{sys}} \cdot i \text{ for } N \gg 1 \quad (4.29)$$

The step to Eq. 4.28 was calculated with the formula for arithmetic series [95]. A small systematic gradient error causes an error in the stitched topography that is approxi-

mately quadratic with the number of subapertures (or with the radius of a specimen).

Even if the tilt parameters are regularized (that is that large deviations from the actual value are "punished" in the minimization), there still will be a quadratic topography error from the integration of the systematic gradient error, however with a smaller amplitude.

Figures 4.15 and 4.16 show the results of the stitching of actually flat subtopographies with a systematic spherical form error (that therefore contains a gradient error in the region of overlap). When the tilt is not fitted (first figure), there is only a local form error in the stitched topography. In the case with a fit of the tilt, the propagation of the gradient error can be seen (second figure).

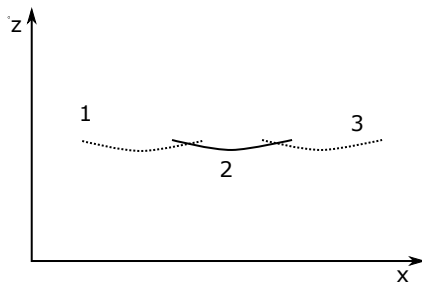


Figure 4.15: Stitching of subapertures with a systematic form error: Fitting the offsets but not the tilts.

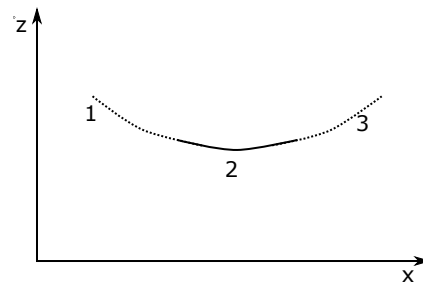


Figure 4.16: Stitching of subapertures with a systematic form error: With offset and tilt as parameters to be determined.

4.4 Advanced stitching with cavity length and angle measurement

In this section, a stitching method for freeform surfaces is presented, that was developed to minimize the propagation of the uncertainties of the subaperture measurements to the global topography. A further aim was to create a simple and linear algorithm, that allows for traceable topography values. When measuring a curved specimen instead of a flat specimen, the Fizeau interferometer has to be aligned to the local surface gradient of the SUT at each measurement position. This minimizes the deviation from the null-configuration of the interferometer.

In addition to subtopographies, the stitching method uses the cavity length between the Fizeau interferometer and the specimen, as well as their angular positions. From the angles between the specimen and the Fizeau interferometer, the tilt of the measured topographies in a global coordinate system can be calculated. Furthermore, the lateral positions of the subtopographies in the global coordinate system can be calculated from the cavity length and the angular positions (knowing the distance $d_{\text{RF-PP}}$ from the pivot point of the interferometer to its reference flat). The accuracy of the relative topography

measurement is much higher than the accuracy of the distances measurement, therefore each absolute distance gets an offset parameter, that will be fitted to minimize the height difference between the topographies of neighbouring subaperture measurements. In principle, this leads to a non-linear equation system, because the angles under which the measurements are performed depend on the surface topography and can be different for each position. The subtopographies would be shifted along the optical axis of the interferometer. However, since the angles between z -direction and optical axis are small, the fit along the optical axis can be approximated by fitting only a height offset in z -direction and neglecting the lateral shift caused by the uncertainty of the absolute distance measurement (see Fig. 4.17). After the positioning of the subtopographies in a common coordinate system, the simple stitching algorithm from Sec. 4.3.1 can be used, with the same system of linear equations as Eq. 4.21. In Fig. 4.18, a flowchart shows how the stitching is performed after the subtopographies have been positioned in a global coordinate system.

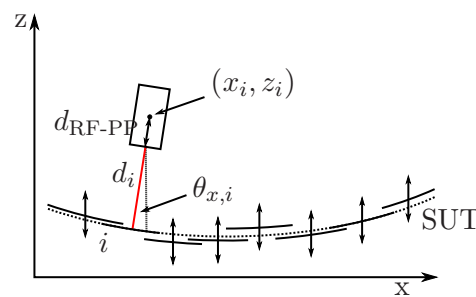


Figure 4.17: The dotted line is the SUT. The interferometer at position (x_i, z_i) is rotated to match the surface slope of the SUT. With the absolute distance d_i and the angle $\theta_{x,i}$, the lateral position of subaperture i can be calculated. The subapertures are slightly shifted along the optical axis of the interferometer (red) due to an inaccuracy of the measurement of d_i . This shift is corrected in approximation by minimizing the height offsets (indicated by the black arrows).

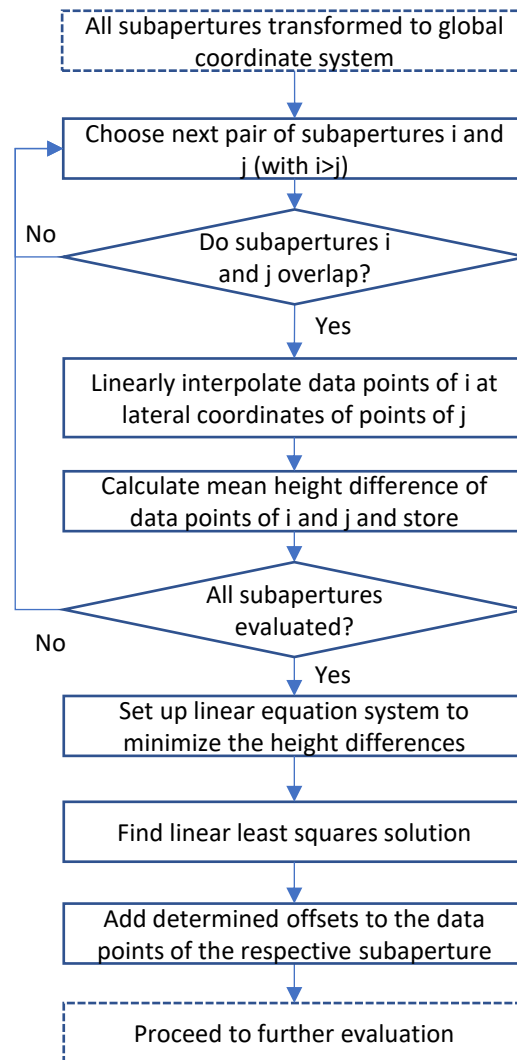


Figure 4.18: Flow chart for the stitching algorithm.

4.4.1 Stitching with fitting of alignment parameters of angle measurement device

Here, a stitching algorithm is introduced that is based on the stitching fitting the offsets only as in Sec. 4.3.1. It can be used for the case, when the two tilt angles θ_x and θ_y between interferometer and specimen are measured with a two-axis measurement device like an autocollimator. If the measurement device is misaligned by a rotation around its symmetry axis with an angle γ , proportions of each tilt angle will be erroneously attributed to the other tilt angle as follows:

$$\theta'_x = \theta_x \cos(\gamma) + \theta_y \sin(\gamma) \quad (4.30)$$

$$\theta'_y = \theta_y \cos(\gamma) - \theta_x \sin(\gamma) \quad (4.31)$$

where θ'_x and θ'_y are the measured tilts including the alignment error. For the autocollimator, the misalignment is a rotation around its optical axis. This is only relevant when measuring tilt angles significantly greater than a few arcseconds because the misalignment γ is usually small. This is the case when the autocollimator not only measures the angle errors of the linear z -stage but when the interferometer is actively rotated with a rotation stage.

Taking resulting height error due to the alignment error into account, the equation system from Eq. 4.21 is expanded to

$$[z_i + o_i + \gamma\theta_{x,i}\Delta x_{ij} - \gamma\theta_{y,i}\Delta y_{ij}] - [z_j + o_j + \gamma\theta_{x,j}\Delta x_{ji} - \gamma\theta_{y,j}\Delta y_{ji}] = 0 \quad (4.32)$$

where Δx_{ij} is the x -distance from the centre of the subaperture i to the centre of the overlapping region with subaperture j . $\theta_{x,i}$ is the tilt measured by the misaligned angle measurement device for subaperture i . For the y -direction, the variables are named accordingly. The parameter for the misalignment γ is also determined by this system of equations. The factors multiplied with γ can be summarized as a single variable:

$$c_{ij} = [\theta_{x,i}\Delta x_{ij} - \theta_{y,i}\Delta y_{ij}] - [\theta_{x,j}\Delta x_{ji} - \theta_{y,j}\Delta y_{ji}] \quad (4.33)$$

The system of equations simplifies to

$$[z_{ij} + o_i] - [z_{ji} + o_j] + \gamma c_{ij} = 0 \quad (4.34)$$

The topography and the alignment parameter for the autocollimator are determined by solving this linear least squares problem. The covariance matrix can be also be determined. This stitching algorithm determining the misalignment of an angle measurement device is applied in Sec. 6.2 having used an autocollimator for the measurement.

4.5 Angle measurement devices

Here, the functional principles of the three types of angle measurement devices used in the form measurement setup that will be presented in Chapter 5 are briefly outlined.

4.5.1 Rotary encoder

Rotary encoders measure relative rotations. Rotary encoders are used in the rotation stage and the goniometer in the experimental setup in Chapter 5. A rotary encoder is mounted to a rotating element of a rotation stage and encodes its rotation as an

electric signal. The electrical signal can be generated by using magnetic, capacitive or optical encoder disks. With a conversion factor, the signal can be converted to an angle representing the relative angular motion of the stage. Further information can for example be found in Ref. [96, 97].

4.5.2 Autocollimator

In this work, an autocollimator [98] is used to measure rotations of the Fizeau interferometer of the setup, when the rotations are small enough to be within its measurement range. The principle of an autocollimator is shown in Fig. 4.19. A spherical wavefront from a point source is directed to a lens via a beam splitter and is collimated. The collimated beam then reaches a plane mirror which is attached to the object whose angular position is to be measured. If the mirror is tilted by the angle α relative to the optical axis, the beam is reflected under the angle $\theta = 2\alpha$. The reflected collimated beam is then focused onto an image sensor by the lens. The position of the focused beam on the sensor and in the focal plane of the lens only depends on the angle of the reflected beam as the focal plane is also the Fourier plane [81]. The angle of the mirror can be determined by

$$\alpha = \arctan\left(\frac{\Delta a}{2f}\right) \quad (4.35)$$

$$\approx \frac{\Delta a}{2f} \quad (4.36)$$

where Δa is the displacement of the focal spot from the position of the optical axis on the image sensor. f is the focal length of the lens. Often, the autocollimator beam is Gaussian shaped. There are also variants that use a pattern of several intersecting lines as illumination, for example with the model *TriAngle* by *Trioptics GmbH* [99]. The centre position of the cross can be determined by using a Hough transformation [100] to find the lines and calculate their intersections and the centre of gravity of the intersection points. Because of the principle of operation, the autocollimator in its basic form only measures the pitch and yaw angle of the mirror.

A guideline to the calibration of autocollimators is given by the *European Association of National Metrology Institutes (EURAMET)* in Ref. [101].

4.5.3 Tiltmeter

A tiltmeter measures its angular position relative to the direction of the earth's gravity. It can for example be attached to a linear stage to measure the tilt of the stage's table at different positions due to imperfections of the stage guideway. It can consist of a pendulum that aligns with gravity. The displacement of the pendulum is measured relative to a fixed element. The measurement can be performed in various ways, for example through a capacitive or optical method. For example, the pendulum can form

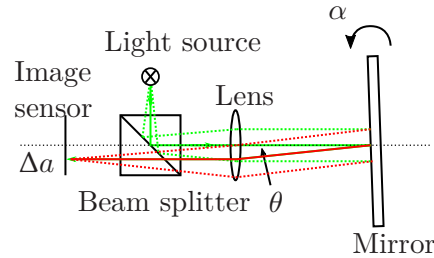


Figure 4.19: Schematic sketch of the principle of an autocollimator. The green rays go from the light source to the mirror, the red rays are the reflected rays. Δa is the displacement of the spot on the image sensor from the optical axis.

a capacitor with the fixed element. The angular position of the pendulum is linearly proportional to the distance between the pendulum and the fixed element in the small angle approximation. For a capacitive tiltmeter, the capacity can be measured and converted to a digital signal. In the ideal case, the pendulum is at rest during the measurement and perfectly parallel to the gravitational force vector, that means there is as little hysteresis of the mechanics as possible. Accuracies below one arcsecond can be achieved. Typically, the accuracy becomes worse with a larger measuring range. Versions with one or two measurement axes are common.

In principle, dynamical measurements with a moving pendulum are possible. Taking the equation of motion for the pendulum into account, the angular movement of the measurement device can be separated. When a tiltmeter is horizontally moved (for

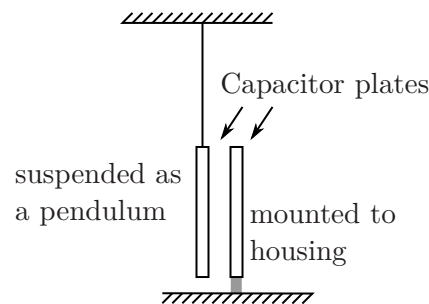


Figure 4.20: Schematic drawing of a simple possible realization of a tiltmeter based on a capacity measurement.

example on a linear stage) over a distance Δx , the direction of gravity changes so that this effect must be corrected (see Fig. 4.21). The change of the direction of gravity for motions much smaller than the earth radius is given by

$$\Delta\theta = \frac{\Delta x}{R_{\text{earth}}} \quad (4.37)$$

The mean earth radius is $R_{\text{earth}} = 6371 \text{ km}$ [102] under the approximation that the

earth's form is a sphere. For a movement of the tiltmeter of 1 m vertical to the direction of gravity, the angular difference is $\Delta\theta = 0.16 \mu\text{rad}$. If this gradient deviation is integrated for a topography measurement, this yields a difference of

$$\Delta z = \int_0^{\Delta x} \frac{x}{R_{\text{earth}}} dx \quad (4.38)$$

For the example of a travel distance of 1 m, this equals a height error of 78.5 nm.

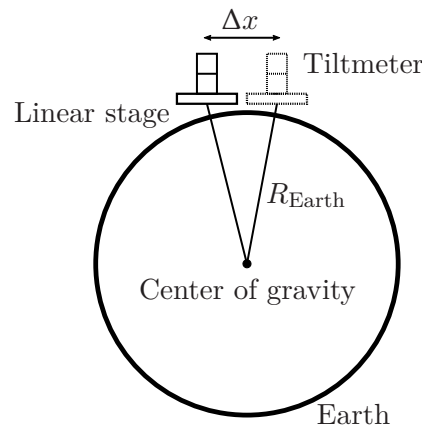


Figure 4.21: Change of gravitational direction when a tiltmeter is laterally moved.

4.6 Geometry of positioning systems and position errors

When using positioning systems for stitching interferometry, an assessment of the occurring position errors is necessary. On the one hand, the position accuracy is limited along their direction of motion of a stage. On the other hand, the guideways of a stage have a limited straightness, which is equivalent to lateral position errors.

4.6.1 Three-axis positioning system

Here, it is discussed what kind of geometric positioning errors occur in a positioning system with three nominally perpendicular linear stages. All in all, there are 21 geometric errors. Each linear stage has a position in space defined by three position variables $\vec{r} = (x, y, z)^T$ and three angles $\vec{\theta} = (\theta_x, \theta_y, \theta_z)^T$. Accordingly, there are 6 position errors $\epsilon_x, \epsilon_y, \epsilon_z, \epsilon_a, \epsilon_b, \epsilon_c$ for each stage (compare Fig. 4.22), that depend on the position of the respective linear stage. For three axes, this yields $3 \cdot 6 = 18$ position errors.

Next, the perpendicularity of the linear stages relative to each other is considered. There are three angles between the direction of motions of the three linear stages called XwY, YwZ, ZwX , and three according errors $(\epsilon_{XwY}, \epsilon_{YwZ}, \epsilon_{ZwX})$. XwY is the angle

between X- and Y-stage, for the other angles the denotation is equivalent as shown in Fig. 4.23. Since individual stage position errors, the angle between the stages can be interpreted as the angle between the best-fit lines through the positions of the respective stages. The errors ϵ then are the deviations from the best-fit line.

A measurement of the 21 geometrical errors is possible. This requires accurate measurement equipment for all relevant errors. With this information, the deviation of the actual position to the target position in a coordinate measurement machine can be calculated with a matrices calculations that is for example found in Ref. [103].

The errors of the single stage can be divided into systematic and statistical errors, that is, accuracy and precision. The important part are the systematic errors, because they are typically larger than the statistical errors and cannot be minimized by repeated measurements. These are mostly the errors of the stage guideways. The stage position along the direction of motion can be very accurate when it is measured with high-accuracy encoders. Parameters as straightness and flatness cannot be measured without further measurement devices.

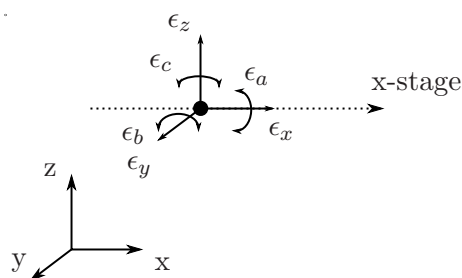


Figure 4.22: The six geometrical position errors of a linear X-stage.

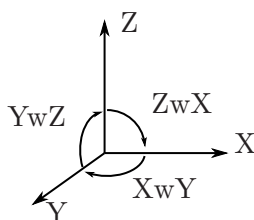


Figure 4.23: This plot shows the definition of the angles between the axes of motion of three linear stages X,Y and Z.

4.6.2 5-axis system with two rotation stages

By adding two rotation stages to a 3-axis positioning system, an areal measurement sensor with one symmetry axis, like a Fizeau interferometer, can completely scan the surface

of an arbitrary specimen. In the 5-axis positioning system, each rotation stage has six geometric position errors just as a linear stage, three rotational and three translational. In comparison to the three-axis system, this gives 12 additional errors. Regarding the alignment, there are two angles for each rotation stage describing the position of the actual rotation axis relative to the coordinate system. This gives four additional geometrical errors, and 16 in total for the two rotation stages.

The order and way in which the rotation stages are mounted to the system has to be taken into account when calculating the actual position of the positioning system. For small angles, the order in which the rotations of the axes are applied is negligible. For larger angles, the order is important.

5 Experimental setup and methods

In this section, the technical realization of the advanced stitching interferometry with cavity length and angle measurement (SCaLA) for moderately curved freeform optics is described. First, the interferometric measurements of the topography and the cavity length are presented. Then, the whole measurement system with a 5-axis positioning system is introduced. It is also discussed how angular guideway errors of different stages are measured. Afterwards, the alignment is briefly summarized and the stability of the angle measurement is evaluated. In the end, two three-dimensional optical angle measurement methods are introduced, which provide opportunities to improve the form measurement system in the future.

5.1 Wavelength-swept interferometry

For the wavelength-swept interferometry, a Fizeau interferometer *VI-direct 10* by *Möller-Wedel GmbH* is used. It has a 10 mm aperture, an image sensor with $1600 \cdot 1200$ pixels and a frame rate of up to 60 fps. Its reference flat has a nominal flatness of $\lambda/30$ peak-to-valley (PV) at a wavelength of $\lambda = 633 \text{ nm}$, which equals 21.2 nm . The pixel pitch projected onto the specimen was determined [104] as $\frac{8.64 \mu\text{m}}{\text{px}}$ (see Section 5.1.1). The effective focal distance was determined as $(77.38 \pm 0.10) \text{ mm}$, measured from the reference surface. In Ref. [104], it has been verified that the distortion of the image due to the optics is less than or equal to one pixel for the whole field of view (see also Section 5.1.1). This will be taken into account in the uncertainty budget.

The laser for the wavelength sweep is a tunable diode laser (model *LaSy* by *TEM Messtechnik GmbH*). It has a tuning range of 300 GHz (or equivalently 0.4 nm) at a centre frequency of $473\,168.530 \text{ GHz}$ (or a centre wavelength of 633.3949 nm in air). The centre frequency is calibrated with an iodine reference cell with a nominal frequency stability of 50 MHz when locked onto the iodine spectrum. The line width is 5 MHz nominally. The frequency during the sweeping is locked onto a built-in Fabry-Perot interferometer [105, 106] with a nominal stability of 100 MHz per 8 hours.

5.1.1 Validation of the interferometric measurement

In Fig. 5.1, the wavelength-dependent intensity at a single pixel during a wavelength sweep is shown. Only a part of the possible sweeping range is depicted. There is a dominant oscillation for the interference of the reflections of the laser beam from the reference surface and the specimen. In the time domain, this frequency component is

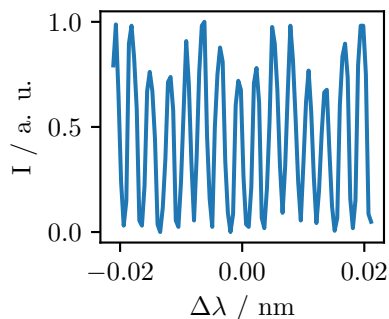


Figure 5.1: Normalized intensity signal in dependence of the wavelength recorded during the wavelength sweep. From Ref. [1].

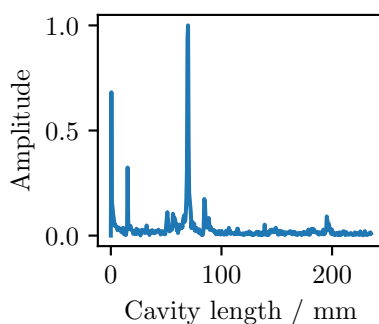


Figure 5.2: Normalized spectrum of the interference signal plotted over the corresponding cavity lengths. From Ref. [1].

superimposed with interferences that correspond to other cavities in the system. In Fig. 5.2, the absolute value of the Fourier transform of the signal shown in part in Fig. 5.1 is plotted. The individual maxima can be clearly distinguished. The cavity length of the highest peak equals the effective focal distance of the interferometer as the measured surface was in focus. The other peaks correspond to the other cavities which include higher reflection orders.

Topography measurement

To verify the flatness of the reference flat, a flat aluminium coated mirror with a nominal flatness of better than $\lambda/100$ was measured. However, the mirror is not calibrated with this accuracy. The measurement was performed with the supplied software for the *Möller-Wedel VI direct 10* interferometer at a fixed wavelength. The specimen is slightly tilted relative to the interferometer so that the software can analyse the spatial interference fringes. The topography can be seen in Fig. 5.3.

The relative topography deviates less than $\pm 0.015 \lambda \approx \pm 10 \text{ nm}$ from a perfectly flat

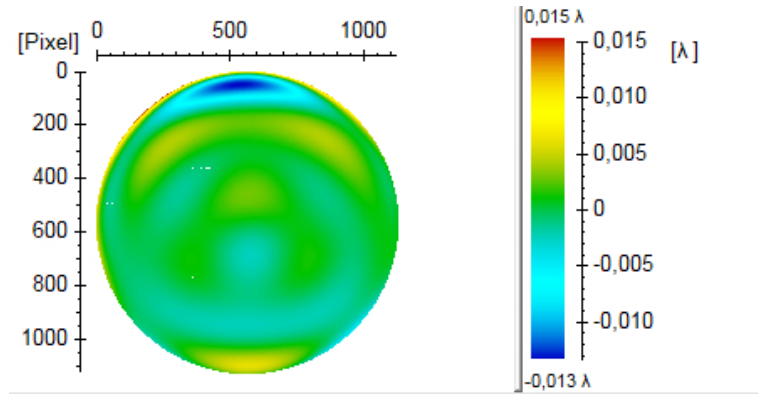


Figure 5.3: Relative topography of the reference flat and the mirror with $\frac{\lambda}{100}$. Screenshot of a measurement from the supplied software.

difference topography. This result is within the specification of the PV flatness deviation of $\frac{\lambda}{30} = 21.2 \text{ nm}$ for the reference flat. The repeatability of the topography measurement was also evaluated. A spherical surface (see Fig. 5.4) was measured 50 times directly in succession, this time with the custom implemented wavelength sweep. The frequency sweep range was 100 GHz. For each of the 50 measurements, offset and tilts were subtracted. The resulting mean standard deviation for a height value at a single pixel is $\sigma = (0.5 \pm 0.1) \text{ nm}$ (see Fig.). The tilts of the topographies have been removed as they are caused by mechanical drifts that can be measured with the angle measurement devices as described in Sec. 5.4.

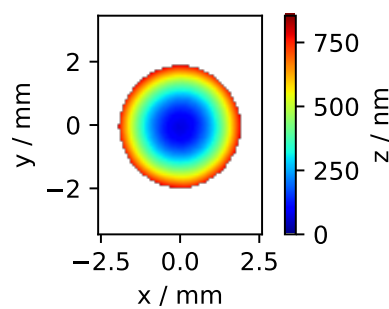


Figure 5.4: Mean of the spherical topography that was measured 50 times

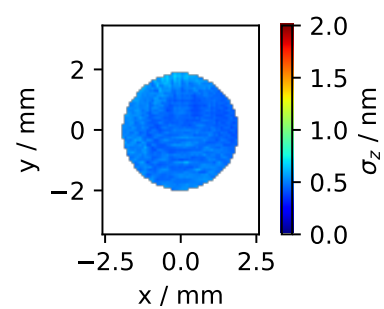


Figure 5.5: Standard deviations for the topography value at each pixel from 50 repeated measurements.

Next, the distortion of the interferometer optics was evaluated. All data, plots and results are adopted from a student project [104]. A chrome on glass structure with cross shaped markers was used as a test target. The accuracy of the structures is $1 \mu\text{m}$. A white light-emitting diode (LED) illuminates the mask from below. The image taken

of the test target in focus is shown in Fig. 5.6. The mask was rotationally aligned to the interferometer pixel rows and columns. The centre positions of the crosses were searched for by using functions from the *python* library of *OpenCV* [107]. Three different methods were applied: Finding the geometrical centre of the crosses, the centre of an enclosing rectangle and of an enclosing circle around the crosses. The results are plotted in Fig. 5.7. All of the calculated centre positions in a row or column differed by a maximum of one pixel or $9.84\text{ }\mu\text{m}$. When using the laser illumination of the setup for interferometry, only an inner circular section of the field of view is used. Therefore, it can be concluded that the lateral position errors due to the distortion are not larger than the error determined for the test target here.

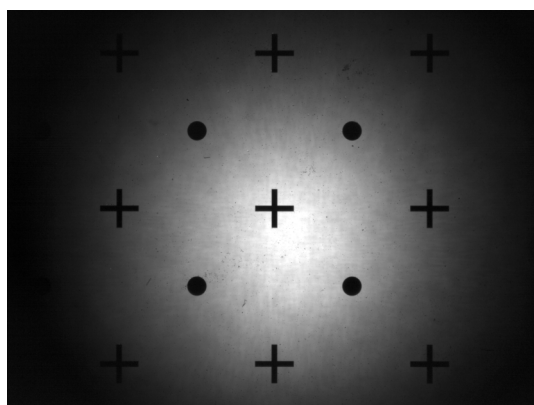


Figure 5.6: Image taken of the test target with a white LED as light source. Image from Ref. [104].

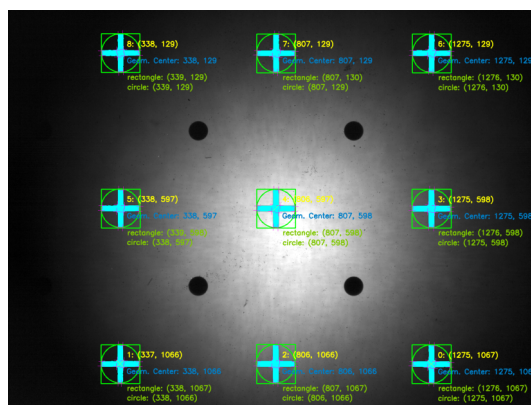


Figure 5.7: Positions of the cross centres localized using OpenCV. Image from Ref. [104].

Cavity length measurement

For the determination of the cavity length, that is, the absolute distance between specimen and reference flat, the same measurement data as for the topography measurement was used. The resolution of the cavity length measurement with zero-padding was $3.6\text{ }\mu\text{m}$. The inner ± 25 pixels around the geometric centre of the valid measuring area on the corresponding row of the image sensor were used to obtain an average cavity length.

To verify the interferometric distance measurement, an experiment was conducted. The corresponding setup is shown in Fig. 5.8. Two 1-inch flat aluminium coated mirrors had been mounted to a vertical linear stage with a micrometer screw. From top, the cavity length was measured with the wavelength-swept interferometry. From below, a laser interferometer (model *Renishaw RLE10*) measured the relative position changes when the linear stage is moved. Ten different positions of the linear stage were approached in steps of 1 mm . Figure 5.9 shows the deviation of the absolute distance measurement and the relative laser interferometer measurement.

The mean of the standard deviations of all distance measurements was $12\text{ }\mu\text{m}$, with 20

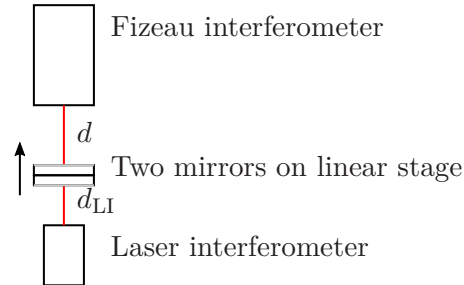


Figure 5.8: Distance measurement: Setup for the comparison of wavelength-swept interferometry and relative measuring laser interferometer.

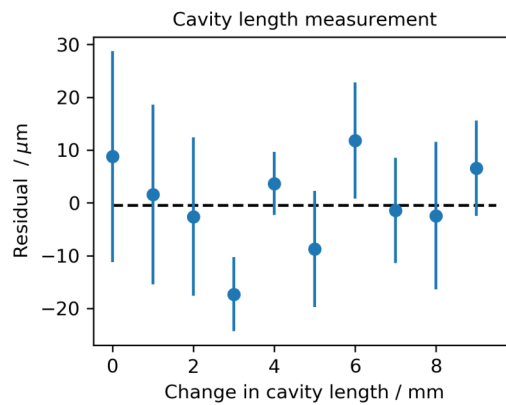


Figure 5.9: Deviation of the cavity length measurements from the relative laser interferometer measurement. The plot shows the mean of 20 repeated measurements with the respective standard deviation. A constant offset has been subtracted. From Ref. [1].

repetitions at each position. The empirical standard deviation is $9.1 \mu\text{m}$ (see Fig. 5.9). The uncertainty of the relative laser interferometer measurement was neglected in this evaluation as it is much smaller than that of the cavity length measurement.

5.2 Positioning system

The purpose of the 5-axis positioning system is to move the Fizeau interferometer relative to the specimen. The interferometer is scanned over the SUT step by step. It has to have the right distance to the SUT so that it is in focus, and its optical axis has to be perpendicular to the SUT in the centre of the field of view. To scan the specimen surface, the positioning system needs five degrees of freedom. The x - and y -stages laterally scan the specimen and the z -stage moves the specimen into the focal plane of the Fizeau interferometer if the topography has significant height differences. Furthermore, there is a goniometer stage and a rotation stage for interferometer. The interferometer is rotated

Table 5.1: Overview over the stages of the 5-axis positioning system. The specifications are nominal values.

Stage	Manufacturer and model	Travel range	Absolute position accuracy
Linear x -stage	<i>PI miCos GmbH</i> , LS-270	1016 mm	not specified
Linear y -stage	Custom built	200 mm	$\pm 2 \mu\text{m}$ (estimated)
Linear z -stage	<i>Newport Spectra-Physics GmbH</i> , M-IMS100V	100 mm	$\pm 2 \mu\text{m}$
Rotation stage (rot- y)	<i>Newport Spectra-Physics GmbH</i> , RV120CC	360°	$\pm 7.5 \text{ mdeg}$
Goniometer stage (rot- x)	<i>Newport Spectra-Physics GmbH</i> , BGS80CC	90°	$\pm 25 \text{ mdeg}$

around the x -axis (causing a change of θ_y) and the y -axis (causing a change of θ_x) to align it to the surface gradient. Each stage is equipped with an internal length or angle encoder. The goniometer stage is mounted on the rotation stage. They share the same pivot point by design. The Fizeau interferometer is mounted on the two stages so that its optical axis contains the pivot point. The realization of the 5-axis positioning system is shown in Fig. 5.10. The stages used are listed in Table 5.1.

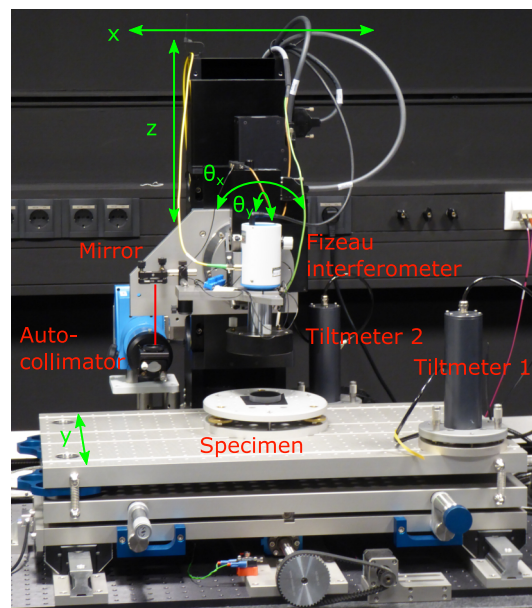


Figure 5.10: Photo of the measurement setup viewed from the front. From Ref. [1].

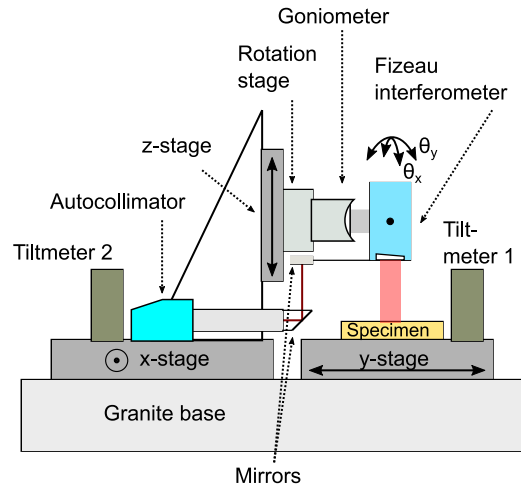


Figure 5.11: Schematic drawing of the setup as shown from the side. From Ref. [1].

5.2.1 Measuring angular guideway errors

The angular guideway errors of the x - and the y -stage correspond to tilts of the subapertures. They are measured with two two-axis tiltmeters, one on each stage. The rotations of both goniometer and rotation stage and their angular position errors can be measured with the autocollimator shown in 5.11. The autocollimator has a mirror attached to deflect its beam in vertical direction to measure θ_x and θ_y of the interferometer. The corresponding angular errors of the vertical z -stage are included in this measurement. The z -stage angular errors are small because only a short travel range is used.

With these angle measurements, the angular relation between specimen and interferometer is established during the form measurement. With the knowledge of the position of the rotation stage and goniometer (measured by the angle encoders or the autocollimator) and the cavity length, the lateral positions of the subapertures are calculated. The angle values measured by the tiltmeters are not included in this calculation because they are small. Their pivot point is unknown but it can be assumed to be close to the guideway of the stage.

Angular positioning errors that are rotations around the z -axis are not measured. However, the gradients of the subaperture topographies in direction of the rotation are small. The angle errors of the system components that are rotations around the z -axis are also small. Therefore, they are neglected.

The angle sensors used are further evaluated in Section 5.4.

5.3 Setup of the form measurement system

First, the alignment of the system is explained. This concerns particularly the alignment of the angle sensors. The positioning system is inherently well aligned due to small

mechanical tolerances. Furthermore, the pivot point of the rotation of the Fizeau interferometer is determined. Then, the safety measures to avoid collisions of components of the system are presented.

Alignment of the system

The Fizeau interferometer and the pixel array of its image sensor have to be aligned to the positioning system. The pixel rows and columns must be parallel to the x - and y -axis, that is to the direction of motion of x - and y -stage. For the alignment, a chrome on glass mask with a quadratic pattern is placed under the Fizeau interferometer (see Fig. 5.12). The x -stage is moved forth and back. The Fizeau interferometer is rotated in its mount, until the features of the grid have a constant y -position while the x -stage is scanning. This can be achieved best when the grid lines are parallel to the x -stage (Fig. 5.13). Then, the y -stage can also be scanned to verify its perpendicularity to the x -stage. The z -stage is sufficiently perpendicular aligned to x -stage due to the mechanical tolerance of its mount.

For the coarse alignment of the tilt angles of the Fizeau interferometer and the specimen stage, spirit levels are used to align them parallel to the direction of gravity. The exact alignment is done during the measurement with the rotation stages.

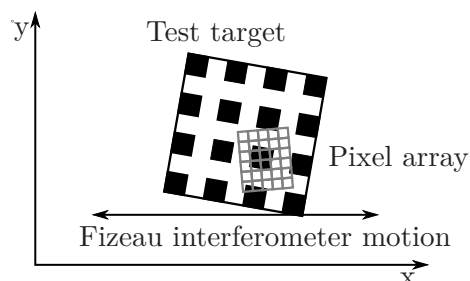


Figure 5.12: Alignment of the Fizeau interferometer, seen from above. Both pixel array of the image sensor and the test target are not well aligned.

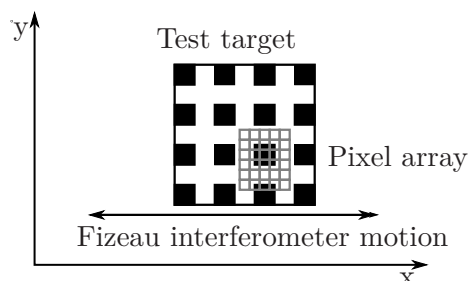


Figure 5.13: After the alignment, the grids of the image sensor and the test target are parallel to the direction of motion. The y -position of the features does not change when moving the Fizeau interferometer in x -direction.

The autocollimator on the x -stage measuring the angular position of the Fizeau interferometer is also aligned with a spirit level. The spirit level is placed on its housing, and the autocollimator is rotated in its mount around its optical axis (see Fig. 5.14). This allows for an accurate alignment of much smaller than 1° relative to the direction of gravity. The remaining misalignment will be described by a parameter to be fitted in the stitching algorithm (see Sec. 4.4.1). The mirror attached to the autocollimator (α_x)

is adjusted so that the position of the beam on the z -stage mirror does not change while moving the z -stage.

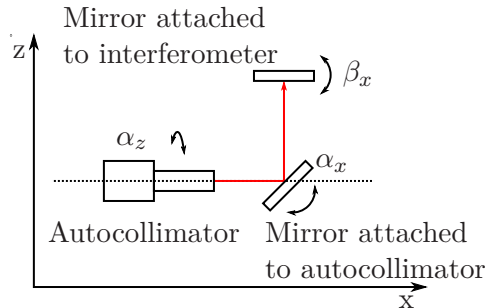


Figure 5.14: Alignment of the autocollimator in its mount (compare to Fig. 5.11).

Determination of the pivot point of the rotation and goniometer stage

The position of the pivot point is necessary to calculate the lateral positions of the subtopographies. A reflective surface with a marker is placed under the Fizeau interferometer. The lateral position of the marker in the field of view of the interferometer is determined, and the distance from the Fizeau interferometer to the specimen is interferometrically measured. Then, the Fizeau interferometer is rotated by a small angle ξ . As shown in Fig. 5.15, the distance from the reference flat to the pivot point can be geometrically calculated. For this, the distance d_{PP-RF} from the common pivot point of both stages to

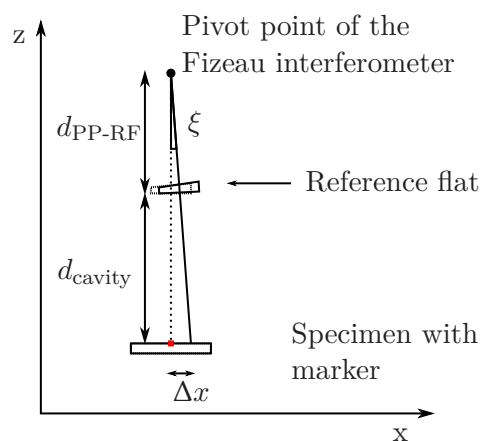


Figure 5.15: Geometrical considerations to determine the distance from the pivot point to the reference flat. With the marker on the specimen, its lateral shift in the interferometer field of view Δx due to the rotation ξ can be measured.

the interferometer reference flat is determined.

For a small rotation of the Fizeau interferometer ξ , the apparent relative lateral move-

ment of the marker Δx (from the point of view of the interferometer) is given by

$$\Delta x = d_{\text{total}} \cdot \sin(\xi) \quad (5.1)$$

$$\Delta x = (d_{\text{PP-RF}} + d_{\text{cavity}}) \cdot \sin(\xi) \quad (5.2)$$

d_{total} is the total distance from the pivot point to the marker. d_{cavity} is the cavity length that can be interferometrically measured. The angle ξ is measured with the angle encoder of the rotation stage. Rearranging the formula yields

$$d_{\text{PP-RF}} = \frac{\Delta x}{\sin(\xi)} - d_{\text{cavity}} \quad (5.3)$$

From the experiment, the distance $d_{\text{PP-RF}}$ is calculated as 120.83 mm.

Its uncertainty can be calculated by an error propagation derived from Eq. 5.3 as

$$u(d_{\text{PP-RF}})^2 = \left[\frac{u(\Delta x)}{\sin(\xi)} \right]^2 + \left[\frac{\Delta x}{\sin(\xi)^2} \cdot u(\sin(\xi)) \right]^2 + u(d_{\text{cavity}})^2 \quad (5.4)$$

The larger the maximum rotation of the Fizeau interferometer, the larger the influence of this uncertainty on the lateral position of a subaperture becomes. For small angles up to 5° , the resulting lateral subaperture position errors are small.

Motion safety measures

Several precautions are taken to minimize the risk of collisions during the measurement, especially of the interferometer with the specimen or the specimen stage. In the software, the positions to be approached are checked if they are within allowed limits. If not, the motion command is not executed. Furthermore, only float or integer values as relative or absolute positions are accepted. In case of an invalid value, the positioning system does not move. The Fizeau interferometer can be equipped with three micro-switches below its lower edge if a collision with the specimen is possible. When the switches are triggered due to contact with an object, an electrical circuit is interrupted and the motion of the z -stage, the goniometer and rotation stage are immediately stopped. The switches are easily triggered so that they do not damage the specimen surfaces. There are also switches for the rotation and goniometer stage that prevent motions outside of the permitted ranges.

When different subapertures of the specimen are measured, the specimen has to be moved into the interferometer focal plane using the z -stage. The gradients of the specimen are small, then large apparent changes in the absolute distance indicate an error of some sort. In this case, the motion of the z -stage is also not performed.

5.4 Angle measurement

The angular relation of the SUT and the Fizeau interferometer is a critical quantity for the uncertainty of the topography measurement. Without a measurement of the angles, guideway errors of the positioning system are interpreted as a proportion of the surface form of the measured specimen. The angular relation is established with two tiltmeters and an additional autocollimator (as it was shown in Fig. 5.11). The first tiltmeter is mounted on the y -stage, which also bears the specimen. The table of the y -stage is sufficiently rigid (made from solid aluminium), so that the first tiltmeter measures the angular orientation of the specimen relative to the direction of gravity. The second tiltmeter is mounted on the x -stage and measures the angular position of the x -table (carrying the tower for the interferometer and the autocollimator) relative to gravity. The autocollimator measures the angle of the mirror attached to the rotatable Fizeau interferometer mount relative to the x -table. Thus, the angular relation between specimen and Fizeau interferometer is completely described by the measurements. If the Fizeau interferometer is not rotated during a measurement (for example for a flat specimen), the measurement with the autocollimator is not necessary, because the mounting of the Fizeau interferometer to the x -stage is sufficiently stiff. As it can be seen, only the tip/tilt angles θ_x and θ_y are measured, because they are the critical angles. An error in these angles directly causes a gradient error in the measured surface form. The error for a rotation around the z -axis does not have to be measured and can be neglected, since it does not affect the surface gradient in the same way as a tilt error. It only rotates the measured subaperture topography by a very small angle. Due to the small surface gradients, these guideway errors cause form errors orders of magnitudes smaller than those caused by tilt errors.

If the rotations are too large to be measured with the autocollimator, the angle positions of the rotation axes can be read out from their internal rotation encoder. They are also very accurate, however they cannot measure guideway errors and misalignment of the stages to each other or to the rest of the measurement system. Also, the angular errors of the z -stage are not measured, however they are small since a short travel range is used.

5.4.1 Short-term stability and repeatability of the angle measurements

Here, the tiltmeters and autocollimators are evaluated with respect to their stability on a short time scale of several minutes. Figure 5.16 shows the signals from both channels (R_x and R_y) of each measurement device over a time of about 20 minutes while the system stands still. The devices are mounted in the measurement system, so the signals contain drifts of the measurement devices themselves as well as drifts of the system and influences of the environment (as vibrations, for example). A polynomial of degree two is

Table 5.2: SEM for the different angle sensors in μrad , and for a different number n of averaged data points. AC: Autocollimator, TM: Tiltmeter.

Measurement device	AC 1		TM 1		tiltmeter (TM) 2	
Channel	1	2	1	2	1	2
SEM ($n=10$)	0.10	0.19	0.30	0.16	0.31	0.15
SEM ($n=100$)	0.07	0.05	0.15	0.07	0.15	0.07
SEM ($n=1000$)	0.02	0.02	0.05	0.02	0.05	0.02

fitted to and subtracted from every signal to remove drifts. Here, only the repeatability of the measurement is investigated, while the drifts will be evaluated in Sec. 5.4.2.

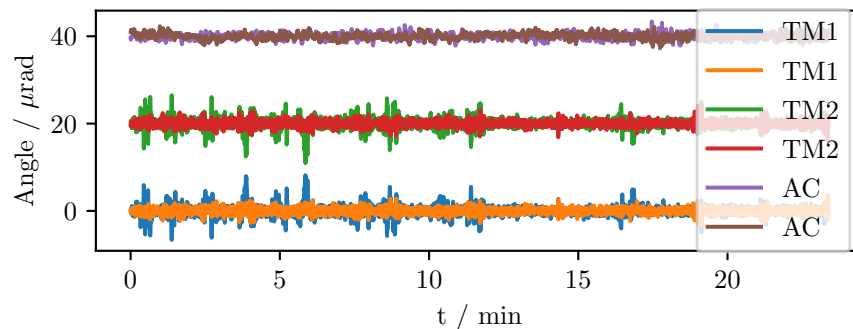


Figure 5.16: Signals of the angle measurement devices during standstill. The measurement frequency is approximately 3 Hz. For better visibility, offsets were added to the signals. For each device, both channels are plotted together. A fitted polynomial of second degree was subtracted from each signal to remove drifts. AC: Autocollimator, TM: Tiltmeter.

The standard error of the mean (SEM) was calculated for each channel starting from $t = 5$ min since some higher-order drifts were still present directly after moving the system. The results are shown in Table 5.2. With an increasing number n of averaged measurements, the SEM decreases for all devices as expected. For an increasing n , the SEM approaches similar values. The tiltmeter signals show simultaneous deflections which are most likely caused by vibrations. The environmental influences also explain why the SEM is not completely proportional to $\frac{1}{\sqrt{n}}$ as it would be expected for normally distributed noise.

5.4.2 Long-term stability of the angle measurements

To investigate the drifts, a similar measurement was performed for about 12 hours recording a value each minute. The signals of the angle sensors are shown in Fig. 5.17. Only the mean value was subtracted from each signal. The drifts are summarized in Table 5.3.

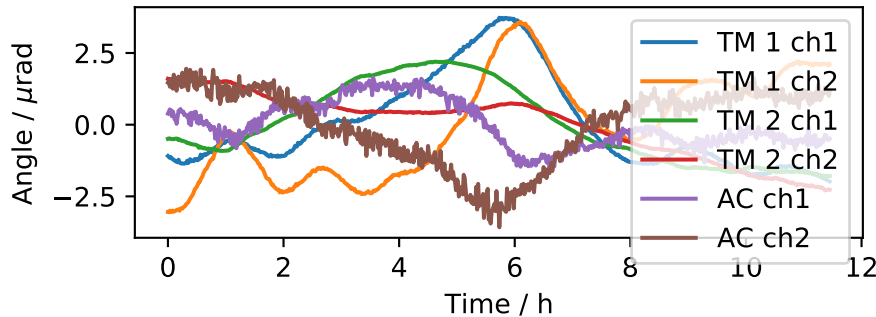


Figure 5.17: Drifts of the angle sensors during a 12 hour measurement.

Table 5.3: Peak-to-valley (PV) drift of the different angle sensors during 12 h in μrad .

Measurement device	AC 1		TM 1		TM 2	
Channel	1	2	1	2	1	2
PV drift	3.2	5.6	5.7	6.6	4.0	3.9

The second channel of the TM1 has the largest peak-to-valley drift with $6.6 \mu\text{rad}$ or $1.4''$. The mean drift of the tiltmeters is about $5.1 \mu\text{rad}$. Apparently, the drifts can be caused by the measurement device itself but also by the mechanical setup. In the former case, it would be a measurement error. In the second case, the measurement would be correct.

5.4.3 Repeatability and absolute accuracy of the rotation stages

The position repeatability and the accuracy are specified by the manufacturer and listed in Table 5.4. The repeatability of the stages is much smaller than the accuracy of the absolute position. The angular position errors of the rotation stage cannot be separated from the surface slope and therefore will propagate into the final surface form. In comparison to the autocollimator, the overall accuracy of the stages is worse, however the measurement range is significantly larger. In principle, the systematic position errors can be corrected by using an external angle measurement device like an angle comparator for calibration. The repeatability gives a lower limit in practice since many repeated measurements would require a long measurement time.

5.5 Alternative rotation sensors

In the following sections, two approaches for optical three-dimensional angle sensors will be presented which have been developed during this work. The optimization and integration into the form measurement system is a task for the future. The first approach is an interferometric roll angle sensor combined with an autocollimator to measure three-dimensional rotations. With this combination, it is possible to measure rotations around

Table 5.4: Accuracies and unidirectional repeatabilities of the rotation stage (360° travel range) and the goniometer stage (90° travel range). The repeatabilities were specified by the manufacturer as $\pm 1.5\sigma$ intervals and are also presented as the corresponding standard deviation.

Property	Rotation stage (corresponding gradient: θ_x)	Goniometer stage (corresponding gradient: θ_y)
Repeatability (typical)	± 0.35 mdeg (1.5σ) [± 4.1 μ rad (1σ)]	± 0.5 mdeg (1.5σ) [± 5.8 μ rad (1σ)]
Repeatability (guaranteed)	± 1 mdeg (1.5σ) [± 11.6 μ rad (1σ)]	- -
Absolute accuracy (typ.)	± 4.0 mdeg (PV) [± 69.7 μ rad (PV)]	± 25 mdeg (PV) [± 435.6 μ rad (PV)]
Absolute accuracy (guaranteed)	± 7.5 mdeg (PV) [± 130.7 μ rad (PV)]	- -

the x -axis from the side of the form measurement setup. This means that an angle measuring device like the autocollimator does not have to travel on the x -stage and can be mounted on the granite base. The second approach is an optical 3D measuring angle sensor similar to an autocollimator. It extends the measurement range of an autocollimator to more than 10° , and adds the possibility to measure the roll angle and the position of a reference element.

5.5.1 Autocollimator with additional optical roll angle sensor

An autocollimator measures the pitch and yaw angle of a reference object, for example a mirror, relative to its optical axis. The roll angle, that is, a rotation around the optical axis, is not measured by the autocollimator. Figure 5.18 shows a possible setup using an autocollimator and a roll angle sensor to determine the angular position of the interferometer. An optical rotation sensor like this is known from Ref. [108]. The rotation sensor and the autocollimator are both mounted to a granite portal or base. This way, only one additional tiltmeter on the y -stage for the setup of Fig. 5.10 is necessary to measure the complete angle relation between interferometer and specimen. However, the measurement range of the autocollimator is still limited so it is only suitable for small angles.

The roll angle sensor consists of a light source and a complementary metal-oxide-semiconductor (CMOS) image sensor. The light source emits diverging light that is

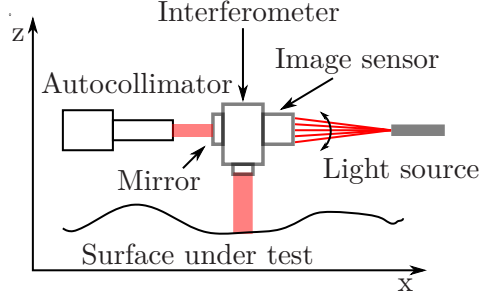


Figure 5.18: Autocollimator (left) and light source for the roll angle sensor (right) are fixed to frame (not shown) on a granite portal or base.

structured along one direction perpendicular to the optical axis as shown in Fig. 5.19. The image sensor is also perpendicularly oriented towards the optical axis, so the lateral pattern of the light is visible (see Fig. 5.20). The structured light can be formed by the interference of coherent light from two point sources with the same intensity. The intensity of the interference pattern in the far field is described by

$$I(\vec{r}_1, \vec{r}_2) \propto \left| \frac{1}{|(\vec{r}_1 + \vec{r}_2)/2|} e^{-i \frac{2\pi(\vec{r}_1 - \vec{r}_2)}{\lambda}} \right|^2 \quad (5.5)$$

where $\vec{r}_1$ and $\vec{r}_2$ are the distances from each point source 1 and 2 to the position $\vec{r}$ on the image sensor where the pattern is evaluated. The first fraction term describes the decrease in intensity of the spherical waves. The distance is averaged over $\vec{r}_1$ and $\vec{r}_2$ because the distance between the point sources is very small compared to $\vec{r}$. The second exponential term describes the interference of the two spherical waves.

The point sources can be realized as the ends of two single mode optical fibres fed by a HeNe laser. The sinusoidal pattern detected by the image sensor can be analysed line-by-line with Fourier analysis. For each line, the dominant frequency is the same, but the phase depends on the angle of the pattern on the sensor. From this data, the angle of the pattern relative to the pixel rows (or columns) can be calculated. The roll angle α_{roll} is given by

$$\alpha_{\text{roll}} = \arctan \frac{\Delta x_{\text{row}}}{\Delta y} \quad (5.6)$$

Δy is the pixel pitch in y -direction in pixels. Δx_{row} is the shift of the interference pattern position in x from row to row in pixels. It can be calculated from the phase difference $\Delta\phi_{\text{ro}}$ of the corresponding spatial frequency of the pattern f_{row} in radian/pixel. The angle α_{roll} is given by

$$\alpha_{\text{roll}} = \arctan \frac{\Delta\phi/(2\pi f_{\text{row}})}{\Delta y} \quad (5.7)$$

A rotation of the sensor or light source other than around the optical axis leads to a different projection of the pattern, so it has to be kept small or needs to be taken into

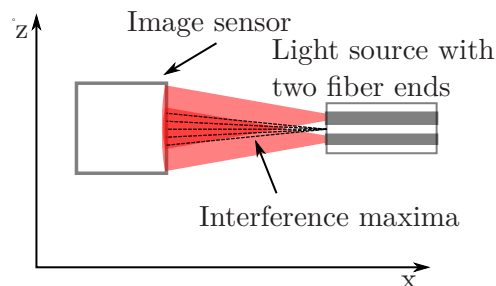


Figure 5.19: A light source consisting of two point sources generates an interference pattern detected by the image sensor. Therefore, relative rotations of the source sensor around the x -axis can be registered.

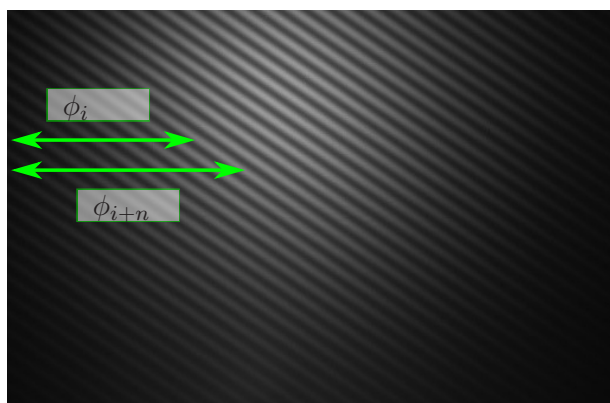


Figure 5.20: Interference pattern detected by the roll angle sensor. Each row has the same spatial frequency. The phase difference from row to row depends on the rotation of the sinusoidal interference pattern.

account. The sensor allows for a resolution smaller than $1''$ due to the phase evaluation of the interference pattern. A larger image sensor is useful to increase the accuracy. With large sensors, not all pixels have to be used for the evaluation, since the precision barely improves from a certain number of pixels on. In case of a rectangular but not quadratic sensor, the orientation of the sensor (with the long side of the sensor rather parallel or perpendicular to the interference pattern) does not change the accuracy. This makes sense since the amount of information stays the same. Also, different lateral frequencies of the interference pattern do not matter, as long as there are enough periods of the signal for the phase determination. The spatial frequency depends on the distance of the light sources to each other and to the image sensor according to Eq. 5.5. The achievable accuracy of a realization was estimated in Ref. [108] as approx. $1.5''$ over 100° . However, an improvement should be possible using a larger sensor and a stabilization of the light source. Another option for the light source is the use of shearing plates

with only one fibre end. The shear plate virtually creates two point light sources. The pattern is more stable, however the maxima stripes have a slight curvature due to the different axial positions of the virtual light sources. This causes non-linearities in the angle measurement. The reason is the different optical path lengths for the light from the point source partially reflected by front and back side of the shearing plate.

A patent has been granted for the approach shown in Fig. 5.18, see Ref. [7].

5.5.2 Three-dimensional optical rotation sensor

A three-dimensional optical rotation sensor (3D-ORS) was developed during this work. The setup is shown in Fig. 5.21. It is a combination of a conventional autocollimator as

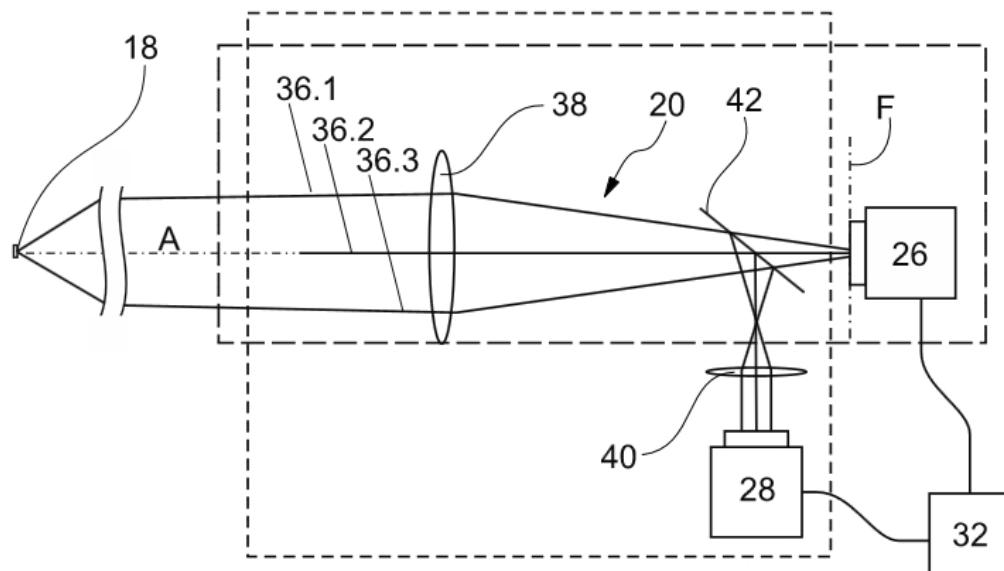


Figure 5.21: Schematic drawing of the three-dimensional optical rotation sensor. The light source (18) emits several rays under different angles and can move relative to the detection unit, which is the rest of the setup. An image sensor (26) is placed in the Fourier plane F of the lens (38), so that the ray positions on the sensor only depend on their incident angles at the lens [rays (36.1,2,3)]. A lens (40) forms an imaging system together with the lens (38), so that the image sensor (28) measures the position of the light source. Figure from Ref. [8].

detection unit and an external structured light source instead of an internally generated beam. The structured light source emits many different rays under different angles, which leave the source from the same origin (see Fig. 5.22). A light source with this property can be realized in many different ways. Here, two shearing plates rotated by 90° relative to each other were illuminated by a single mode fibre end. The light source is attached to the object whose relative angular position is to be measured. The angle between

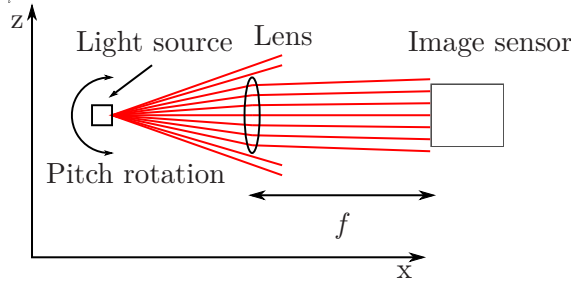


Figure 5.22: Principle of the three-dimensional rotation sensor. A light source attached to an object to be measured generates a three-dimensional intensity distribution which is shown as diverging rays. Through the lens, the rays reach the image sensor which is in the Fourier plane of the lens. The position on the image sensor only depends on the ray angle relative to the lens, but not on its position. When the light source is rotated (for example the pitch rotation), the ray positions on the image sensor can be tracked and give the angular motion according to Eq. 5.8. The angular aperture of the light source is much larger than the one of the lens and is the only limit of the measurement range for pitch and yaw rotation.

the different rays is small enough, that several rays are detected by the autocollimator detection unit. If the light source is rotated by an angle that is large enough that one ray leaves the measurement range, another ray from the other side of the range will enter the measurement range. Thus, the measurement range is iteratively expanded and its size is given by the angular aperture of the light source. The roll angle motion of the light source can also be determined from a set of rays. In this case, the rays rotate around the optical axis of the detection unit. The different motions of the maxima on the image sensor and their interpretation are shown in Fig. 5.23. Each maximum is tracked with sub-pixel resolution. The separation of roll-, yaw- and pitch-motions can be solved with means of linear algebra with a principal component analysis [109]. In a mathematical sense, the motion of the maxima on the image sensor is a proper rigid transformation. This means, that only translations and rotations of the interference pattern on the image sensor take place, but there is no scaling or shearing of the interference pattern. Lateral movements of all maxima together equal yaw- and pitch-motions. Rotations of all spots around the centre of the image sensor equal roll-motions. The change in roll angle $\Delta\alpha_{\text{roll}}$ can be directly calculated from the principal component analysis. The angle of relative rotation of the yaw (or pitch) angle is given by

$$\Delta\alpha_{\text{yaw}} = \frac{\Delta x}{f} \quad (5.8)$$

where Δx is the displacement of the rays in x -direction and f the focal length of the lens. For the pitch angle, this formula is equivalent along the y -axis. The evaluation algorithm of the 3D-ORS was developed and tested in a Master thesis [110]. Its individual axes

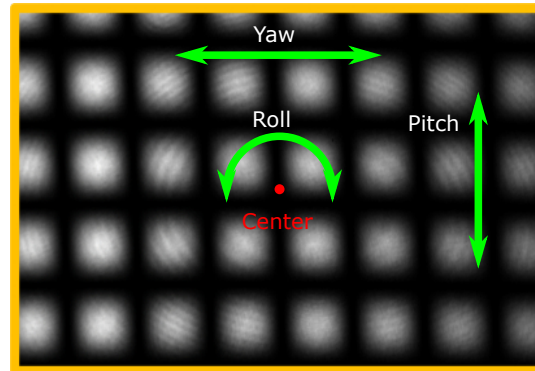


Figure 5.23: Interference pattern as registered by the image sensor of the 3D-ORS. Each maximum is being tracked by means of image processing. The arrows show the movement of the maxima for the corresponding relative motion of light source and detection unit. Image modified from Ref. [110].

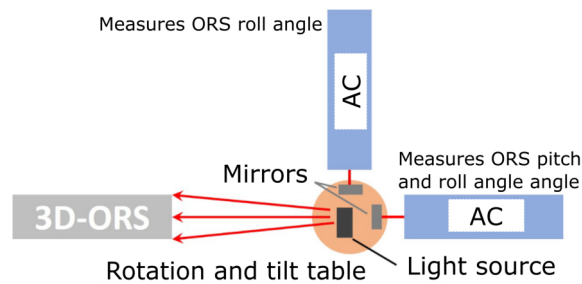


Figure 5.24: Setup for the test of the 3D-ORS. The autocollimator is placed in one of the two shown positions. When the table is rotated or tilted, the autocollimator (AC) mirror is readjusted when the autocollimator is about to leave the measurement range. Image modified from Ref. [110].

have been calibrated against an autocollimator (see Fig. 5.24). For this, a mirror for an autocollimator and the light source of the 3D-ORS were placed together on a tilt table. The table was tilted and measurements were taken with both devices. The mirror of the autocollimator was readjusted every time just before the angle measured by the autocollimator was about to leave the measurement range. Then, the difference of the autocollimator and the 3D-ORS measurements was evaluated. A calibration factor for the 3D-ORS was fitted, so that the linear components of the difference were corrected. The angle difference for the yaw angle is $3.8''$ RMS over 11.8° and for the pitch angle $8.9''$ over 6.9° . The errors are probably caused to a large extent by the readjustment of the mirror. A possible explanation for the higher RMS error of the pitch angle in comparison to the yaw angle is that the shorter side of the CMOS image sensor is used in this direction. For the roll angle, the RMS difference compared to the autocollimator is $20.1''$ over 7° . This is larger than for yaw and pitch angle and can be explained in the

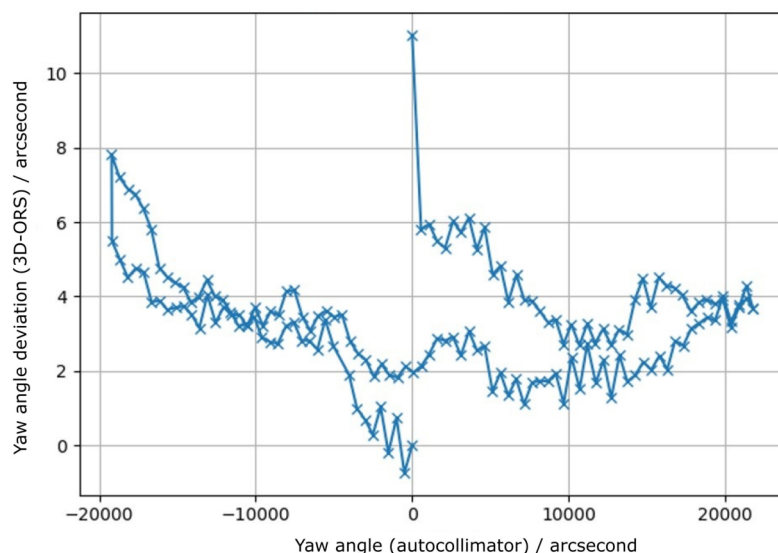


Figure 5.25: Difference of the 3D-ORS measurement to an autocollimator measurement for the yaw angle over a range of 11.8° . The rotation was done in both directions, back and forth. The RMS of the residuals is $3.8''$. Image from Ref. [110].

following way: For yaw and pitch, the lever for the motion of the maxima on the sensor is the focal length of the lens. For the roll angle, the lever is the distance of the maxima on the sensor which is much smaller than the focal length. Also here, the accuracy can be improved by increasing the sensor size. The results from Ref. [110] can be found in Fig. 5.25, 5.26 and 5.27. The measurement range is larger than 12° for pitch and yaw axis, while for the roll axis it is in principle infinite.

A remaining task is the traceable calibration of the 3D-ORS. Each axis can be calibrated separately with an angle comparator, as it is usual done for conventional autocollimators [111]. The perpendicularity of pitch and yaw axis is then ensured by the tolerance of the pixel positions on the image sensor. For the calibration of spatial angles, a three-dimensional calibration setup with three 3D-ORS is feasible. The light sources for each 3D-ORS move together on a rotation table. The alignment of the detection units can be reconstructed by solving a system of linear equations for the measured angles with the alignment and calibration factors as parameters to be determined. A similar setup for autocollimators was presented in Refs. [112, 113]. It is called *spatial angle autocollimator calibrator*. The ideas can be transferred to the 3D-ORS. Autocollimators are typically calibrated using standards for plane angles. In this paper, a mathematical model is presented on how to calibrate a spatial angle measuring device with two plane angle standards. In the form measurement setup in Fig. 5.10, the 3D-ORS could replace the conventional autocollimator in the future. A simple approach to improve the accuracy would be a larger image sensor as it increases the number of detected maxima

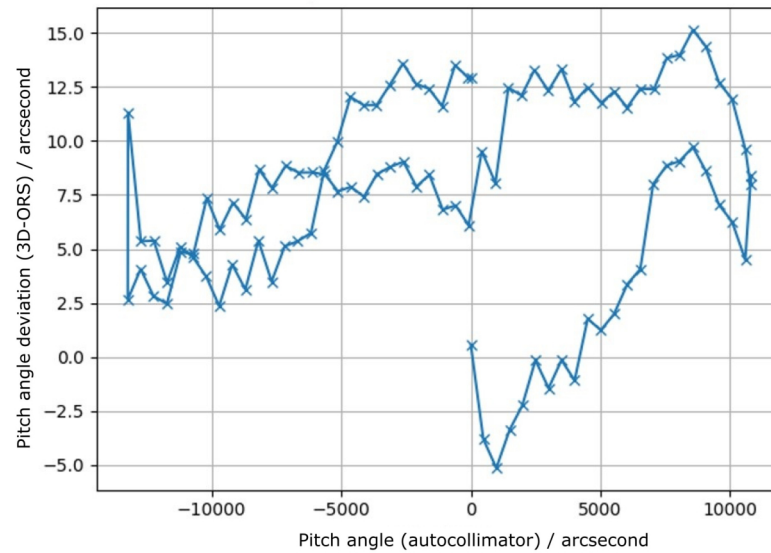


Figure 5.26: Difference of the 3D-ORS measurement to an autocollimator measurement for the pitch angle. The measurement range was 6.9° , also moving back and forth. The RMS of the deviations amounts to $8.9''$. The larger residual in comparison to Fig. 5.25 could arise because the pitch axis is in direction of the short edge of the image sensor, hence less maxima are averaged for this rotation. Image from Ref. [110].

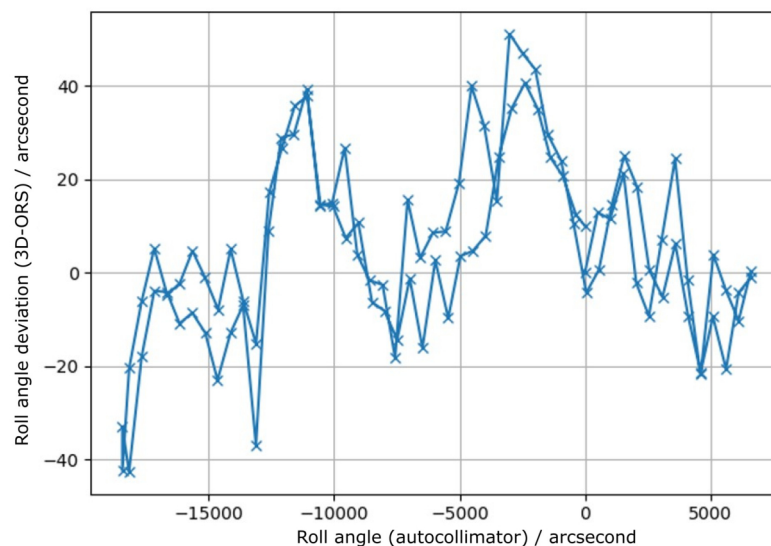


Figure 5.27: Difference of the 3D-ORS measurement to an autocollimator measurement for the roll angle over a range of 7° . The RMS of the residuals amounts to $20.1''$. The residuals are larger than for the other two rotation axes, because the lever arm of the rotation is the diagonal of the image sensor, and not the much larger focal length of the lens as for the other two. Image from Ref. [110].

to be averaged. The 3D-ORS can be expanded with a second image sensor that has a lens in front of it that forms an imaging optic with the autocollimator lens (as in Fig. 5.21). Then, also the position of the light source can be measured.

A patent has been granted for this sensor, see Ref. [8].

5.6 Measurement procedure

In this section, the measurement procedure for the stitching interferometry with cavity length and angle measurement is presented. First, some manual steps for the preparation of the measurement have to be performed. Then, the rest of the measurement is automated.

First, the specimen is placed on the specimen table of the y -stage. The interferometer is manually positioned so that it is at the first subaperture position at one corner of the specimen and that interference fringes are visible. Also, the cavity length between specimen and interferometer must be coarsely adjusted. The fine alignment will be done by the measurement algorithm. The tuning range for the laser also has to be chosen. The shorter it is, the faster the measurement will be completed, however, the cavity length measurement can be less accurate. For the measurement procedure, it can be chosen if the autocollimator shall be used or the rotations encoder of the rotation stages. Furthermore, the area where the specimen is located has to be passed to computer, as well as the desired step size in x - and y -direction. The step size for the measurements presented here was always 5 mm. Finally, the measurement positions are calculated from the entered parameters.

The program flow chart for the automated measurement is shown in Fig. 5.28. Figure 5.29 shows the possible path of the subaperture system on a spherical specimen. The data evaluation, i.e. the stitching is performed as described in the Theory Section 4.4 (for a measurement without autocollimator) or Sec. 4.4.1 (for a measurement with autocollimator).

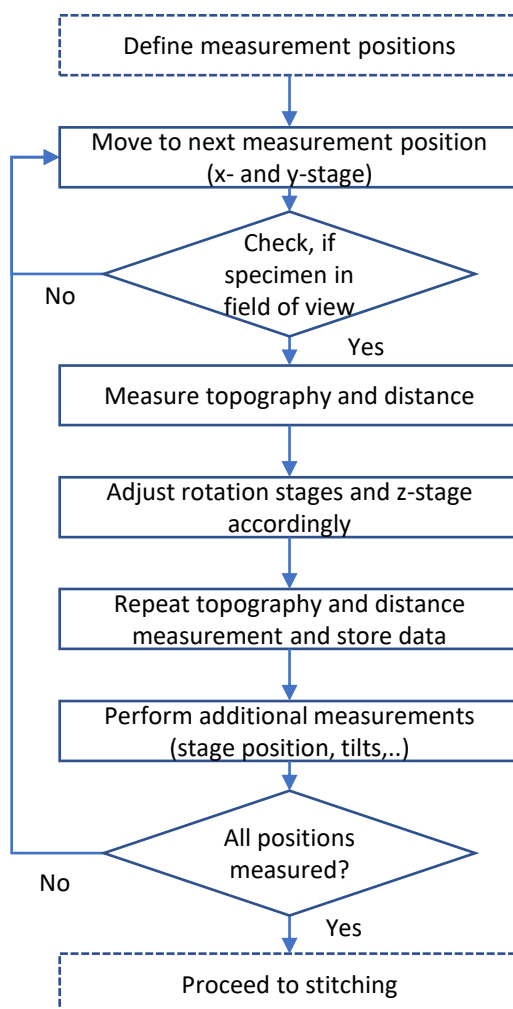


Figure 5.28: Program flow chart for the measurement. The flow chart starts from the point where all manual preparations are completed.

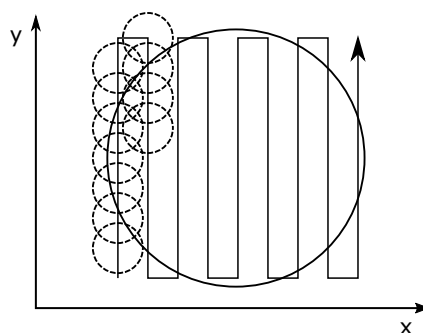


Figure 5.29: Raster path of the interferometer on a spherical specimen. From Ref. [1].

6 Measurements

In this chapter, the capabilities of the presented advanced stitching interferometry setup and its versatility for different types of specimens will be demonstrated. First, a flat specimen was measured to evaluate the performance of the measurement method without rotations of the Fizeau interferometer. This especially shows the current functionality and limits of the tiltmeters measuring the angular errors of the linear stages. Secondly, a small spherical specimen was measured using the autocollimator to track the rotations of the Fizeau interferometer. This proves the accuracy of the angle measurement and the resulting ability to measure the radius of curvature with high accuracy. Subsequently, an almost flat specimen with 230 mm in diameter and a radial chirp structure was (partially) measured to calculate the experimental instrument transfer function of the form measurement setup. Finally, a car side window was measured to show the suitability of the measurement system for large moderately curved freeform specimens. The rotation encoders of the rotation and goniometric stage were used instead of the autocollimator. For each measurement, there is a comparison with the result of another measurement system.

6.1 Flat surface

The measurement of a flat surface, representing the simplest surface form possible, is a verification for the correction of the angular errors of the x - and y -stages. It is also a test for the accuracy of the tiltmeters and the Fizeau interferometer, whose errors will propagate into the topography.

The autocollimator is not used in this measurement, because the rotation stages are not moved. Therefore, the angular relation between the tiltmeter on the x -stage and the Fizeau interferometer is fixed. Also, the z -stage is not necessary to correct the distance between interferometer and specimen, as the distance only changes by a few ten micrometers (due to small alignment tolerances). This does not have a significant effect on the reconstructed surface form when measuring a flat wavefront. The cavity length measurement is also irrelevant as the Fizeau interferometer always measures from the same direction. Consequently, the lateral position of the subapertures is given by the x - and y -stage position.

A photo of the specimen is shown in Fig. 6.1. It is the transmission flat of another Fizeau interferometer with an aperture of 4 inch. 384 subaperture measurements were performed. The effective aperture had a diameter of 8 mm. The step width was 5 mm and the positions were set on a rectangular grid. Before the measurement, the Fizeau interferometer was aligned to the specimen with its optical axis perpendicular to the sur-



Figure 6.1: Image of the specimen: The transmission flat of a 4 inch Fizeau interferometer.

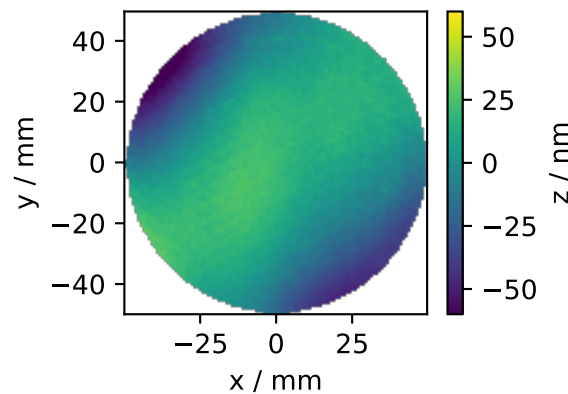


Figure 6.2: Topography as reconstructed from 384 subaperture measurements.

face at the first measurement position. An effective interferometer pixel pitch of $17.3\text{ }\mu\text{m}$ was chosen. Of those, only every 50th pixel was used in the evaluation to search for overlaps with at least 100 pixels. The sub-sampling speeded up the computational time without significantly changing the result. 1075 overlaps were found. The topography is shown in Fig. 6.2. The reconstructed topography has a PVr of 105 nm and an RMS height of 21 nm . The form appears shaped like an oblique astigmatism. The standard error of the regression for the stitching (or residual standard error) according to Eq. 4.21 was 3.6 nm . This is the RMS value of the remaining height differences between the overlapping subapertures after the stitching.

The topography was also measured with a 4-inch *Zygo* Fizeau interferometer in a single measurement. The result is shown in Fig. 6.3. The PVr is 33.4 nm and the RMS height 6.8 nm . There are some deviations between the two measurements as the PVr of the stitching interferometry measurement is significantly larger than the one of the full aperture measurement with the *Zygo* interferometer. Also the form of the surface is different. The deviations are most likely caused by unknown systematic angle errors.

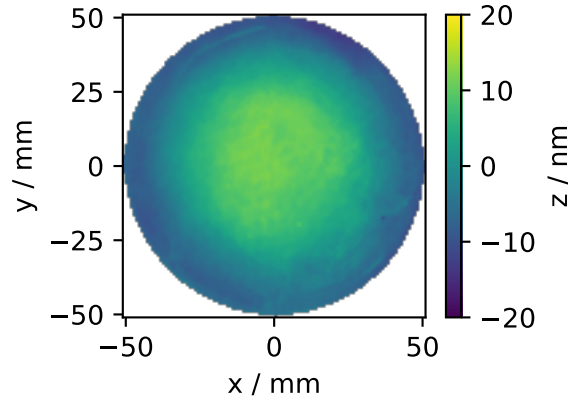


Figure 6.3: Topography of the 4 inch transmission flat as measured with a *Zygo* interferometer.

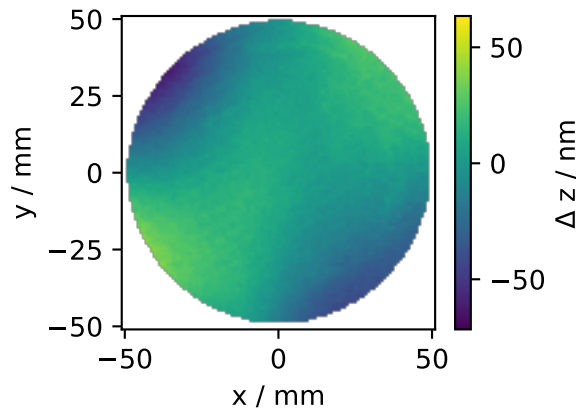


Figure 6.4: Difference of the stitched topography and the measurement with the 4-inch interferometer. The difference is the largest at the edge of the topography.

This issue is further examined in the simulations in Chap. 8.2.1.

In a Zernike polynomials fit, the spherical component and the oblique and vertical astigmatism terms are given by $C_2^0 = -19.6 \text{ nm}$ and $C_2^{-2} = -27.6 \text{ nm}$ and $C_2^2 = 1.7 \text{ nm}$ for the stitched topography. For the *Zygo* measurement, they are $C_2^0 = -10.9 \text{ nm}$ and $C_2^{-2} = 0.6 \text{ nm}$ and $C_2^2 = -0.4 \text{ nm}$. In the *Zygo* measurement, there is almost no astigmatism present. The RMS difference between both measurements is 18.7 nm , and the PVr of the difference is 121.8 nm (see Fig. 6.4). The relative rotations of the topographies were not fitted because the topography from the *Zygo* interferometer is approximately rotationally symmetric. It can be concluded, that the errors in the stitched topography are on a scale in the order of few tens of nanometres and dominate the topography. For an interferometric form measurement, this is a significant deviation. Possible solutions are proposed in Chap. 8.2.1.

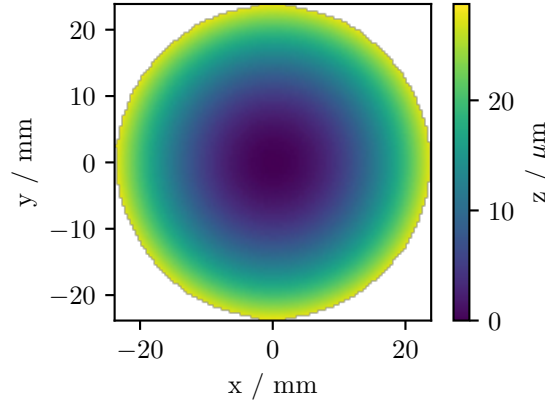


Figure 6.5: Topography of the spherical specimen with a nominal radius of curvature of $r_C = 10$ m. The PV height is $28.74\mu\text{m}$. From Ref. [1].

6.2 Spherical surface cap

The second measurement was performed on a specimen with 50.8 mm in diameter and a spherical topography with a nominal radius of curvature of $10\text{ m} \pm 1\%$ as given by the manufacturer (*Eksma optics*). The surface irregularity is specified as less than $\frac{\lambda(=633\text{ nm})}{10} = 63.3\text{ nm}$. The maximum necessary rotation of the Fizeau interferometer amounts to about $\pm 0.14^\circ$ at the edge of the specimen and can be measured with the autocollimator. This experiment is of interest with regard to the determination the radius of curvature and the residuum after subtracting a best-fit-sphere.

The measurement consists of 99 subapertures and took two hours. The step size was 5 mm and the inner 8 mm of the interferometer aperture were used. The rotation stages as well as the z -stage were repositioned for each subaperture to achieve the best possible alignment regarding a minimization of the number of fringes and the surface being in focus. The stitching algorithm with the determination of the alignment parameter for the rotation of the autocollimator around the optical axis was used. It returned a misalignment of 0.158° . Here, the standard error of the regression for the stitching according to Eq. 4.34 is $SE_R = 2.8\text{ nm}$. From the original lateral resolution in the measurement with $17.3\mu\text{m}$, every 50th pixel was used for evaluation. Overlaps with at least 100 pixels were used, which resulted in 170 overlaps in total.

Figure 6.5 shows the reconstructed topography of the spherical specimen minus tilts and offset. A sphere was fitted to the data points within a radial coordinate of 24 mm because the slope decreases at the edge of the specimen. It yielded a radius of curvature of 9.9936 m. This falls well within the specification. Then, the determined best-fit sphere was subtracted from the topography leaving the residual shown in Fig. 6.6. As it can be seen, the residual is to a great extent rotationally invariant. This is reasonable as this specimen was manufactured with a rotating tool. The residual has a RMS height

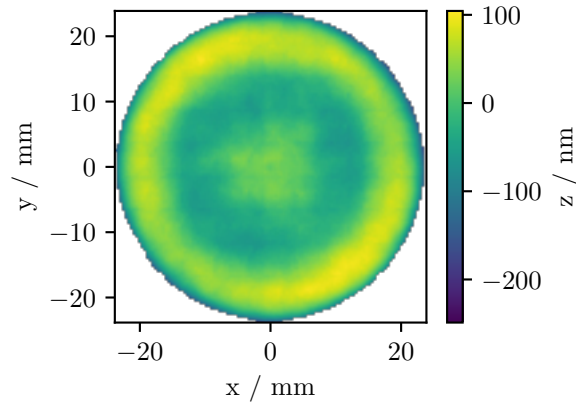


Figure 6.6: Residual of the specimen topography after subtraction of the best-fit sphere. Modified from Ref. [1].

of 57 nm. The surface irregularity is specified as 63.3 nm by the manufacturer. This also agrees well with the measurement. In Figure 6.7, the topography minus the best-fit sphere is shown for the stitching without autocollimator alignment parameter. To evaluate the influence of the alignment parameter in more detail, the respective first 36 Zernike polynomials of the topographies have been removed in Fig. 6.8.

A profile through the centre of the specimen was measured with a chromatic confocal sensor on a coordinate measurement machine (CMM). For details on the CMM measurement see Appendix 11.1. The lateral step width was 100 μm . A Gaussian filter with $\sigma = 100 \mu\text{m}$ was applied to reduce noise. The radius of curvature was determined with a fit of a polynomial of second degree to the profile. The resulting value for the radius of curvature is 9.983 m with a standard deviation of 4 mm from the covariance matrix of the fit. The application of the Gaussian filter changes the radius of curvature by less than 1 mm. The best-fit spherical profile was subtracted from the profile, see Fig. 6.10. The residual has again a large proportion of symmetrical components, as in the interferometric measurement. Also, it has a comparable peak-to-valley height.

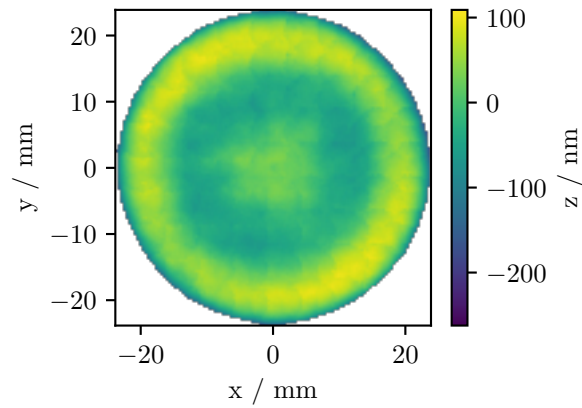


Figure 6.7: Topography resulting from the stitching without the adjustment parameter. More pronounced artefacts at the edges of the subapertures are visible. The artefacts have peak-to-valley heights of up to 30 nm. Modified from Ref. [1].

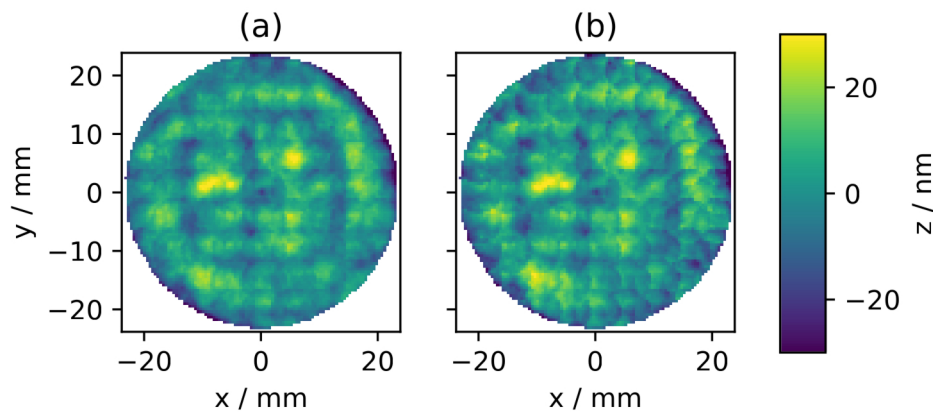


Figure 6.8: Topography minus the first 36 Zernike polynomials: (a) with alignment parameter, (b) without alignment parameter for the autocollimator. In (b), the tilts in tangential direction can be seen. From Ref. [1].

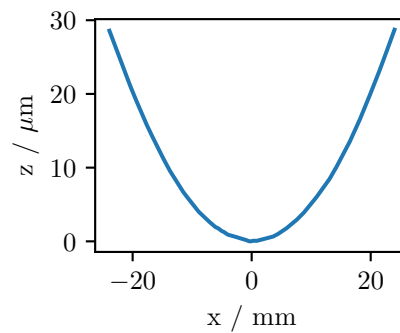


Figure 6.9: Profile through the centre of the spherical topography measured by a chromatic confocal sensor on a coordinate measurement machine. From Ref. [1].

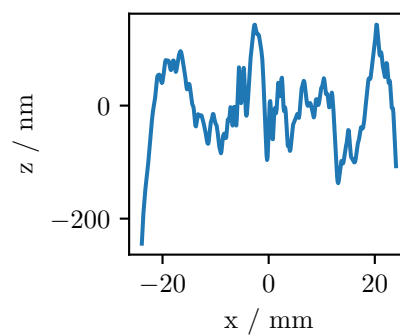


Figure 6.10: Profile from Fig. 6.9 minus a best-fit sphere. The local maximum of the topography seen in the stitching interferometry measurement in the centre is also visible. However, it is superimposed with straightness errors of the CMM. From Ref. [1].

Repetition of the measurement

The repeatability of the interferometric measurement was briefly investigated. The measurement from before had been repeated a week after the first measurement. The first measurement yielded a radius of curvature of 9.993 56 m, and the repeated one a radius of curvature of 9.9889 m. The difference of both values is 4.65 mm. The difference of the topographies minus their respective best-fit-sphere is plotted in Fig. 6.11. As it can be seen, the difference of the two topographies increases with increasing distance from its centre. The form resembles the astigmatism term of the Zernike polynomials. The RMS difference is 19 nm. The peak-to-valley height of the topography difference is 136 nm.

In conclusion, a certain agreement between the two repeated measurements can be seen. However, the difference between the two measurements is similar to the absolute deviation of transmission flat from the Zygo measurement (see Fig. 6.4). This suggests, that the same effect, presumably a systematic angle error, is causing an astigmatic form error in the measurement.

A comparison of the radii of curvature is shown in Fig. 6.12. The respective uncertainties shown in the plot are derived in Sec. 7.3.

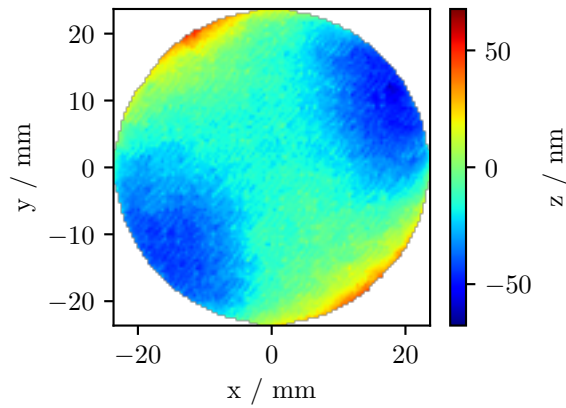


Figure 6.11: Difference of the two repeated measurements of the spherical specimen with their respective best-fit-sphere subtracted.

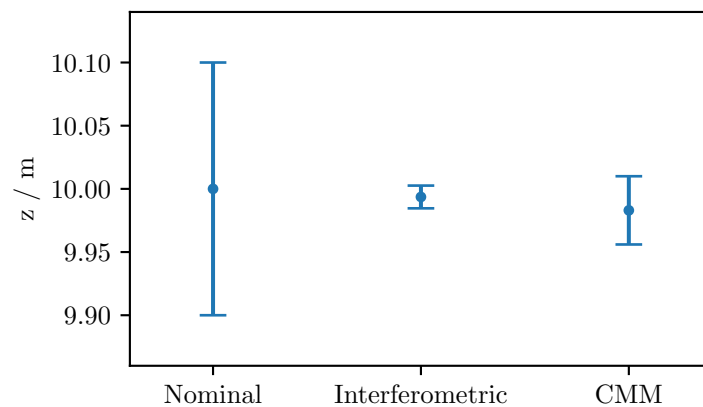


Figure 6.12: Difference of the two repeated measurements of the spherical specimen with their respective best-fit-sphere subtracted. For the CMM and the interferometry measurement, the error bars are the standard uncertainty, for the nominal value it is the tolerance range. Image from Ref. [1].

6.3 Specimen with chirp structure

This specimen has a nearly flat surface form (about $1.3\text{ }\mu\text{m}$ PV height) with a chirp structure in radial direction. An image of the specimen can be seen in Fig. 6.13. The spatial frequency increases with increasing distance from the specimen centre. The nominal amplitude is 50 nm and constant over the whole specimen. It is made of copper coated with nickel. The specimen has eight marks on its edge to define different profiles, that can be measured and compared. The spatial wavelength of the structure in radial direction first decreases from 2.5 mm to 1 mm . Then, it decreases to a spatial wavelength of $19\text{ }\mu\text{m}$ with a smaller gradient. The position dependent spatial wavelength is shown in Fig. 6.14. Due to the chirp structure, the specimen is well suited to determine the instrument transfer function (ITF) [114] of a form measurement system, that means how well structures can be measured depending on their spatial frequency. The measurement area was limited to a section of the specimen, that covers the whole x - range with 45 subapertures and three subapertures in y -direction. All spatial frequencies are covered for the x -direction while also having a short measuring time. The step size was 5 mm again. The surface gradient did not have to be tracked with Fizeau interferometer on the rotation stages, hence the autocollimator was not used either. The measurement took 1 hour 40 minutes.

From the original measurement data with an effective pixel pitch of $17.3\text{ }\mu\text{m}$, all pixels were used for the evaluation in order not to limit the resolution. 211 overlaps were found with at least 100 pixels. The residual standard error for the solution of the systems of linear equations Eq. 4.21 was 1.7 nm . The stitched topography can be seen in Fig. 6.15. From the topography, a profile was extracted at $y = 0$ by linearly interpolating the measurement data with a resolution of $7.3\text{ }\mu\text{m}$. It is shown in Fig. 6.16. In Fig. 6.17 and 6.18, sections of the profile are shown with magnification. The profile is well resolved. At the positions -102 mm and 102 mm , a small height displacement can be seen. In Fig. 6.19, a measurement of the same specimen from Ref. [115] is shown. The measurement was done with the *traceable multiple sensor method* from Ref. [60]. The height displacement at around $+102\text{ mm}$ from the centre can also be seen. Therefore, it can be concluded that it is an artefact on the topography.

Next, the ITF of the form measurement was extracted from the complete profile. Therefore, the lower frequency components of the profile were subtracted by applying a Gaussian filter with $\sigma = 100\text{ px} \equiv 400 \cdot 7.3\text{ }\mu\text{m} = 2.92\text{ mm}$ to the signal and subtracting the convoluted signal from the original signal. Then, the signal was analysed in several sections. A section had the length of 500 px , and the step size was 100 px . For each section, a Fourier analysis was performed to retrieve the dominant wavelength in the structure and its amplitude. The results are plotted in Fig. 6.20.

The ITF of the measurement system is shown in Fig. 6.21, starting from a spatial wavelength of 1 mm . As it can be seen, the signal is resolved until a wavelength of $200\text{ }\mu\text{m}$. The nominal amplitude of the chirp structure is 50 nm which the instrument

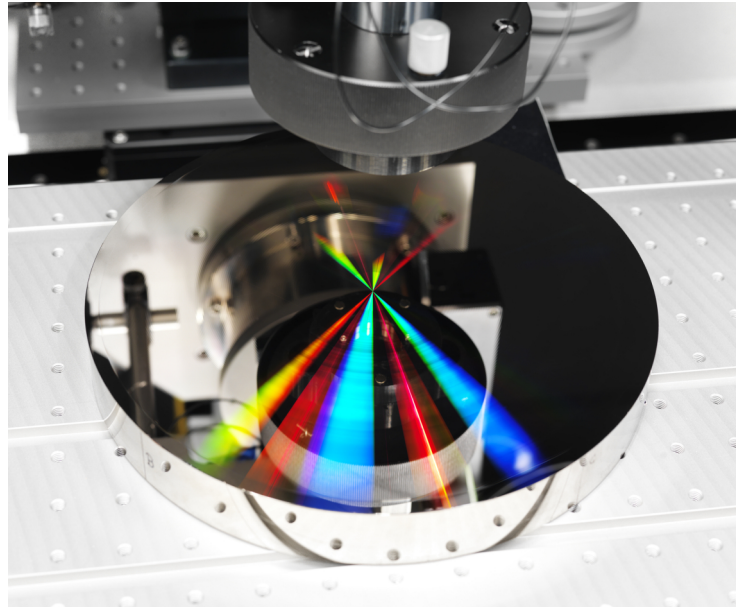


Figure 6.13: Image of the chirp specimen that has been measured from left to right and from marker A to B. The measurement area consisted of 3 times 45 subapertures. On the image, the specimen is differently rotated than for the measurement. Marker A is not visible.

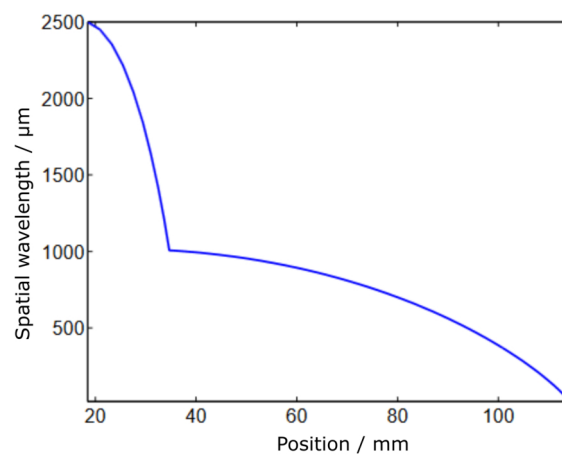


Figure 6.14: Design of the chirp structure: Structure wavelength versus radial position. Fig. from Ref. [115].

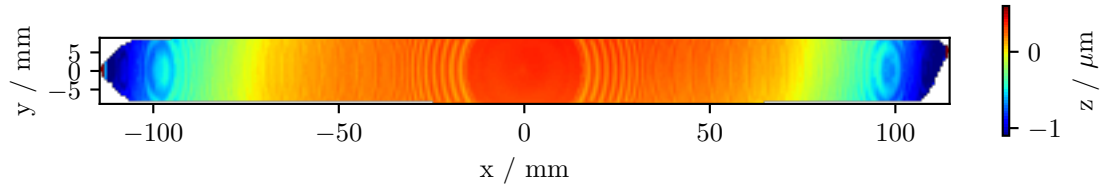


Figure 6.15: Stitched topography of the measured area of the specimen. At the side ends, aliasing occurs.

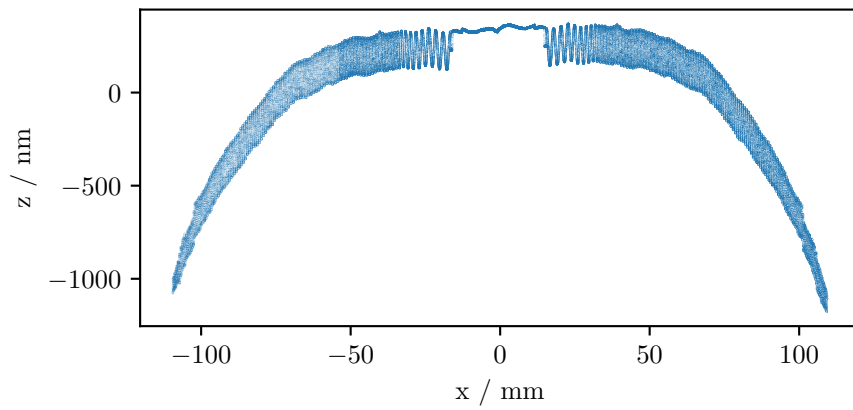


Figure 6.16: x -profile through the centre of the specimen from the data in Fig. 6.15 to achieve a better visibility of the structure.

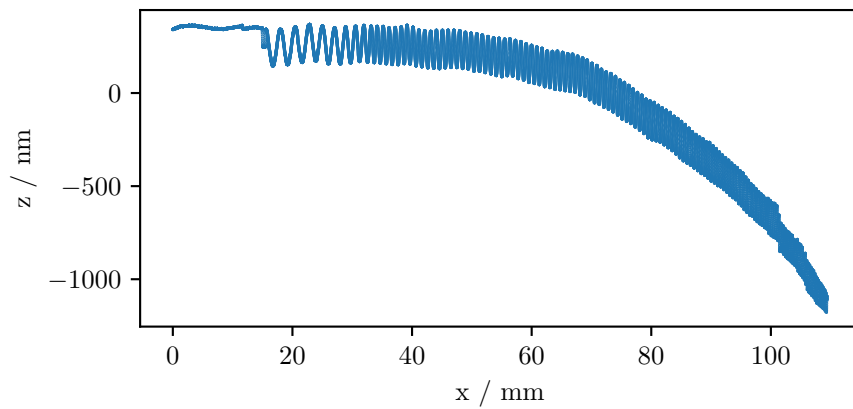


Figure 6.17: Right hand side of the x -profile in Fig. 6.16.

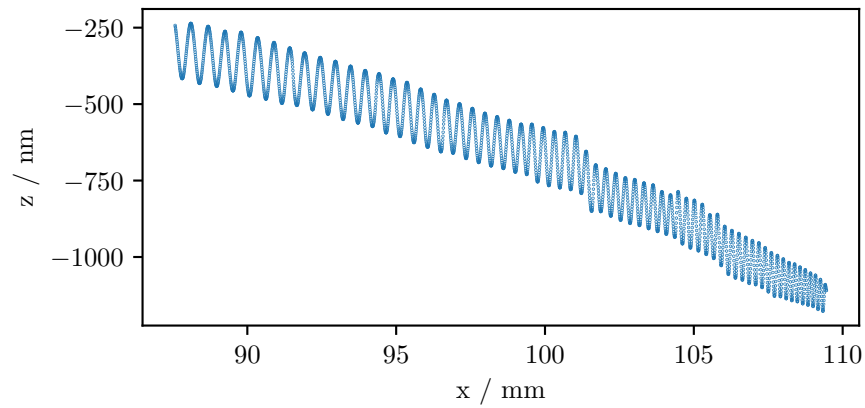


Figure 6.18: Section on the far right of the profile in Fig. 6.17.

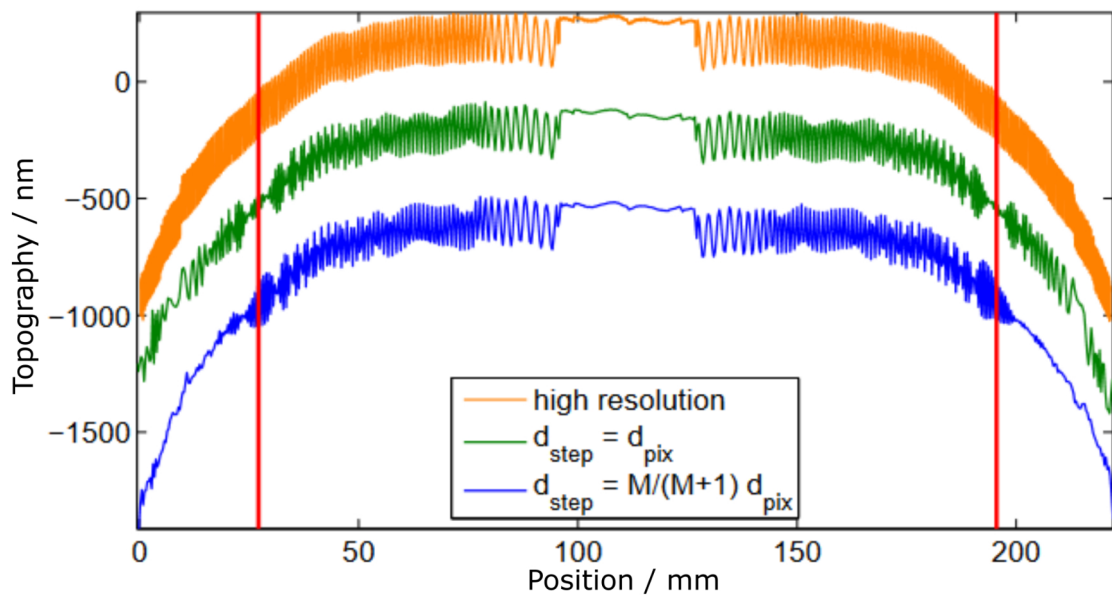


Figure 6.19: Plot of the x -profile through the centre of the chirp specimen. Figure from Ref. [115]. The measurement was done with the traceable multiple sensor method [60], a self-referencing stitching interferometry method. At position 20 and 210, the same artefact as in Fig. 6.16 is visible. (In the original work, several measurement modes of the TMS method were compared. For this comparison with the measurement here, only the orange high resolution profile is of interest.)

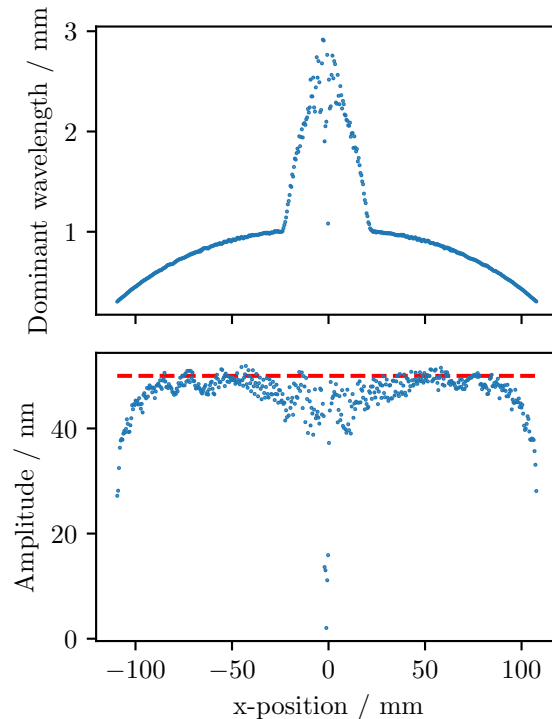


Figure 6.20: Upper plot: Dominant spatial wavelength as obtained by Fourier analysis (compare to Fig. 6.15). For the instrument transfer function (ITF), the outer flat "tails" are analysed later. Close to the centre around $x = 0$, the data points are not meaningful because the determination of the spatial wavelength is inaccurate (apparent from the scattered data points). Lower plot: Corresponding amplitude for the dominant spatial wavelength.

can completely resolve for large wavelengths. For smaller wavelengths, the transferred amplitude decreases. Since this effect occurs on the whole profile the alignment of the measurement system can not be the cause, as this would only cause a decreased amplitude in the overlaps of two or more subapertures. Therefore, the interferometer is the limit for the resolution due to its small numerical aperture. The ITF can be improved by using an interferometer with a higher resolution and when the system is very well aligned. For form measurements, the resolution is more than sufficient.

6.4 Freeform surface: Small window pane

In this section, a measurement of a freeform surface is presented. The specimen is a small rear side window pane of a car (see Fig. 6.22). The window has a size of $230 \text{ mm} \cdot 500 \text{ mm}$ and a surface height of 4 mm . It has a changing radius of curvature over its entire length. The radii of curvature go down to about 2 m . The angles by which the Fizeau interferometer has to be rotated are too large to be measured with the

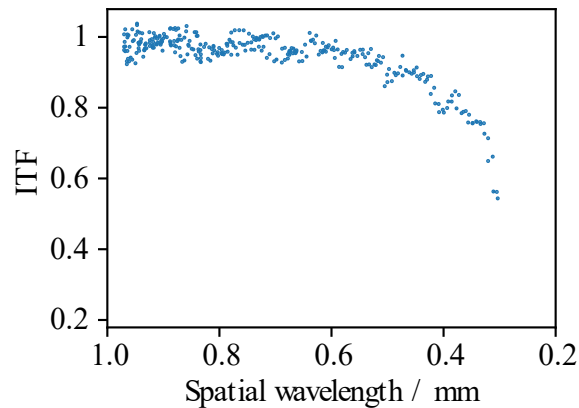


Figure 6.21: Instrument transfer function of the measurement system. The ITF is normalized to the nominal amplitude of the chirp structure of 50 nm. It can be seen that the ITF decreases to shorter wavelengths as it can also be seen in Fig. 6.18. The reason is the small numerical aperture of the interferometer.

autocollimator. Therefore, the internal rotation encoders of the rotation and goniometer stage were used. They have a good accuracy, however, they cannot measure errors due to misalignment of the axes due to the tolerances. The maximum angle range for this measurement is $\pm 2.7^\circ$ in direction of its short edge. The specimen was placed on foamed plastic and it was waited some time until the specimen did not sink in further. The pure measurement time (without interruptions) was a little less than three days. 3130 subaperture measurements were performed.

The unwrapping took 40 minutes, the further data evaluation about 20 minutes. The effective pixel pitch in the measurement was $17.3 \mu\text{m}$. After the phase unwrapping, only every 50th pixel of the Fizeau interferometer was used for the stitching to reduce the amount of data to be processed. The stitched topography is shown in Fig. 6.23. The residual standard error is 84 nm for the stitching according to the equation system in Eq. 4.21. The distribution is shown in Fig. 6.24. The distribution is symmetrical but has a larger kurtosis than a Gaussian distribution.

The measurement was compared to a measurement from Ref. [116], where profiles of the specimen were measured with a chromatic confocal sensor on a coordinate measurement machine. The specimen had been placed on the same foam material with additional optical cleaning cloths added in between. The profile from the coordinate machine measurement with the highest curvature (marked in Fig. 6.22) was fitted into the topography presented here. Therefore, the topography was linearly interpolated at the coordinates of the CMM profile. The position and rotation of the CMM profile in the topography was sought after with the Nelder-Mead method minimizing the RMS difference between the profile and the interpolated topography. Tilts and offsets were removed before the RMS value was calculated. The termination criterion for the fit was

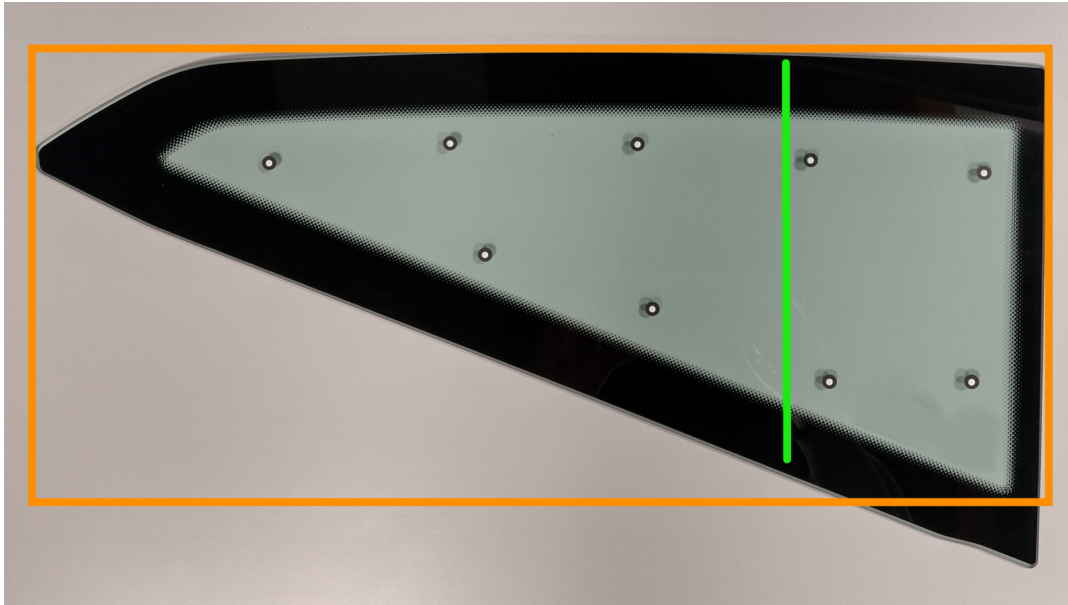


Figure 6.22: Photo of the window pane as it was measured interferometrically and with the CMM. Several markers are glued onto the specimen surface. The orange rectangle gives the approximate measurement range of the interferometric measurement. The green line is the approximate position of a profile from Ref. [116] that will be used for a comparison. From Ref. [1].

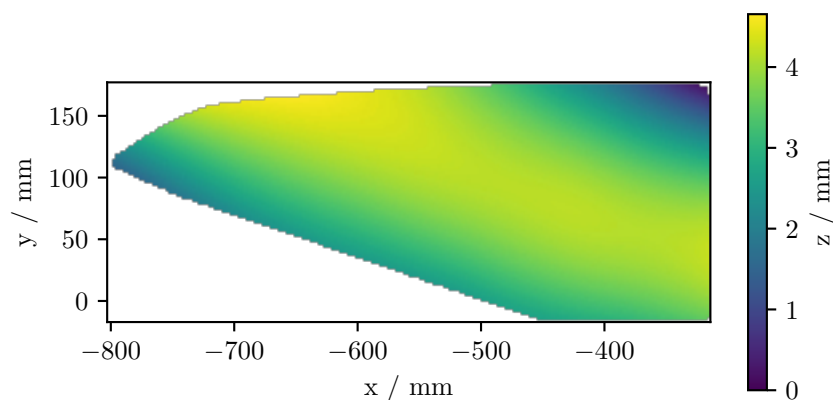


Figure 6.23: Topography of the window pane. The corner at the lower right is cut off, because it is about 30 mm larger than the system's measurement range in y -direction. From Ref. [1].

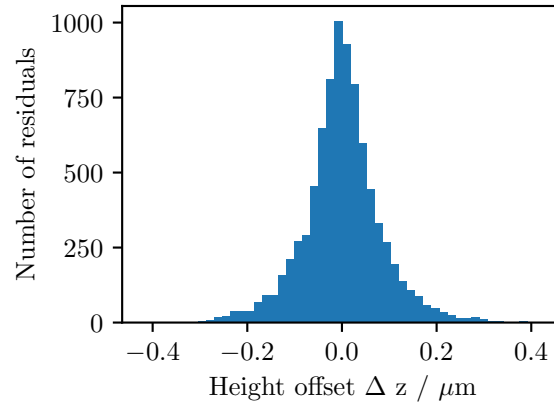


Figure 6.24: Distribution of the residuals from the stitching of the subapertures for the freeform specimen.

a change of less than 1 nm of the residuals from one to the next iteration.

The profile from the CMM measurement and the corresponding profile of the topography are plotted together in Fig. 6.25. The radius of curvature for the CMM measurement is 2.7185 m, and for the interferometric measured profile 2.7172 m. The difference is radius of curvature is 1.5 mm or $6 \cdot 10^{-4}$ relative. Both profiles with their individually fitted polynomial of second order subtracted are presented in Fig. 6.26. The difference of the profiles is shown Fig. 6.27 where also the RMS difference is calculated.

The influence of gravity on the tiltmeters and thus on the topography can be calculated according to Eq. 4.38 as

$$\delta z_{\text{gravitation}} = \int_{x_{\min}=0}^{x_{\max}=500 \text{ mm}} \frac{\Delta x}{R_{\text{Earth}}} dx \quad (6.1)$$

The resulting change in gravity direction is less than 20 nm for 500 mm of travel range. It has not been considered here because it is small compared to the uncertainties for the measurement in Sec. 7.4.4.

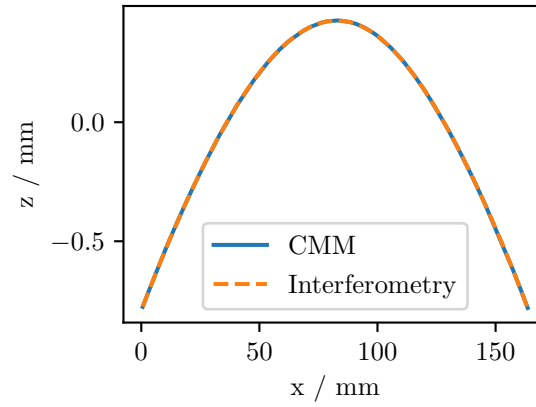


Figure 6.25: Profile measured with a CMM and the fitted corresponding profile from the interferometric measurement. From Ref. [1].

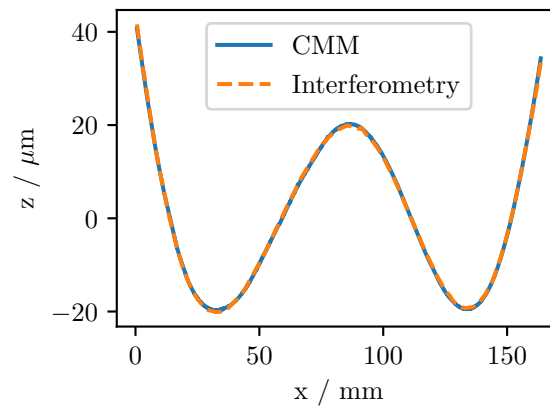


Figure 6.26: Both profiles from Fig. 6.25 with their respective polynomial of second order subtracted. From Ref. [1].

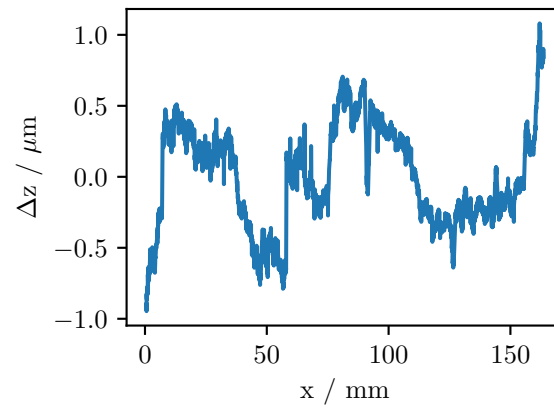


Figure 6.27: The difference of the profiles of from Fig. 6.25. The second order polynomials fitted to the profiles were not subtracted. The RMS of the topography difference is 370 nm. From Ref. [1].

7 Traceability and uncertainty budget

A traceability chain for the topography measurement will be presented. Then, an uncertainty budget according to the *Guide to the Expression of Uncertainty in Measurement* [20] will be established. It will include the uncertainty of single subaperture measurements, and how they propagate into the complete topography due to the stitching.

Following, it will be evaluated how a certain topography uncertainty can be reinterpreted as an uncertainty of the radius of curvature. This is interesting for specimens that have a spherical base form with added freeform contributions. The contributions to the uncertainty budget are analytically derived for a more comprehensive understanding. Later in Chapter 8, Monte Carlo simulations of the measurements will be presented and compared to the uncertainty budget derived here.

7.1 Traceability

The measurement system and the stitching algorithm are designed in a way that allows for traceable topography measurements. Each partial measurement result contributing to the topography must be traceable to SI units. A possible traceability chain is depicted in Fig. 7.1 and will be discussed in the following text. It is important to note that not all components of the form measurement setup are calibrated yet so this represents a possibility to achieve traceability.

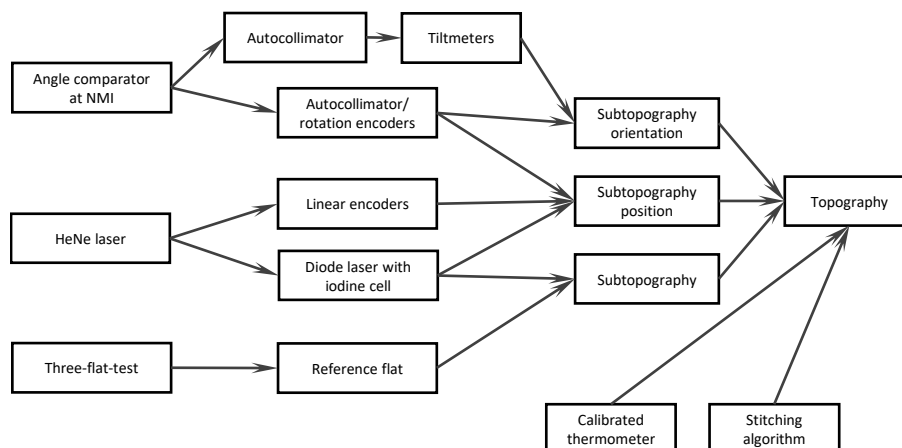


Figure 7.1: Possible traceability chain for the presented stitching interferometer measurement system. From Ref. [1].

The central wavelength of the laser is locked on an iodine reference cell, that can be traced to the wavelength of HeNe laser as a primary standard [117] using, for example, a calibrated wavelength meter. The relative wavelength during the wavelength sweep is measured with a Fabry-Perot interferometer in the laser. The accuracy of the internal wavelength reference can also be traced back to the HeNe laser with a calibrated wavelength meter.

The flatness of the reference flat of the Fizeau interferometer can be calibrated with another reference flat, which has been subject to a three flat test (see for example Ref. [118]).

The lateral coordinates of the Fizeau interferometer have been calibrated with two different chrome on glass masks with a quadratic pattern and line pattern whose dimensions are traceable.

The autocollimator can be calibrated with an angle comparator [119] as primary standard at a national metrology institute. Both angles of the autocollimator are calibrated independently.

The tiltmeters can be calibrated using a calibrated autocollimator as a secondary standard. Each axis of the tiltmeter is also independently calibrated against one of the autocollimator axes.

The positions of the linear encoders of the linear axes can also be traced back to a HeNe laser. The rotation encoders of the rotation stages can be traced to an angle comparator.

A calibrated thermometer ensures that the temperatures stay within a range for which the individual components are calibrated. The choice of the stitching algorithm also influences how the final topography is stitched and therefore how the uncertainty budget is derived.

7.2 Uncertainty of the topography measurement

The uncertainty budget describes the uncertainty of the topography measurement and will be applied for all measurements presented in Sec. 6. It includes statistical errors and unknown (and therefore uncorrected) systematic errors. Known systematic errors are of course always corrected.

First, a model equation for the height difference between two different points of the topography is introduced. It takes into account the topography of the subapertures and the path between the two points including several subapertures. It is a comprehensive two-dimensional simplification of the stitching. This model gives a maximum uncertainty estimation which will be further explained.

After that, the effects of different uncertainty contributions for the topography are discussed. These contributions include contributions for single subapertures as well as contributions involving several subapertures between two points of the topography. Many of the contributions are later not explicitly used as they are too small, or implicit.

itly contained in the stitching residuals. Nevertheless, they are presented because they allow to calculate achievable uncertainty contributions for individual components of the measurement system.

Finally, the topography uncertainties for the measured specimens will be calculated, using only the relevant contributions for each specimen.

7.2.1 Model equation

The topography $z(x, y)$ can be interpreted as a relative height between the point $z_k(x, y)$ and a fixed reference position $z_r(x_r, y_r)$. If both positions are in different subapertures, there will also be height offsets introduced by the subapertures in between called Δz_s . They are summed up on the path between the two points of the topography. Δz_s is the height difference introduced by the topography of a subaperture s itself and by its offset determined during the stitching. The height offset may include a height residual between two neighbouring subapertures. The topography is then given by:

$$z(x, y) = z_k(x, y) - z_r(x_r, y_r) + \sum_{s=1}^{N_{\text{path}}} \Delta z_s \quad (7.1)$$

where N_{path} is the number of subapertures between the points $z(x, y)$ and $z_r(x_r, y_r)$.

The uncertainty is given by

$$u(z(x, y))^2 = u(z_k(x, y))^2 + u(z_r(x_r, y_r))^2 + u\left(\sum_{s=1}^{N_{\text{path}}} \Delta z_s\right)^2 \quad (7.2)$$

It is assumed, that is the first two terms are equal and position independent: $u(z_k(x, y)) = u(z_r(x_r, y_r))$. It follows that

$$u(z) = 2 u(z_k)^2 + u\left(\sum_{s=1}^{N_{\text{path}}} \Delta z_s\right)^2 \quad (7.3)$$

In this work, an upper limit for the uncertainty is calculated. Therefore, two points will be chosen which are separated from each other as far as possible on the topography. For a circular shaped specimen with radius R , these would be, for example, opposite points on the edges at $x = -R$ and $x_r = R$ with $y = 0$. A possible refinement to obtain an accurate uncertainty estimate is presented in the chapter 10.2 Outlook.

In the next sections, different uncertainty contributions are discussed.

7.2.2 Uncertainty contribution of global offsets and tilts

In the final topography, global offsets do not play any role, because the offset can be arbitrarily chosen. The same applies for global tilts because they are also subtracted from the final topography. Therefore, topography errors that are linear proportional to the lateral coordinates will also be removed by the removal of the tilts and are not

included in the uncertainty budget. In terms of a polynomial describing the surface error on a 2D profile, only the terms of second and higher order contribute to the topography uncertainty budget. Tilts and offsets would only matter, if two topographies (for example front and back surface of a transmitting optic) were measured relatively to each other. This is not the case here.

7.2.3 Uncertainties of the height

Here, topography uncertainty contributions are discussed that directly cause an uncertainty in the height, that is, the z -value of the topography.

Short-term repeatability of the Fizeau interferometer topography measurement

The short-term repeatability of the Fizeau interferometer was determined in an experimental way. In Sec. 5.1.1, the topography of a spherical mirror was measured 50 times directly one after the other. The standard deviation of the height value for a pixel was 0.5 nm in average. This includes short-term thermal and mechanical stability, wavelength fluctuations of the laser and changes of the refraction index of the surrounding air.

The standard deviation of the final topography is rather small, because it is formed from many different interferograms during the wavelength sweep. This has an averaging effect. As the chosen wavelength range for the sweep for different experiments varies, this value is conservatively estimated as $u_{\text{Fiz repeatability}}(z) = 1 \text{ nm}$.

Distortion of the Fizeau interferometer optics

In Section 5.1.1, the imaging optics of the Fizeau interferometer were tested. A chrome on glass mask had been imaged with the Fizeau and a white light illumination. The deviation of the detected coordinates from the ideal coordinates was less than 1 px (at highest resolution) for all detected corners. With a maximum aperture of 10 mm and a pixel pitch of 1px $\hat{=}$ 8.64 μm at highest resolution, this is a relative error of $\epsilon_{\text{Distortion}} = 8.7 \cdot 10^{-4}$ for the lateral coordinates. If this distortion is applied to a subaperture with a radius of $R_{\text{subaperture}}$ and a topography with a maximum surface gradient of θ_{max} , the maximal resulting height error is

$$u_{\text{Distortion}}(z) = \epsilon_{\text{Distortion}} R_{\text{subaperture}} \theta_{\text{max}} \quad (7.4)$$

This is an upper limit for the uncertainty contribution due to the distortion of the optics.

Pixel pitch

The lateral position is given by the positions of the positioning system and the effective pixel pitch of the Fizeau interferometer projected to the SUT. The pixel position was determined by measuring a chrome on glass mask with a rectangular structure with a

period length of $200\text{ }\mu\text{m}$. The pixel distance was $8.64\text{ }\mu\text{m}$. Its standard uncertainty was estimated as $\epsilon_{\text{pixel pitch}} = 0.03\text{ }\mu\text{m}$.

With a maximum gradient of the topography θ_{max} and a subaperture radius $R_{\text{subaperture}}$, this can be converted to a maximum height uncertainty contribution of

$$u_{\text{pixel pitch}}(z) = \epsilon_{\text{pixel pitch}} \theta_{\text{max}} \quad (7.5)$$

Retrace error

The retrace error was described in Sec. 4.2.9. It occurs when a Fizeau interferometer is used to measure a surface in non-null-configuration so that the common path principle is violated. The retrace error for different setups and under different conditions has been investigated theoretically and experimentally [120, 71].

To get an estimate for the uncertainty budget, a form measurement of a flat mirror with a flatness error of $\lambda/20$ at $\lambda = 633\text{ nm}$ was performed at different tilt angles, that is with different number of interference fringes. The offset and the tilts were fitted and subtracted from the topographies for all tilt angles. Then, the null configuration measurement was subtracted from the topographies in the non-null configuration. The topography differences are showing in Fig. 7.2, 7.3 and 7.4. The resulting difference can be attributed to the retrace error. The RMS deviations from the null configuration can be seen in Table 7.1. The uncertainty $u_{\text{retrace error}}(z)$ can be estimated for each specimen with the RMS difference for largest number of fringes occurring in this measurement. In the measurements in this work, it is always less than 10 fringes.

In Fig. 7.5, a cross section of the retrace error is shown for better visibility for 10 and 50 fringes.

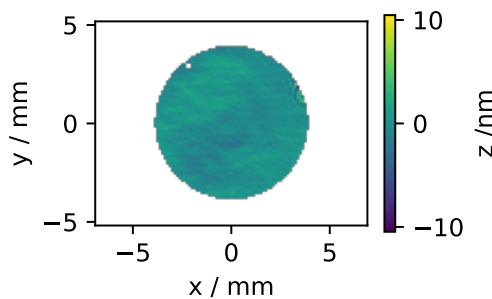


Figure 7.2: Difference between a topography from a measurement with 10 fringes and the null-configuration (tilts subtracted).

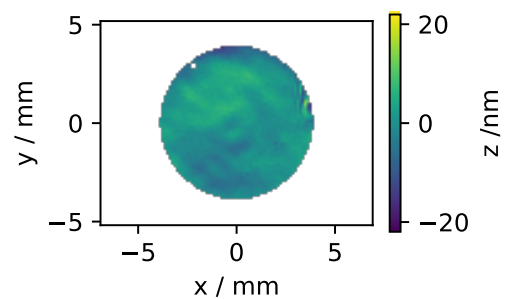


Figure 7.3: Difference between a topography from a measurement with 50 fringes and the null-configuration (tilts subtracted).

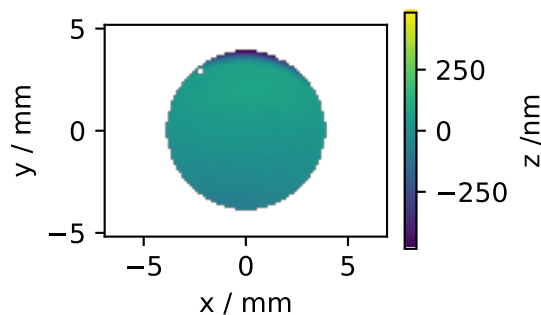


Figure 7.4: Difference between a topography from a measurement with 100 fringes and the null-configuration (tilts subtracted).

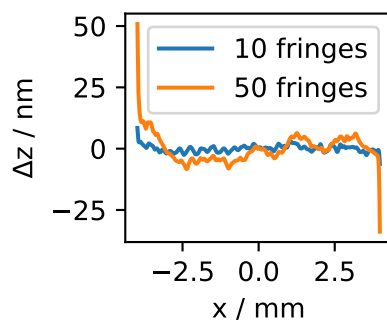


Figure 7.5: Retrace error of the Fizeau interferometer at $y = 0$. The plot shows the difference between a measurement with less than one fringe, compared to the number of fringes stated in the legend. For 10 fringes, the retrace error is very small. For 50 fringes, it is larger, in the range of ± 10 nm (PV) if the edges are not considered.

Table 7.1: Experimentally determined retrace error.

Number of fringes at $\lambda = 633$ nm	RMS deviation to null-configuration
10	2.1 nm
50	4.4 nm
100	96.7 nm

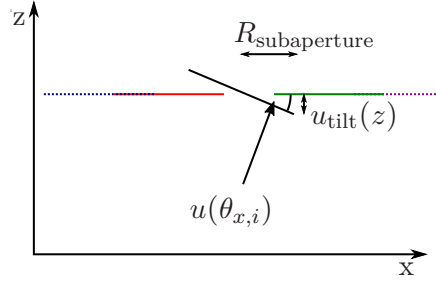


Figure 7.6: The central subaperture has a tilt error. The other subapertures (in colour) are fixed in height by their surrounding subapertures (y-direction not shown). The uncertainty for the height values in this subaperture is given by its radius multiplied with the tilt uncertainty.

Tilt error of the single subaperture

When the uncertainty of the tilt for a subaperture is $u_{\text{tilt}}(\theta)$ (see Fig. 7.6), then the maximum height uncertainty for this subaperture is

$$u_{\text{angle}}(z) = R_{\text{subaperture}} u(\theta) \quad (7.6)$$

7.2.4 Uncertainty of the lateral position

If the lateral coordinates of topography data points have an uncertainty, then it can be reinterpreted as a topography uncertainty by multiplying the coordinate error with the surface gradient in a first order approximation. Next, uncertainty contributions resulting from position uncertainties are introduced. Some of these will be neglected in the uncertainty budgets for the measurements but are presented anyway.

Positioning system

For the positioning system, it is useful to first calculate the uncertainty for the lateral position. In the next step, it can be converted to an uncertainty of the height. For the x -direction, the combined uncertainty is derived from the considerations about the geometrical errors of a positioning system in Sec.4.6. It is given by

$$\begin{aligned} u_{\text{positioning system}}(x)^2 = & u_{x\text{-stage}}(x)^2 + u_{y\text{-stage}}(x)^2 + u_{z\text{-stage}}(x)^2 \\ & + u_{\text{rot-x}}(x)^2 + u_{\text{rot-y}}(x)^2 \\ & + u_{\text{perpendicularity z}}(x)^2 + u_{\text{perpendicularity y}}(x)^2 \end{aligned} \quad (7.7)$$

The first term is for the encoder accuracy, and this position is measured accurately. The second and third terms in the first line describe the straightness of the stages

moving perpendicular to the x -axis. The next two terms in the second line describe the x -position uncertainty of the rotation and goniometer stage (due to wobble and eccentricity). The terms in the last line describe the uncertainty due to a deviation from ideal perpendicularity of the linear y - and z -stage. These terms linearly depend on the position of the respective linear stage. Non-relevant contributions were not included in the equation (causing only offsets).

The uncertainty for the deviation of a rectangular angle $u(XwZ)$ is multiplied with the longest distance $Z_{\max}$ which the z -stage travels. This yields for the z -stage:

$$u_{\text{perpendicularity } z}(x) = u(XwZ) \cdot Z_{\max} \quad (7.8)$$

The contributions for the y -stage is equivalent.

Abbe error of first order

The Abbe error [121] of first order (also called sine error) describes the positional error that occurs when the length or position of an element is measured along a line that does not coincide with the length of interest and if there are tilt angle errors present. h_z is the distance between the line where the position is measured and the line where it is of interest. For a specimen placed on a linear stage, this is the distance between the linear encoder and the specimen surface.

If the stage has an uncertainty for the angle of $u(\theta_{x,x\text{-stage}})$, this leads to an additional x -position uncertainty of

$$u_{\text{Abbe}}(x) = u(\theta_{x,x\text{-stage}}) \cdot h_z \quad (7.9)$$

Abbe error of second order

Abbe errors of second order (also called cosine errors) account for the non-parallelism of a linear encoder and the direction of movement. If the angle between direction of motion and the linear encoder is ζ and has an uncertainty $u(\zeta)$, the uncertainty for the position is

$$u_{\text{Abbe,2nd}}(x) = x \cdot [1 - \cos(u(\zeta))] \quad (7.10)$$

Due to the cosine function, this uncertainty is of little relevance. As an example: For $u(\zeta) = 0.1^\circ$, the position uncertainty for 1 m travel range is $u_{\text{Abbe,2nd}}(x) \approx 1.5 \mu\text{m}$.

Repeatability of the cavity length measurement

The uncertainty of the cavity length (or absolute distance) measurement $u(d)$ has no effect on the subaperture topography itself, but it propagates into the stitched topography. For the stitching, the individual topographies from the subaperture measurements are laterally positioned in a global coordinate system using the absolute distance and the angular position of the Fizeau interferometer. An uncertainty of the absolute distance

d is projected to the lateral coordinates according to

$$u_d(x) = \theta_{\max} u(d) \quad (7.11)$$

By multiplication with the maximum gradient of the topography $\theta_{\max}$, this leads to a maximum height uncertainty of

$$u_d(z) = \theta_{\max}^2 u(d) \quad (7.12)$$

Due to the small gradients and quadratic dependence on the gradients, this contribution is small for moderately curved specimen. The uncertainty $u_d(z)$ has not been taken into account as it is corrected by the stitching algorithm.

There are also analytical calculations on how mechanical drifts influence the cavity length measurement in Ref. [122].

7.2.5 Uncertainty of the path along the subapertures

Along the path $\sum_{s=1}^{N_{\text{path}}} \Delta z_s$, uncertainty contributions for Δz are summed up or integrated. Several contributions are presented here.

Integrated statistic uncertainty of the angle measurement

For each subaperture measurement, the angular positions of the interferometer and the specimen are measured. Each angle measurement has a statistical uncertainty. The topographies are stitched together and errors propagate. This is shown in a 2D model in Fig. 7.7. The angle measurement for the i -th subaperture has an angle error of $\Delta\theta_i$. The resulting height error for this subaperture is

$$\delta z_i = \Delta\theta D_{\text{subaperture}} \quad (7.13)$$

where $D_{\text{subaperture}}$ is the diameter of the subapertures. The integrated height error for all subapertures along a path is

$$\epsilon_h = \sum_i^N \Delta x_i \Delta\theta_i \quad (7.14)$$

where Δx is the distance between the centres of the overlapping regions of neighbouring subapertures. $\Delta\theta_i$ is a statistical value for each subaperture and can be reduced to $\frac{\Delta\theta}{\sqrt{n}}$ by performing n repeated angle measurements for each subaperture.

For the whole specimen, the uncertainty from this contribution can be obtained by using the variance of a *Wiener process* [123], which is a generalization of the *Brownian motion* [124]. The generalized Brownian motion describes the motion of a particle which moves each time step with a step size drawn from a normal distribution. If the time step is identified with the number of the subaperture measurement i , and δz_i with the step

size, both problems are equivalent on an abstract level.

The uncertainty at the edge is given by

$$u(z) = \sqrt{N_{\text{subapertures}} D_{\text{subaperture}}} \frac{u(\theta)}{\sqrt{n_{\text{measurements}}}} \quad (7.15)$$

Examples for the random walks and the respective variance are shown in Fig. 7.8.

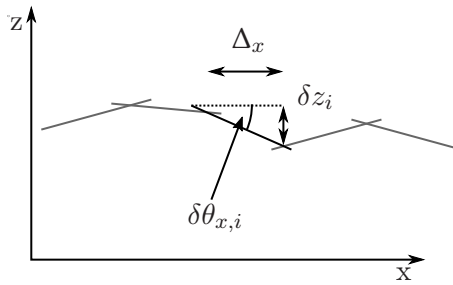


Figure 7.7: Illustration of the random walk-like behaviour of stitched subapertures with tilt errors drawn from a normal distribution.

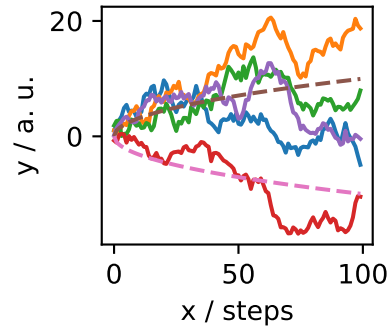


Figure 7.8: Five exemplary results (solid lines) and the standard deviation (dotted lines) of the Wiener process modelling Brownian motion as an analogy for the statistical angle error.

Integrated unknown systematic uncertainty of the angle measurement

In principle, the systematic error in dependence of the measurement position can take any form which is in accordance with a calibration certificate. Here, the lowest polynomial order relevant, an unknown systematic angle error linear with the x -position, will be evaluated. It is assumed that the angle error changes linearly with the x -position¹. The error is defined as

$$\epsilon(x) := \frac{\epsilon_{\text{PV}}}{x_{\text{max}}} x \quad (7.16)$$

where ϵ_{PV} is the peak-to-valley angle error.

To convert it to a topography uncertainty, the angle error $\epsilon(x)$ is integrated over the specimen from $x = 0$ to x_{max} .² The resulting topography error contribution will also be a peak-to-valley value. To convert it, it is assumed that it has a uniform distribution and

¹In principle, the angle error should be mostly dependent on the measured angle. However, the measured angle depends on the topography of the specimen (for the rotation encoders or the autocollimator) or the angular error of the stages (for the tiltmeters). These quantities in turn depend on the position on the SUT, so the dependence on the x -position is a good approximation.

²A continuous integral is chosen for simplicity. Theoretically it is step-wise summation of angle error multiplied with the subaperture diameter.

can be converted to a standard uncertainty according to GUM [20] by a multiplication with $\frac{1}{\sqrt{12}}$ (see Appendix 11.1.3). The uncertainty contribution for the height is then given by

$$u_{\text{Integrated sys. angle error}}(z) = \frac{1}{\sqrt{12}} \int_0^{x_{\max}} \epsilon(x) dx \quad (7.17)$$

With Eq. 7.16, the integral results in

$$u_{\text{Integrated sys. angle error}}(z) = \frac{1}{\sqrt{12}} \left(\frac{1}{2} \epsilon_{\text{PV}} x_{\max} \right) \quad (7.18)$$

This error also contains the tilt error of the single subaperture from Section 7.2.3. For small specimen, it is a good approximation to just use the tilt uncertainty of the single subaperture for z_r and z_k .

Stitching residuals

First a scenario is considered, in which only a profile of the specimen topography is stitched, that is, a row of 1 times n subapertures. Stitching the subapertures according to Eq. 4.21, there is one more offset to be determined than there are overlaps of subapertures. Therefore, there is an exact solution with exception for a global offset and the height differences between all subapertures can be adjusted to zero.

If there are m times n subapertures to be stitched (with $n, m > 1$), the system of linear equations is overdetermined, because there are more overlaps between neighbouring subapertures than offsets to be fitted. A solution can be found as the linear least squares solution. If there are topography, tilt and position errors present in the subapertures, height offsets between the subapertures will remain even after stitching. These residuals are an implicit measure of other occurring errors that were described before. In principle, it is possible to create scenarios in which the residuals can be zero while there are form and tilt errors present. However, this would require that the errors are correlated and is to be deemed as unlikely.

The topography uncertainty due to stitching is estimated by

$$u_{\text{stitching}} = \sqrt{N_{\text{path}}} \text{SER} \quad (7.19)$$

where $\sqrt{N_{\text{path}}}$ is the number of subapertures along the path of interest and SER is again the standard error of the residuals. For a large number of subapertures, this stitching error is assumed to include all the error contributions presented before. This includes for example the integration of statistical and unknown systematic angle errors and the uncertainty of the cavity length measurement.

7.3 Uncertainty of the radius of curvature

To determine the uncertainty of the radius of curvature, a new approach is introduced. The uncertainty of the topography was investigated in Section 7.2. It was estimated as the uncertainty of the height difference between two points of the topography which are the furthest away from each other. It was assumed, that the height errors integrate up along the way between the two topography points. If it is in addition assumed, that also all other topography uncertainties continuously accumulate along the path, this accumulation can be interpreted as a random walk in one dimension. The peak-to-valley height of this random walk can be drawn from a normal distribution that represents the uncertainty of the height difference of the points farthest from each other. By adding this error signal to an ideal topography and fitting a polynomial to determine the radius of curvature from its second order term, the effect of the topography on the radius of curvature can be estimated. This can be done as a Monte Carlo simulation for many different error signals and PV heights drawn from the normal distribution.

The Monte Carlo simulation to estimate the uncertainty of the topography is performed in the following way: In one repetition of the simulation, the height error between those two furthest away points is drawn from the normal distribution $N(0, \sigma = u(z))$ as δz_i . i is the i -th repetition of the Monte Carlo simulation. Then, a random walk with normally distributed step size with $\sigma = 1$ is drawn as $\delta h(x)$. Its PV height can be determined as δh_{PV} . The random walk is then normalized in its height to δz_i yielding

$$\delta h'(x) = \delta h(x) \frac{\delta z_i}{\delta h_{PV}} \quad (7.20)$$

This signal $\delta h'(x)$ generated from the random walk is added to the ideal profile, for which the effects of the topography error on the radius of curvature are to be investigated. Then, a polynomial of second degree is fitted to the topography with errors. The constant and linear term give offset and tilt. The quadratic term gives the radius of curvature according to Eq. 11.4. Since the offset and the linear term of the error signal do not influence the radius of curvature, only the form of the error signal of second order or higher can change the radius of curvature.

Also, the normalization of the generated error signal leads to different standard deviations of the effective step size of the random walk:

$$\sigma_{\text{eff}} = [\sigma = 1] \cdot \frac{\delta z_i(x)}{\delta h_{PV}} \quad (7.21)$$

As an example, this method is applied to the spherical specimen from Sec. 6.2. The topography uncertainty is given as $u(z) = 34.1 \text{ nm}$ (this is an advance on Sec. 7.4.2). Error signals from random walks are drawn and normalized to a PV value from a normal distribution with $\sigma = u(z)$. Five examples of those error signals are shown in Fig. 7.9. The signals are added to the ideal topography, and a polynomial of second degree is

Table 7.2: Coverage intervals for the radius of curvature $r_C = 10$ m from 10000 simulations fitting the radius of curvature of an ideal spherical profile with different topography error signal.

Coverage probability	coverage interval
68.27%	$[r_C - 2.9 \text{ mm}, r_C + 2.9 \text{ mm}]$
95.45%	$[r_C - 9 \text{ mm}, r_C + 9 \text{ mm}]$

fitted. The results for the radius of curvature for 10000 repetitions of this procedure are shown in Fig. 7.10. The distribution has a higher skewness than that of a Gaussian distribution. The coverage intervals for this distribution are given in Table 7.2.

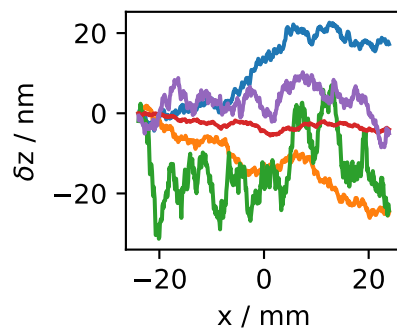


Figure 7.9: Graph with five exemplary error signals for the topography generated by a random walk and with a PV value normalized to different values drawn from $N(0, \sigma = u(z))$.

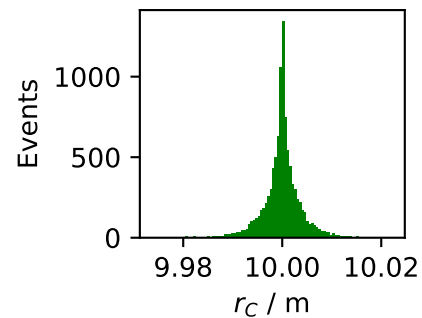


Figure 7.10: Histogram with the best-fit radii for the spherical topography with 10000 different error signals. Image from Ref. [1].

7.4 Uncertainty budgets for the measured topographies

Here, uncertainty budgets will be presented for the specimens measured in Chapter 6 by using selected relevant uncertainty contributions from Sec.7.2. It is important to note that the estimated topography uncertainties are upper limits. The uncertainty contributions that affect the single topography are taken into account twice because, when a height difference is formed, the points are generally in two different subapertures.

Table 7.3: Uncertainty budget for the topography of the flat specimen. The contributions labelled with “*” are taken into account twice.

Quantity	Uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
Reference flat*	21 nm	Rectangular	1	6.1 nm
Distortion*	8.64 μm	Normal	$\theta_{\max} \approx 0$	0
Lateral position*	5.8 μm	Normal	$\theta_{\max} \approx 0$	0
Integrated angle	6.9 μrad	Rectangular	$\frac{1}{2}R_{\text{specimen}}$	50 nm
Stitching	3.6 nm	Normal	$\sqrt{N_{\text{overlap}}}$	15.7 nm
Combined uncertainty				52.8 nm ($k = 1$)

7.4.1 Flat surface

Here, the uncertainty budget for the measurement in Sec. 6.1 is presented. The derivation of the contributions was done in Sec. 7.2.

The reference flat has a nominal flatness of ≤ 21 nm (PV). This leads according to Ref. [20] to a standard uncertainty of $u_{\text{RF}} = 6.1$ nm.

For the flat surface, lateral errors of the position are negligible as the surface gradients are very small. The only surface gradients originate from the flatness error of the surface. The distortion of the imaging optics is negligible for the same reason.

The distortion of the Fizeau interferometer amounts to a maximum of 8.64 μm , but since the topography is approximately flat, an error of the lateral coordinates leads to a zero uncertainty contribution.

The uncertainty of the lateral position $u(x)$ is estimated by using the straightness $\pm 10 \mu\text{m}$ of the y -stage resulting in $u(x) = \frac{10 \mu\text{m}}{\sqrt{3}} = 5.8 \mu\text{m}$. Since the gradient is approximately zero, the corresponding uncertainty contribution is also approximately zero.

The uncertainty of the angle measurement was estimated to be 0.25'' for each tiltmeter. The PV error is estimated to 1'' which is the complete range of the 95% ($k = 2$) coverage interval. An integration according to Eq. 7.17 (or respectively Eq. 7.18) leads to a contribution of $u_{\text{Integrated angle}}(z) = 50$ nm.

For the stitching, there are $N_{\text{overlap}} = 19$ overlaps in one direction with a standard error of the regression of $SE_R = 3.6$ nm. This leads to an uncertainty of $u_{\text{stitching}} = 15.7$ nm.

The contributions for this specimen are listed in Table 7.3. The uncertainty for the topography measurement is given by $U(z) = 105.6 \text{ nm} (k = 2)$. The value of the measurand then normally lies within the attributed coverage interval with a probability of approximately 95%.

7.4.2 Spherical surface

This is the uncertainty budget for the measurement of the specimen with a spherical surface in Sec. 6.2. Its radius of curvature is $r_C = 10$ m. With an effective radius

Table 7.4: Uncertainty budget with the most important contributions for the topography measurement of the spherical specimen. The components for $u(z_k)$ marked with * are taken into account twice to account for the factor in Eq. 7.3. The estimates of all quantities are zero. From Ref. [1].

Quantity	Uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
Reference flat*	21 nm	Rectangular	1	6.1 nm
Distortion*	8.64 μm	Normal	$\theta_{\max} = \frac{R_{\text{subaperture}}}{r_C}$	4 nm
Lateral position*	8.2 μm	Normal	$\theta_{\max} = \frac{R_{\text{specimen}}}{r_C}$	20.5 nm
Angle*	2.1 μrad	Normal	$R_{\text{subaperture}}$	8.4 nm
Stitching	2.8 nm	Normal	$\sqrt{N_{\text{overlap}}}$	8.4 nm
Combined uncertainty				34.1 nm ($k = 1$)

of the subaperture of $R_{\text{subaperture}} = 4\text{ mm}$, it leads to an uncertainty contribution of $u_{\text{distortion}} = 4\text{ nm}$ for the image distortion.

In the measurement, the z - and y -stage were used moving perpendicularly to the x -stage. Both have a standard uncertainty of $u_{\text{straightness}} = 5.8\text{ }\mu\text{m}$ (compare last section for the flat specimen). The uncertainty of the lateral position is calculated as $u_{\text{straightness}}(x) = \sqrt{2} \cdot 5.8\text{ }\mu\text{m} = 8.2\text{ }\mu\text{m}$. The uncertainty contribution for the topography is $u_{\text{lateral}}(z) = 20.5\text{ nm}$.

As this specimen has a small diameter, the integrated angle errors are not considered. Instead, the tilt error of the single subaperture is used as an approximation (see Section 7.2.5). In addition to the two tiltmeters, the autocollimator was used. Each device has a standard uncertainty of $0.25''$, leading to a combined uncertainty of $u(\theta_x) = 2.1\text{ }\mu\text{rad}$. Through a multiplication with the subaperture radius $R_{\text{subaperture}} = 4\text{ mm}$, the uncertainty contribution for the topography is $u_{\theta_x}(z) = 8.4\text{ nm}$.

A profile through the topography consists of ten subapertures and nine overlaps. With a standard error of the residuals of $SE_R = 2.8\text{ nm}$ for the stitching, this leads to a contribution of $u_{\text{stitching}} = 8.4\text{ nm}$.

The contributions are listed in the Table 7.4. The uncertainty for the topography measurement is given by $U(z) = 68.2\text{ nm}(k = 2)$. The value of the measurand then normally lies within the attributed coverage interval with a probability of approximately 95%. The smaller uncertainty in comparison to the flat specimen is mainly due to the smaller diameter.

7.4.3 Chirp structure

For the specimen with the chirp structure, the important uncertainty contributions are equivalent to those of the transmission flat. In addition, the chirp structure has rather large local gradients, so that effects as the alignment of the Fizeau interferometer, the

Table 7.5: Uncertainty budget for the chirp specimen. ("*": Taken into account twice.)

Quantity	Uncertainty	Distribution	Sensitivity coefficient	Uncertainty contribution
Reference flat*	21 nm	Rectangular	1	6.1 nm
Distortion*	8.64 μm	Normal	$\theta_{\max}$	16.3 nm
Lateral position*	5.8 μm	Normal	$\theta_{\max}$	11.0 nm
Integrated angle	6.9 μrad	Rectangular	$\frac{1}{2} D_{\text{specimen}}$	229.1 nm
Stitching	1.61 nm	Normal	$\sqrt{N_{\text{overlaps}}}$	10.7 nm
Combined uncertainty				230.3 nm ($k = 1$)

stages and the errors of the pixel pitch and of other aberrations of the interferometer optics play a role. The maximum surface gradient due to the chirp structure can be calculated by

$$\theta_{\max} = \left[\frac{\partial}{\partial x} \left(A \sin \left(\frac{2\pi}{\lambda_{\text{spatial}}} x \right) \right) \right]_{x=0} = A \frac{2\pi}{\lambda_{\text{spatial}}} \quad (7.22)$$

With a nominal amplitude of $A = 50 \text{ nm}$ and a minimum spatial wavelength that was measured $\lambda_{\text{spatial}} = 167 \mu\text{m}$ (see Fig. 6.20), it gives $\theta_{\max} = 1.88 \cdot 10^{-3}$ (at the maximum at $x = 0$). With this gradient, the estimations for the maximum resulting topography uncertainty can be calculated.

The distortion leads to an uncertainty of $u_{\text{Distortion}}(z) = 16.3 \text{ nm}$. For the y -stage x -straightness error, this leads to an uncertainty of $u_{\text{lateral}}(z) = 11 \text{ nm}$. The integrated angle error is calculated according to Eq. 7.18, also with $1''$ as PV error for each tiltmeter. The specimen diameter is 230 mm. This yields a contribution of $u_{\text{Integrated angle}}(z) = 229.1 \text{ nm}$. The residual standard error from the stitching is $SE_R = 1.61 \text{ nm}$. With 45 subapertures in a row and 44 overlaps, this gives an uncertainty contribution of $u_{\text{stitching}} = 10.7 \text{ nm}$. The contributions are summarized in Table 7.5.

7.4.4 Freeform surface

For this specimen, the accuracy of the rotation and the goniometer stage is the most important uncertainty contribution. The specimen is large and the topography consists of 3130 subapertures and a large number of subapertures in both x - and y -direction. Therefore, it is assumed that all errors are implicitly contained in the stitching error.

The standard error of the residuals is $SE_R = 84 \text{ nm}$. The profile that was compared with the CMM measurement consists of 34 subapertures. This gives an uncertainty of $u_{\text{profile}}(z) = 483 \text{ nm} (k = 1)$.

There are 100 subapertures on the long edge of the specimen, that means 99 overlaps. The resulting uncertainty is $u_{\text{long edge}}(z) = 836 \text{ nm} (k = 1)$.

8 Simulations

Two different measurement systems are simulated in this chapter. First, the presented measurement system and process is simulated. The uncertainty of the topography measurements of the flat, spherical and freeform specimen will be retrieved from Monte Carlo simulations. For the spherical specimen in Section 6.2, the uncertainty of the radius of curvature is also evaluated. Moreover, the simulations are a theoretical verification of the measurement and evaluation method.

Second, the form measurement system under development for specimen up to 1.5 m in diameter will be virtually evaluated in regard of the accuracies that can be achieved depending on the system specifications. The virtual specimens are a flat surface and two paraboloids with different curvature. The paraboloids can represent primary telescope mirror segments.

The simulations focus on the uncertainties of the mechanical components. Concerning the interferometer, the wavelength sweeping interferometry and the optical system are not simulated. However, resulting errors for topography and distance measurement are included, for example a systematic form error of the reference flat.

Lateral position errors are modelled by taking the ideal topography from the target coordinates plus the lateral error. Height errors (for example the systematic form error of the interferometer reference flat) are added to the subtopographies. Absolute distance errors are modelled by giving the according lateral shift to the subtopography. The height component of the absolute distance error is fitted during the stitching and therefore irrelevant. Tilt errors are simulated by adding a linear tilt to the subtopography and applying a lateral shift to the subtopography. This shift depends on the distance of the specimen to the pivot point of the interferometer.

The simulations are a method to calculate the uncertainty of a measurement result according to the supplementary S1 of the Guide to the Expression of Uncertainty in Measurement [21].

8.1 Modelling

This section describes how the virtual measurements are performed, how errors are taken into account, and how large these errors are for the stitching interferometry system from Chapter 5.

8.1.1 Virtual subaperture measurements and errors

The model of the simulation is purely geometric. In the simulation, the input topography to be virtually measured is expressed as a function that gives the z -value of the

topography at the coordinates (x, y) . The positions to be measured are defined as in the real experiment by defining an area containing the virtual specimen and the step size for the subaperture measurements. Also, the aperture size and the pixel pitch are given. At each position that is virtually measured, the z -values of the topography are calculated. The lateral shift of the subapertures due to the rotation of the interferometer is not included in the simulation. The shift is small so that this simplification is not a fundamental difference. The positioning errors of the rotation stages are still taken into account.

Angle errors in general are also included in the simulation. The resulting tilt errors for the topography gradients θ_x and θ_y are considered by adding tilted planes to the ideal subtopography. This is a small angle approximation for the rotation of the subtopography. Also, the lateral displacement of the subapertures due to the angle errors is calculated geometrically. Therefore, the distance from the pivot point of the interferometer to the surface is multiplied with the angle error.

To simulate the lateral errors (for example the straightness of a linear stage or the lateral shift resulting from an angle error), the topography values are not drawn from the target position (x, y) , but from the position including the lateral position errors $(x + \delta x, y + \delta y)$.

For errors that only affect the z -value of the topography (like the systematic reference flat error), the corresponding error is added to the value $z(x + \delta x, y + \delta y)$.

The lateral coordinates, where the topography heights are taken from, are given by

$$x' = x_{\text{target}} + \delta x_{\text{straightness stages}}(y, z) + \delta \theta_x(x) \cdot d_{\text{pivot point}} + \delta x_{\text{random}} \quad (8.1)$$

x_{target} is the target position for the x -position. $\delta x_{\text{straightness stages}}(y)$ is the unknown systematic x -straightness error of the stages at their respective position in y or z . $\delta \theta_x(x) \cdot d_{\text{pivot point}}$ is the tilt error $\delta \theta_x(x)$ of the subtopography, leading to a lateral shift by the multiplication with the distance to the pivot point $d_{\text{pivot point}}$. δx_{stat} is the statistical position error. For the y -coordinate, the equation is obtained by exchanging x and y .

The height values for the data points of the subtopographies can then be calculated as

$$z' = z(x', y') + (\delta \theta_{x, \text{sys}} + \delta \theta_{x, \text{stat}}) \cdot x_{\text{relative}} + (\delta \theta_{y, \text{sys}} + \delta \theta_{y, \text{stat}}) \cdot y_{\text{relative}} + \delta z_{\text{reference flat}}(x_{\text{relative}}, y_{\text{relative}}) + \delta z_{\text{stat}} \quad (8.2)$$

x_{relative} and y_{relative} are the effective pixel positions relative to the centre of the interferometer field of view. $\delta \theta_{x, \text{sys}}$ is the systematic tilt error of the subtopography in x -direction (same for y). $\delta \theta_{x, \text{stat}}$ is the random tilt error of the subtopography (equivalent for y). $\delta z_{\text{reference flat}}$ is the unknown systematic form error of the reference flat. δz_{stat} is the statistical error for each pixel.

The interaction of the lateral errors and the height errors are neglected as the topogra-

phies are only moderately curved. Errors that are removed by the fitting or have no effect anyway are not included in the simulation to keep the model simple. An example would be the z -error of a linear stage.

Statistical errors for the angles and positions are assumed to be normally distributed and independent, so that their standard deviation can be added up according to the laws of error propagation. One value drawn from a normal distribution with the accumulated standard deviations represents the statistical error of several components together.

The systematic errors are modelled by generating an error signal with random walks. One error signal is generated for one relative position dimension, for example for the relative x -position from the specimen to the Fizeau interferometer. If there are several contributions to a value, for example the angle error from two tiltmeters, the PV errors are combined by taking the root of their square sum:

$$PV_{\text{total}} = \sqrt{\sum_{n=1}^{N_{\text{contributions}}} PV_n^2} \quad (8.3)$$

The error signals are then normalized to the total PV value corresponding to the respective systematic error. This means that the PV height is constant for all repetitions of the simulation, but the form changes each time.

A flow chart of the data generation is shown in Fig. 8.1. The virtual measurement data is then evaluated in the same way as the real measurement data.

Unknown systematic form error of the reference flat

The two-dimensional systematic error of the reference flat is generated in three steps. First, a two-dimensional array with random numbers is generated with one value for each pixel of the interferometer image sensor (see Fig. 8.2). Along one dimension, the drawn values are cumulatively summed up. This gives a new array (see Fig. 8.3). Then, the values of the new array are cumulatively summed along the other direction of the array to form the array that describes the surface error (Fig. 8.4). Since every value of the array is a sum of normally distributed values, each value itself belongs to a normal distribution. In the third step, the PV of these values is determined. Each element of the array is then multiplied with $\frac{PV_{\text{nominal}}}{PV_{\text{array}}}$, so that the PV height of the arrays equals the one specified for the reference flat.

In Table 8.1, the estimates for the unknown systematic errors are given. The values for the stages are given for the whole travel range. When only a part of the travel range is used, the PV error for this section is typically smaller as for the whole range.

The straightness errors are based on the manufacturer specifications. The form error of the reference flat is based on the specifications but was also verified in the measurement from Fig. 5.3. The retrace error is not included in the simulation as it is rather small compared to the others (see Table 7.1 for up to ten fringes). The PV value for the autocollimator is taken from the calibration certificate for the whole range of the

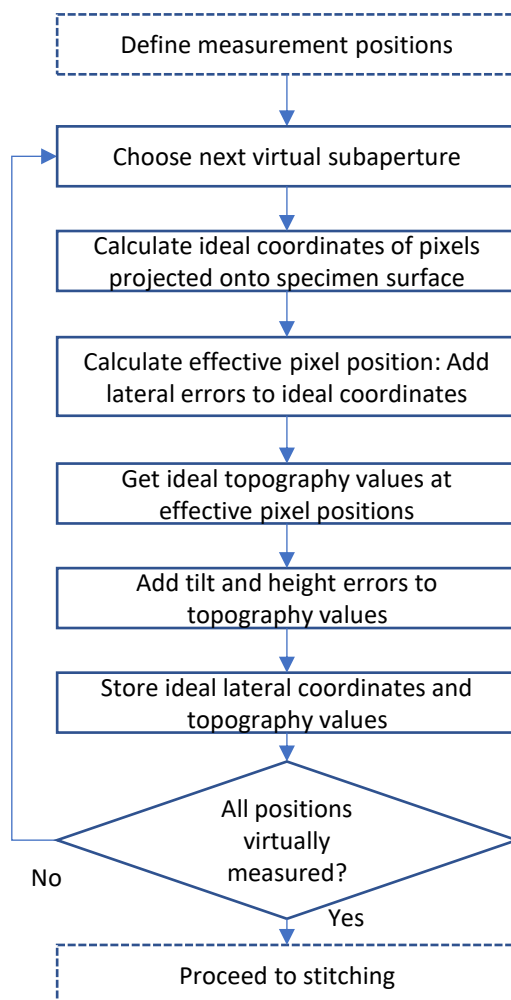


Figure 8.1: A flow chart showing how the virtual measurement data is generated.

expanded uncertainty with $k = 2$. For the tiltmeter the same value is taken as an estimate. When the errors are combined for several measurement devices, they are added according to Eq. 8.3. For example, if only two tiltmeters are used, the PV error for one angle is $\sqrt{2 \cdot (1'')^2}$.

The unknown systematic error of the rotation stages (without using the autocollimator) for the simulation of the car side window will be given in the respective section as they are only used there.

The values for the statistical errors are given in Table 8.1. The values for θ are the tilt errors for all measurement devices combined. They are based on the measurements in Section 5.2 showing that they are achievable. The standard uncertainties for the x - and y -position are also estimated together for all stages. The value for the cavity length measurement is conservatively estimated based on the experiment in Sec. 5.1.1. The

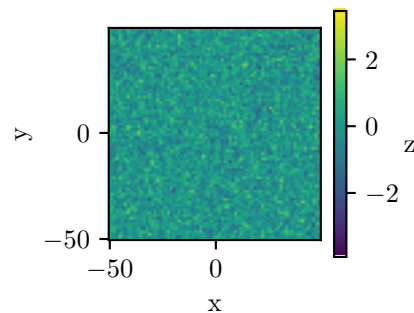


Figure 8.2: Array of normally distributed random numbers with the same number of pixels as the image sensor.

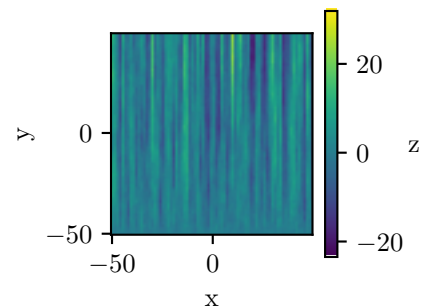


Figure 8.3: Each column is the cumulated sum from the values in Fig. 8.2 from bottom to top.

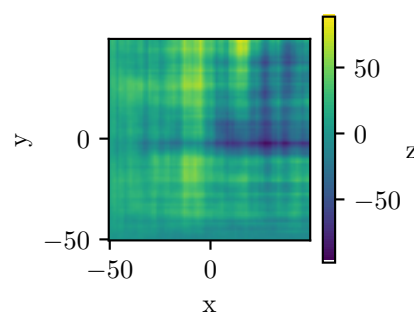


Figure 8.4: The lines are the cumulated sums of the array in Fig. 8.3 from left to right.

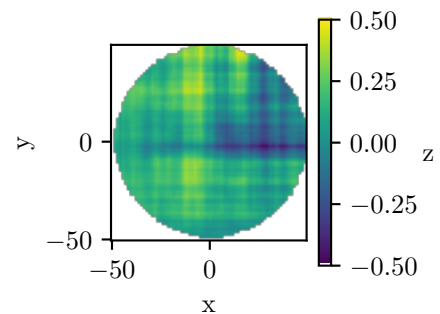


Figure 8.5: One realization of the unknown systematic form error of the reference flat: The aperture of the interferometer is cut out from the array in Fig. 8.4 and normalized to the nominal PV height of the form error (here 1).

repeatability of the Fizeau interferometer is not considered because it amounts to only 0.5 nm. This is small compared to the other contributions.

Table 8.1: Estimates for the individual unknown systematic errors used in the simulations.

Quantity	Value
x -/ y -stage: y -/ x -straightness error	20 μm PV
z -stage: x -/ y -straightness error	20 μm PV
Systematic form error transmission flat	21 nm PV
θ_x, θ_y (Autocollimator/ Tiltmeter)	1'' PV

Table 8.2: Combined standard deviations for dimensional quantities contributing to an error.

Quantity	Value
x -/ y -position	0.1 μm
θ_x, θ_y	0.1 μrad
Cavity length measurement	30 μm

8.2 Virtual experiments

Several goals are pursued with the virtual experiments. First, the measurement principle and the stitching algorithm are verified. Second, the achievable uncertainties with the current setup are estimated. This also gives a comparison with the analytical uncertainty budget from Sec. 7.2. Furthermore, the main uncertainty contributions are identified. A perfect agreement between experiment, simulation and the uncertainty budget is not expected, as many input quantities are estimated and are therefore an approximation of the real properties of the setup. Furthermore, the underlying model for the uncertainty budget is based on assumptions and a simplified version of the simulations. However, the values are expected to be close with the budget overestimating the uncertainty.

The distance from the specimen to the interferometer is chosen as 200 mm, which is approximately 3 mm more than the value in the experiment.

8.2.1 Flat specimen

First, the Monte Carlo simulation for the flat specimen from Sec. 6.1 is performed. The topography is defined as perfect flat with $z(x, y) = 0$. For the flat specimen, a rotation of the Fizeau interferometer with the goniometer and the rotation stage is not necessary. The dominant error source is the systematic error of the reference flat and the angle errors. The pixel pitch is chosen to be 191.5 μm which matches the experiment from Sec. 6.1. The parameters are summarized in Table 8.3. One result for a topography from the simulation is shown in Fig. 8.6.

The straightness error of the stages plays no role as the topography is perfectly flat and therefore a lateral shift does not change the topography. The angle errors of the tiltmeters and the form errors of the reference flat affect the result.

Table 8.3: Parameters for the Monte Carlo simulation virtually measuring the flat specimen.

Number of simulations N	1000
Effective pixel pitch	191.5 μm
Errors	See Table 8.1 and 8.2

Table 8.4: Results of the Monte Carlo simulation for the flat specimen with their sample standard deviation.

Mean standard deviation	Mean 95% coverage interval	Mean peak-to-valley error
$(21 \pm 8) \text{ nm}$	$(37 \pm 13) \text{ nm}$	$(144 \pm 36) \text{ nm}$

The results of the Monte Carlo simulation are summarized in Fig. 8.7 and Table 8.4. The corresponding histograms are shown in Fig. 8.7. The distributions are close to normal distributions but have a slight skewness.

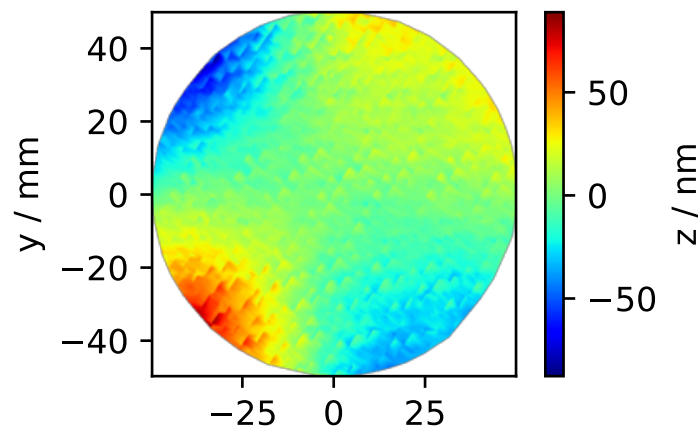


Figure 8.6: This plot shows a typical result of the reconstructed topography of the (virtual) flat specimen. It is also the residual after subtraction of the ideal topography. As it can be seen, the form deviation has components that resemble the astigmatism term of the Zernike polynomials. However, the form of the residual is different for each repetition of the simulation. The high frequency topography error components stem from tilt errors of neighbouring subtopographies.

As the form deviation in Fig. 8.6 is astigmatism-like, the oblique (C_2^{-2}) and vertical (C_2^2) astigmatism terms were evaluated with a Zernike fit. In Fig. 8.8, the absolute value $\sqrt{(C_2^{-2})^2 + (C_2^2)^2}$ was plotted against the orientation of the astigmatism $\frac{1}{2} \arctan\left(\frac{C_2^{-2}}{C_2^2}\right)$ for $N = 1000$ runs of the simulation. There is an accumulation of data points at around $\pm 45^\circ$. This corresponds to the observation of topography errors with an oblique astigmatism in the real experiments.

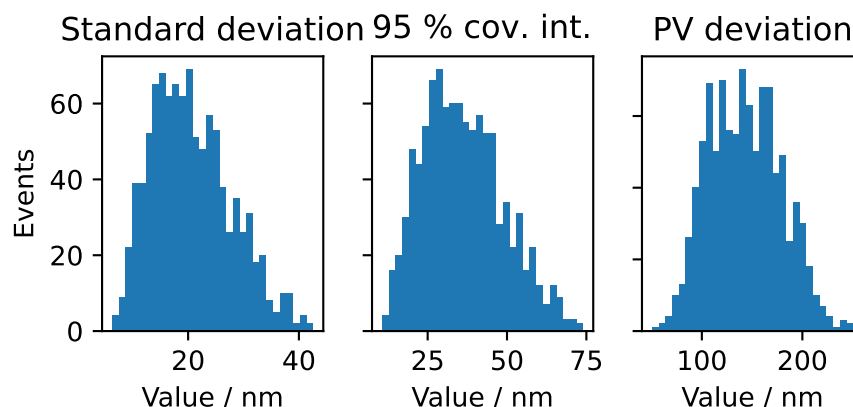


Figure 8.7: Histograms for 1000 repetitions of the simulation for the virtual flat specimen. The 95% coverage interval value is absolute value the upper and lower border. The interval is determined as the symmetric interval around the target topography containing the respective fraction of data points.

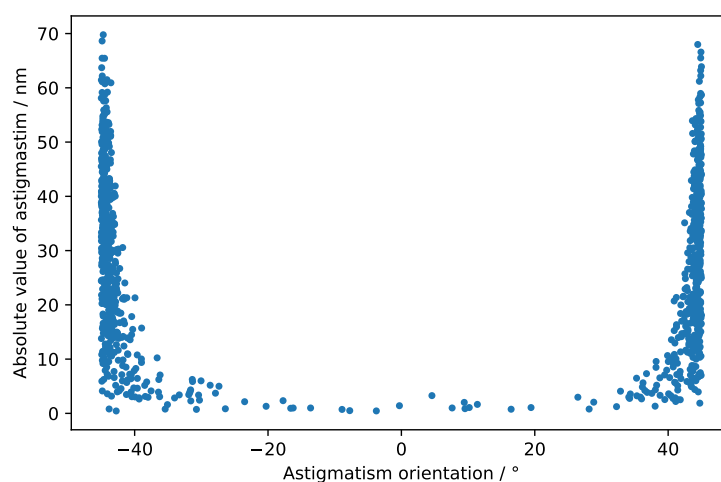


Figure 8.8: Absolute value and orientation of the astigmatism terms for 1000 repetitions of the simulation for the virtual flat specimen.

The virtual experiment was repeated without the systematic angle errors. The virtually measured topography is shown in Fig. 8.9. It has a RMS deviation of 3.5 nm. The overall form error has vanished, while only local form deviations remain. A conclusion of these simulation is, that systematic angle errors lead to topography errors in the form of an oblique astigmatism after stitching.

This implies, that the measurement of the transmission flat in Chap. 6.1 has a topography error also caused by a systematic angle error. There are at least two ways to overcome this effect. First, the remaining systematic errors of the angle measurements can be minimized and calibrated. The alignments of the angle measuring devices to the

coordinate system are also relevant. This approach will nevertheless reach a physical limit, for example due to the long-term stability of the measurement devices. Second, further independent variables can be introduced into the stitching algorithm, that describe systematic angle errors for all angle measuring devices, and their alignment (for example the alignment of a tiltmeter to the coordinate system). This improvement suggestion applies to all following simulations.

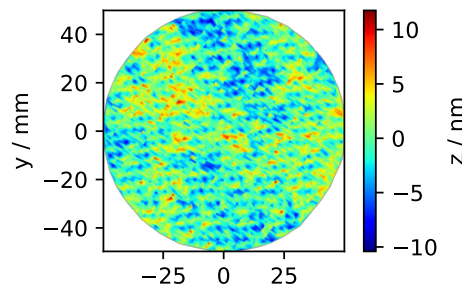


Figure 8.9: Repetition of virtual measurement of the transmission flat without systematic angle errors. Only local measurement errors remain.

8.2.2 Spherical specimen

Now, the measurement of the sphere specimen from Sec. 6.2 is simulated. The input topography of the spherical surface is described by a paraboloidal approximation of the ideal spherical function:

$$z(x, y) = \frac{x^2 + y^2}{2r_C} \quad (8.4)$$

with the radius of curvature $r_C = 10$ m.

In addition to the simulation with the flat specimen, the unknown systematic errors from the autocollimator are taken into account as stated in Table 8.1, and according to Eq. 8.3. The main source of error for this specimen are the angular errors. The parameters for this simulation (see Table 8.5) are again chosen in accordance with the experiment (Sec. 6.2) as shown in Table 8.1 and 8.2. The fitting of the alignment parameter for the roll angle of the autocollimator is not included in the simulation. Implicitly, a perfect alignment or a perfect determination of the alignment parameter is assumed. An exemplary topography and its difference to the input topography is shown in Fig. 8.10 and Fig. 8.11. The results for this Monte Carlo simulation are shown in Table 8.6 and in Fig. 8.12.

The topography deviation is smaller compared to the virtual flat specimen, because the diameter is smaller. The autocollimator brings additional uncertainty contributions, but they are small.

Also, the radius of curvature was determined with a best-fit sphere just as in the experimental section. The 95% coverage interval around the mean value of 10 m is given

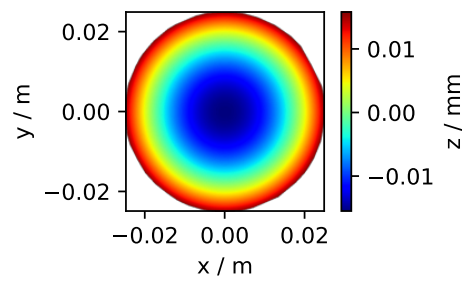


Figure 8.10: Reconstructed topography of the virtual spherical specimen for one cycle of the Monte Carlo simulation.

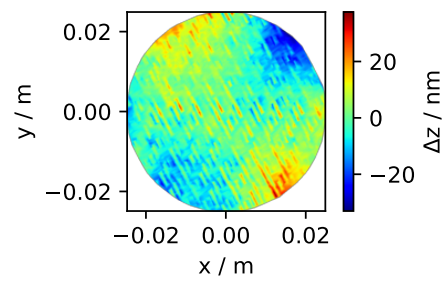


Figure 8.11: Reconstructed topography minus input topography. Offset and tilts are subtracted. The form depends on the error signal created for the systematic errors.

Table 8.5: Parameters for the Monte Carlo simulation virtually measuring the spherical specimen.

N	1000
Effective pixel pitch	191.5 μm
Errors	See Table 8.1 and 8.2

by $[-7.4\text{ mm}, 7.4\text{ mm}]$. The standard deviation of the radius of curvature is 4.6 mm.

Table 8.6: Results of the Monte Carlo simulation for the topography of the spherical specimen.

Mean standard deviation	Mean 95% coverage interval	Mean peak-to-valley error
$(14 \pm 5)\text{ nm}$	$(25 \pm 8)\text{ nm}$	$(101 \pm 24)\text{ nm}$

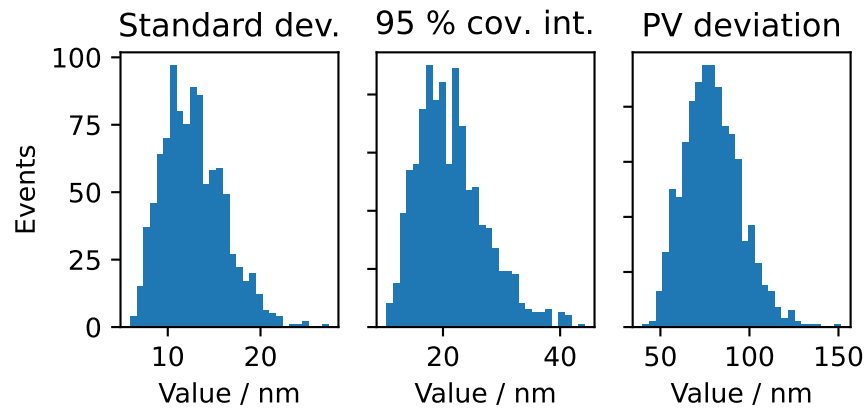


Figure 8.12: Histograms for 1000 repetitions of the simulation for the virtual spherical specimen. The 95% coverage interval value gives the absolute value of the upper and lower border for the symmetric 95% interval around the ideal topography.

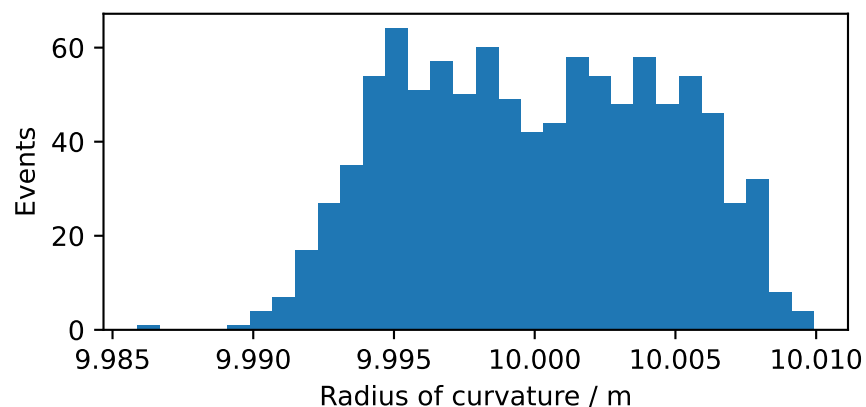


Figure 8.13: This histogram shows the radius of the best-fit spheres for the virtually measured topographies in 1000 simulations.

8.2.3 Freeform surface

For the simulation of the freeform surface (the car side window), the measured topography is used as the ideal virtual input topography. The measurement data is sampled as input topography with a spacing of 1 mm in x -direction and 390 μm in y -direction and interpolated with cubic splines. The topography is virtually measured and the different errors are added to the subapertures topography to determine the deviation of the output topography from the input topography.

For this specimen, the systematic angle errors of the rotation stages are important contributions. Instead of the autocollimator, the rotation encoders were used in the measurement. They are not as accurate as the autocollimator. The goniometer (chang-

Table 8.7: Parameters for the Monte Carlo simulation virtually measuring the freeform specimen.

N	302
Effective pixel pitch	191.5 μm
Errors	See Table 8.1 and 8.2
Unknown systematic errors for the rotation stages (PV)	
Rotation stage (rot-y)	15mdeg for 360° (nominal) 1.98 μrad for 2.7° (estimated)
Goniometer stage (rot-x)	50mdeg for 90° (nominal) 51.3 μrad for 3.6° (estimated)
Statistical errors for the rotation stages	
θ_x and θ_y	0.1 μrad (1σ)

Table 8.8: Results of the Monte Carlo simulation for the topography of the car side window.

Mean standard deviation	Mean 95% coverage interval	Mean peak-to-valley error
(439 $\pm$ 146) nm	(756 $\pm$ 249) nm	(3054 $\pm$ 666) nm

ing θ_y) and the rotation stage (changing θ_x) have different absolute accuracies. The goniometer is less accurate. The accuracies are given by the manufacturer for the complete travel range. As an estimate, they are linearly scaled down to the travel range used. An overview over the parameters used in the simulation is given in Table 8.7.

One reconstructed topography can be seen in Fig. 8.14, and the deviation to the input topography in Fig. 8.15. The statistical results are summarized in Table 8.8.

8.3 Comparison of the uncertainties from the uncertainty budget and the simulation

Two approaches were used to estimate the uncertainty: The analytical uncertainty budget based on a simplified model of the measurement and stitching process in Chap. 7, and the Monte Carlo simulations in Chap. 8. Furthermore, they can be compared with experimental results. In the following section, the comparisons are summarized.

For the 4-inch transmission flat as specimen, the results are summarized in Table 8.9. The experimental values are for the difference between the stitching interferometry and the single aperture measurement. The experimental values are a bit smaller than the values from the simulation. The uncertainty budget value is in the same order of magnitude but overestimates the values from the simulations by a factor of about two

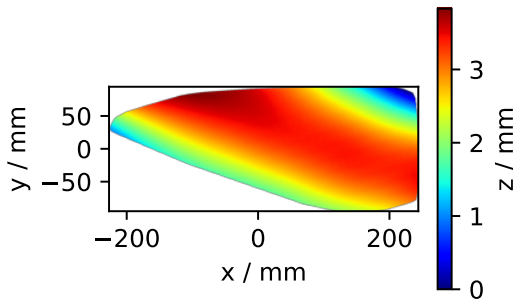


Figure 8.14: Reconstructed topography from the virtual measurement of the car’s side window.

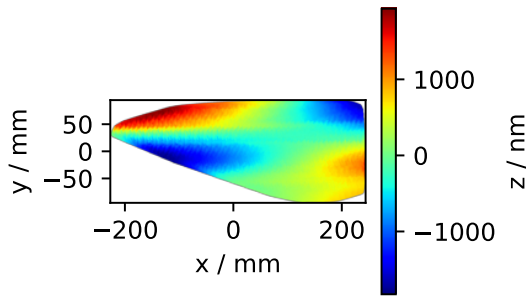


Figure 8.15: Reconstructed topography minus input topography. The form of the residual changes for every repetition.

Table 8.9: Comparison of measurement, simulation and estimated uncertainty for the flat specimen.

Deviation	Experimental	Simulation	Uncertainty budget
Standard deviation	18.3 nm	(21 ± 8) nm	52.2 nm
95% coverage interval	37.1 nm	(37 ± 13) nm	104.4 nm

to three.

For the spherical specimen, the values of simulation and uncertainty budget are compared in Table 8.10. Here, too, the uncertainty budget overestimates the simulation by a factor of approximately two to three. Next, the results for the radius of curvature from uncertainty budget and simulation are compared (see Table 8.11). The coverage intervals and standard deviation of simulation and uncertainty are close to each other. The standard deviation of the simulation is larger, while the coverage interval is smaller. This can also be comprehended on the basis of the different form distributions in Fig. 7.10 and 8.13 for the radius of curvature.

The reason for the different distributions lies within the underlying models. In the simulation, unknown systematic and statistical errors were separately modelled. For the simpler 2D model for the uncertainty estimation from the budget, systematic errors were converted to normal distributions according to GUM 1 (see Appendix Sec. 11.1.3).

Table 8.10: Comparison of measurement, simulation and estimated uncertainty for the spherical specimen.

Deviation	Experimental	Simulation	Uncertainty budget
Standard deviation	-	(13 ± 4) nm	34.1 nm
95% coverage interval	-	(22 ± 6) nm	68.2 nm

Table 8.11: Comparison of measurement, simulation and estimated uncertainty for the radius of curvature of the spherical specimen.

Deviation	Simulation	Uncertainty budget
Standard deviation	± 4.6 mm	± 2.9 mm
95% coverage interval	± 7.4 mm	± 9.0 mm

Table 8.12: Comparison of measurement, simulation and estimated uncertainty for the whole freeform specimen (car side window).

	Experimental	Simulation	Uncertainty budget
Profile with 34 subapertures:			
Standard deviation	-	(439 ± 146) nm	836 nm
95% coverage interval	-	(756 ± 249) nm	1672 nm
Whole specimen:			
Standard deviation	370 nm	-	483 nm
95% coverage interval	645 nm	-	966 nm

Then the error signal was modelled with a random walk with normalized height to a value drawn from a normal distribution.

In Table 8.12, the results for the freeform surface (car side window) are shown. The main uncertainty contribution for this measurement is apparently the absolute accuracy of the goniometric stage (see Table 8.7).

For the profile of the freeform specimen, the standard uncertainty is about 100 nm larger than the RMS difference between CMM and interferometric measurement. It is a good agreement that the RMS difference lies within the standard uncertainty. The profile was also measured in direction parallel to the plane of rotation of the goniometric stage. It is also important to note, that these values might be closer because the CMM measurement has a large uncertainty itself. For the whole specimen, the uncertainty is calculated with $N = 99$ overlaps for the long side of the specimen is most likely overestimated. It amounts to about two times the uncertainty from the simulation. A distinction of the stitching residuals for the overlap in x - and y -direction was not made for the uncertainty budget. However, the simulation takes the directional dependence of the component accuracies into account.

8.4 Simulation of a measurement system for specimens up to 1.5 m in diameter

In this chapter, the large form measurement system being set up at PTB is simulated. The measurement and evaluation methods presented in this work will be applied. One of the main optical sensors will be a Fizeau interferometer with an aperture diameter of

100 mm.

A photograph of the positioning system (manufacturer: *LT Ultra Precision Technology GmbH*) is shown in Fig. 8.16. The linear x - and y -stages are air-bearing and have a travel range of 1.5 m. Two rotation stages and a mechanical z -stage with 100 mm travel range are attached to the gantry stage (y -stage). In comparison to the setup in Chap. 5, the goniometer was replaced with a rotation stage. Also, the specimen table on the y -stage can be rotated with an additional third rotation stage. The positioning system can be equipped with two tiltmeters on x - and y -stage, equivalent to the presented setup, and an optical angle measurement device on the y -stage for the angles of the rotation stages if necessary. The positioning system is set up in an air-conditioned box to achieve constant environmental conditions.



Figure 8.16: Positioning system of the form measurement system for large specimens at PTB.

The goal of the simulations is to estimate the achievable accuracies. It is assumed that the specimen surface is 400 mm away from the pivot point. The final distance is not yet set and depends on how the interferometer will be mounted. The values for the systematic and statistical errors in the simulation are given in Table 8.13 and 8.14. The absolute positioning accuracy of the rotation stages is specified as $\leq 10''(\text{PV})$ and the repeatability as $\leq 5''(\text{PV})$. Both rotation stages have high accuracy rotation encoders with a resolution $\leq 0.01''$. Therefore it is assumed, that the stages can be calibrated with an external angle measurement device to achieve better accuracies. Two scenarios are examined for each specimen with systematic angle errors of $1''$ and $0.1''(\text{PV})$ for the travel range used and for all contributions to the tilt error combined. The statistical

Table 8.13: Systematic errors for the interferometric measurement system for large specimens up to 1.5 m in diameter, according to manufacturer specifications.

Quantity	Value
x straightness error	5 μm PV
y straightness error	10 μm PV
z straightness error	10 μm PV
Form error reference flat Fizeau	21 nm PV

Table 8.14: Estimated statistical errors for the simulation of the large form measurement system for all contributions combined.

Quantity	Statistical uncertainty (1σ)
x -/ y -position	0.1 μm
Gradients θ_x, θ_y	0.05 μrad
Cavity length measurement	30 μm

errors are estimated as 0.05 μrad . This is lower than before because the environmental conditions will be more stable and the stages are more accurate.

8.4.1 Large flat specimen

A flat specimen with a spherical form component of 100 nm peak-to-valley is the first specimen to be virtually measured. The spherical component of the input topography has a radius of curvature of 2812500 m. The statistical and systematic errors from Table 8.13 and 8.14 are applied. The rotation stages are not used in this virtual experiment, so the range of the assumed systematic angle errors can be completely filled by the errors of the tiltmeters.

The lateral step size between measurement positions is 70 mm, the effective interferometer aperture is 95 mm. This results in 22 steps in x - and y -direction each. An effective pixel pitch of 2 mm is set because for this flat specimen not all of the pixels are necessary for an accurate stitching. $N = 1000$ repetitions are performed for the Monte Carlo simulation. In Fig. 8.17, one topography with a systematic angle error of 0.1'' is shown. Figure 8.18 and 8.19 show the residuals of two reconstructed topographies for a systematic angle error of 0.1'' and 1''. The statistics for the deviations of input topography and reconstructed topography are shown in Fig. 8.20 and Table 8.21. A standard deviation of 32 nm and 280 nm respectively are achieved for systematic angle errors of 0.1'' and 1'' (PV) respectively.

In Table 8.15, the results for the two different unknown systematic angle errors are summarized. The increase of the systematic error from 0.1'' to 1'' (PV) also strongly increases the error of the reconstructed topography. For the systematic error of 0.1'', the topography is visible, but partially overlaid by an astigmatic error. The sensitivity to

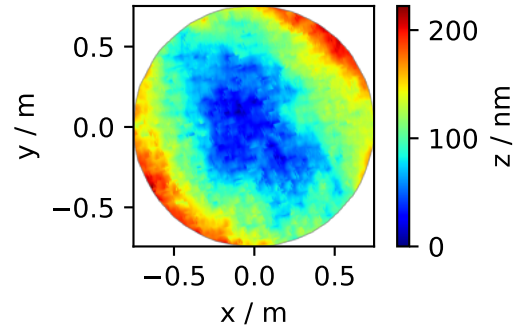


Figure 8.17: Reconstructed virtual topography for a systematic angle error of $0.1''$ PV.

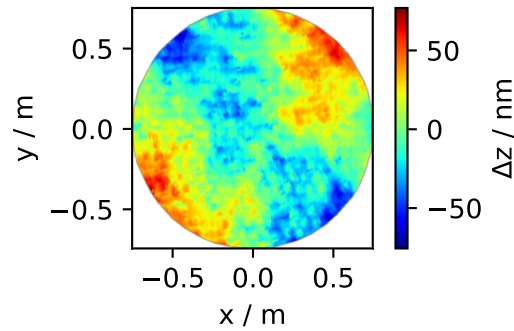


Figure 8.18: Residuals of a reconstructed topography minus the input topography for a systematic angle error of $0.1''$ PV.

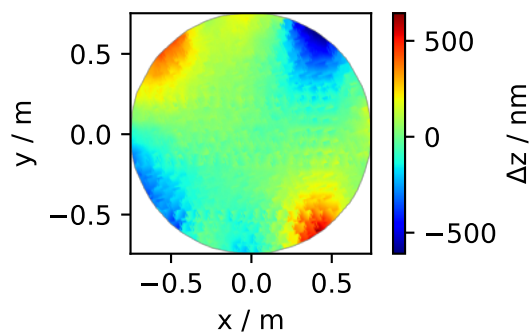


Figure 8.19: Residuals of a reconstructed topography minus the input topography for a systematic angle error of $1''$ PV.

the angle errors can be understood by the fact that the angle errors have a large lever which is the subaperture radius. The minimization of this error is therefore critical to achieve small topography uncertainties at a fraction of the wavelength. Of course, this also applies to the statistical angle errors. As the gradient of the topography is almost

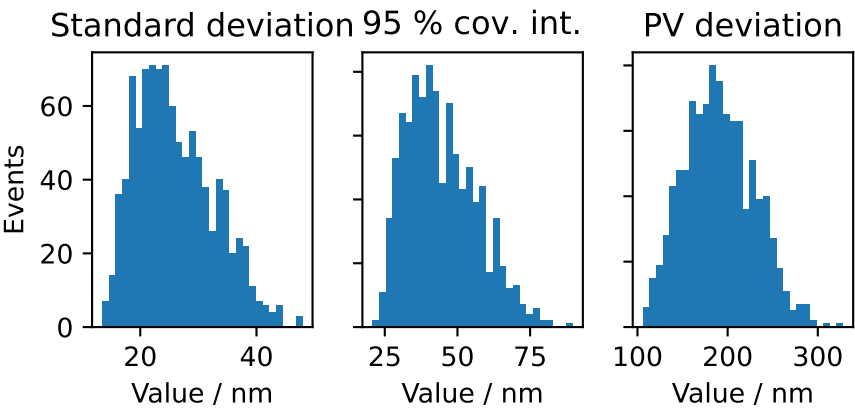


Figure 8.20: Deviation from input to output topography for the flat specimen for 1000 simulations with 0.1'' (PV) systematic angle error. The value of the 95% interval is the absolute value of the upper/ lower interval limit.

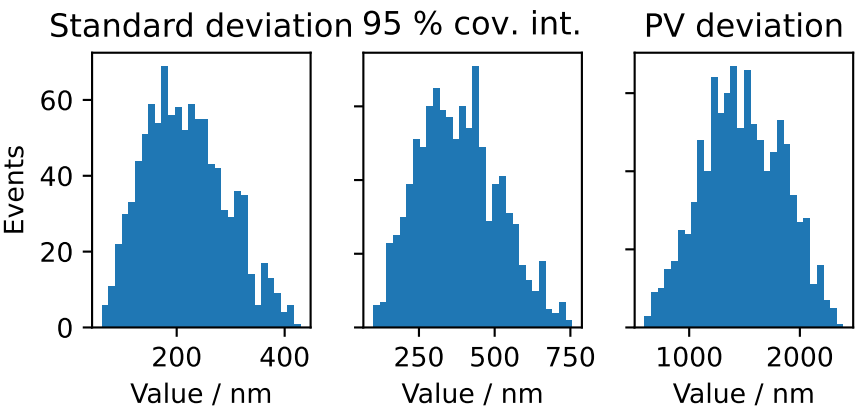


Figure 8.21: Deviation from input to output topography for the flat specimen for 1000 simulations with 1'' (PV) systematic angle error.

Table 8.15: Statistics for the deviation of the reconstructed topography to the input topography for the virtual flat specimen with 1.5 m diameter.

Deviation	Systematic angle error (PV)	
	0.1''	1''
Standard deviation	(26 ± 7) nm	(217 ± 76) nm
95% coverage interval	(45 ± 13) nm	(381 ± 133) nm
PV deviation	(191 ± 39) nm	(1482 ± 361) nm

zero in the lateral directions, lateral positioning errors do not play a crucial role. Also, the absolute distance measurement error is effectively irrelevant since the interferometer

only measures in z -direction.

As for the virtual measurement in Chap. 8.2.1, this virtual experiment was repeated without the systematic angle errors. The virtually measured topography is shown in Fig. 8.9. The RMS deviation from the input topography is 13.2 nm. As before, only local form deviations remain, showing that the minimization or fitting of the systematic angle errors is a necessary step to achieve a high accuracy measurement is possible.

Given the long-term drifts of about 5 μ rad for the tilt-meters and the autocollimator used in the real measurements (see Chap. 5.4.2), a calibration of the large system using the existing angle measuring devices seems not feasible to achieve angle errors $< 0.1''$. Therefore, only an adaption of the stitching algorithm including systematic errors seems to be a possible way to achieve a measurement uncertainty well below a wavelength. In

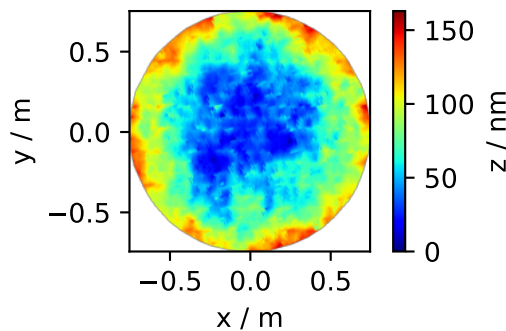


Figure 8.22: Repetition of virtual measurement of the flat with spherical component from Fig. 8.17 without systematic angle errors. The spherical form is now clearly visible.

the next Sec. 8.4.2, a non-flat specimen is virtually measured.

8.4.2 Paraboloid with 70 m in radius of curvature

The virtual specimen (or input topography) is a paraboloid with a vertex radius of curvature of 70 m. This form resembles the primary telescope mirror segment as described in Sec. 2.1.1. The used angular range of the rotation stages is $\approx \pm 0.62^\circ$. Hence, it is too large for a typical autocollimator. In the simulation, the rotation encoders of the positioning system are used to measure the rotations. (Another possible solution is the angle measurement device presented in Section 5.5.2 after some further development.)

The Fizeau interferometer has an aperture diameter of 4 inch. For this experiment, the height of the specimen is given by

$$h(r) = \frac{r^2}{2r_C} \quad (8.5)$$

where r is the radial coordinate. At a wavelength of $\lambda = 633$ nm, there are $n_{\text{fringes}} = \frac{h(r_{\text{aperture}})}{\lambda/2} \approx 113$ circular fringes present. The fringes can be resolved by a typical inter-

ferometer, but retrace error must be corrected if necessary. For this simulation, it is assumed that this is the case.

The Monte-Carlo simulation of the system will be performed and analysed with $N = 500$ repetitions. The pixel pitch for the stitching is also 2 mm and the effective interferometer aperture is 95 mm. The subaperture step size is 70 mm. The uncertainties are the same as in the last chapter. The systematic angle errors are now interpreted as the combined error of tiltmeters and rotation stages. For a curved specimen, also the lateral position errors will have an effect on the topography.

Figure 8.23 shows the reconstructed topography with a systematic angle error of $0.1''$ with tilts subtracted. In Fig. 8.24 and 8.25, the difference of the reconstructed and the input topography for the two different systematic angle errors are shown. The statistical evaluation of the Monte Carlo simulation can be found in Fig. 8.26 and 8.27. The deviations are summarized in Table 8.16.

The topography deviations strongly scale with the systematic angle error. This shows that the angle still is the main uncertainty contribution. In comparison to the flat specimen (compare Table 8.15), the topography errors are barely larger. As mentioned before, the lateral errors scale with the gradients of the topography and increase for a more strongly curved specimen. The gradient is, however, still very moderate. The systematic angle errors of the subapertures are added to the ideal topography but they do not scale with the slope of the specimen. Therefore, they are in principle the same for the flat and curved specimen when the same PV angle error is assumed. In a real experiment, the angle accuracy will most likely be worse if a larger angular range of the rotation stages or angle sensors is used. To conclude, with a minimization and/or fitting of the systematic angle errors it will be possible to achieve small measurement errors for the paraboloid with $r_c = 70$ m. As for the flat specimen in Chap. 8.4.1, a RMS measurement error well below a wavelength seems possible, if the stitching can be adapted to minimize the remaining systematic angle errors to below $0.1''$.

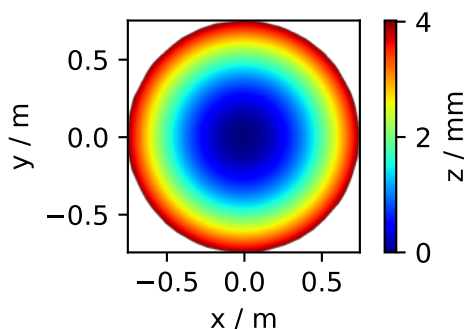


Figure 8.23: Reconstructed topography of the paraboloid with $r_c = 70$ m.

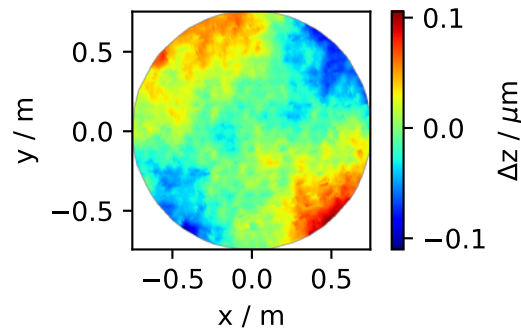


Figure 8.24: Virtually measured topography minus input topography for a systematic angle error of $0.1''$.

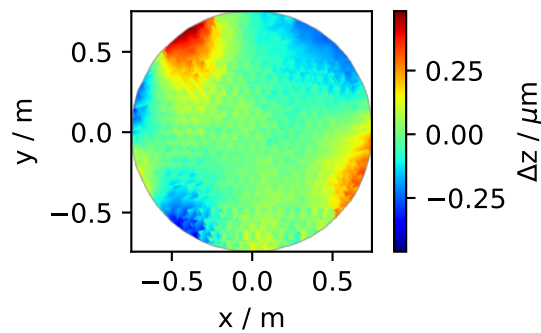


Figure 8.25: Reconstructed topography minus input topography for a systematic angle error of $1''$.

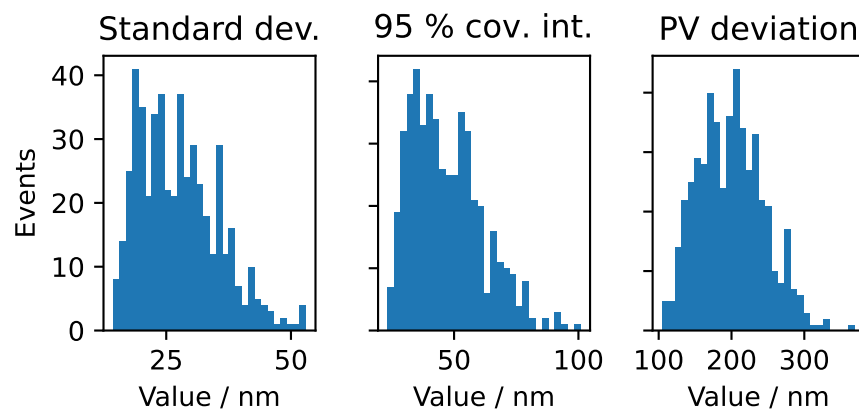


Figure 8.26: Statistics of the topography errors for a systematic angle error of $0.1''$ for the paraboloid with $r_c = 70$ m.

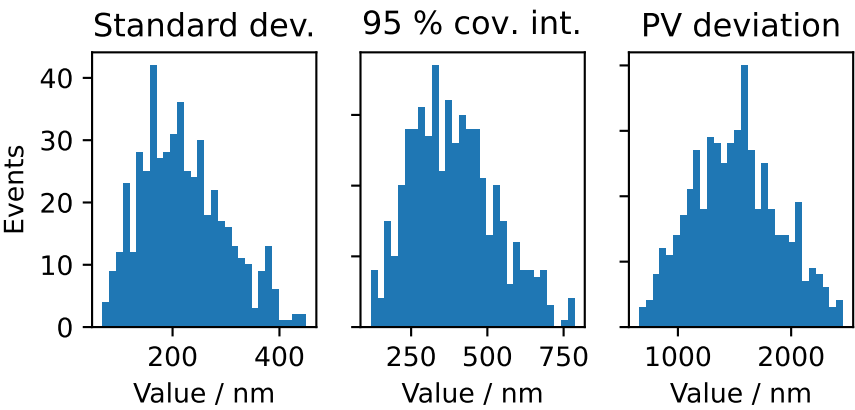


Figure 8.27: Statistics of the topography errors for a systematic angle error of 1'' for the paraboloid with $r_c = 70$ m.

Table 8.16: Comparison of the reconstructed topography errors for the paraboloid with $r_C = 70$ m and 1.5 m in diameter.

Deviation	Systematic angle error (PV)	
	0.1''	1''
Standard deviation	(28 ± 8) nm	(221 ± 79) nm
95% coverage interval	(48 ± 15) nm	(388 ± 139) nm
PV deviation	(200 ± 46) nm	(1513 ± 385) nm

8.4.3 Paraboloid with 10 m radius of curvature

A paraboloidal specimen with 1.5 m in diameter and 10 m in radius of curvature is at the limits of specifications for the specimen given in Sec. 3. The maximum rotations of the Fizeau interferometer are $\pm 4.3^\circ$ in tip and tilt direction. The subaperture step size, the pixel pitch and the interferometer aperture diameter are the same as in the simulation of the paraboloid with $r_C = 70$ m.

Several assumptions are made about the Fizeau interferometer. First, it is assumed that it can laterally resolve the interference fringes. At the edge of an aperture with a radius of 50 mm, this could be reached by having a effective pixel pitch of approximately 30 μ m projected on the specimen surface. This equals 0.47 fringes or 150 nm height difference per pixel at 633 nm and $r_C = 10$ m. Then, the Nyquist theorem is fulfilled. With an equidistant pixel pitch, this would require a sensor with at least 3334 pixels in each direction. Furthermore, the retrace error would have to be corrected sufficiently. For the simulation, it is presumed that the assumptions are fulfilled. In practice, this is a challenging task.

All parameters for the Monte Carlo Simulation are the same as for the paraboloidal specimen in the last section, with exception of the radius of curvature. The reconstructed

topography can be seen in Fig. 8.28. Examples for the difference of the input and reconstructed topography are shown in Fig. 8.29 and Fig. 8.30. The statistical results of the Monte Carlo simulation are presented in Table 8.17.

In comparison with the paraboloid with $r_c = 70$ m in Sec. 8.4.2, the deviations of the reconstructed topography from the input topography are larger, especially for the smaller systematic angle error of $0.1''$ PV. Since only the radius of curvature was changed while all errors are the same as in the last Section 8.4.2, the increase of the error can be solely attributed to the increase of radius of curvature. The reason is that topography errors caused by lateral position errors scale with the radius of curvature. In addition, it is obvious that it is more complicated to achieve the same accuracy of the angle measurement for a smaller radius of curvature than for a larger one. Compared to

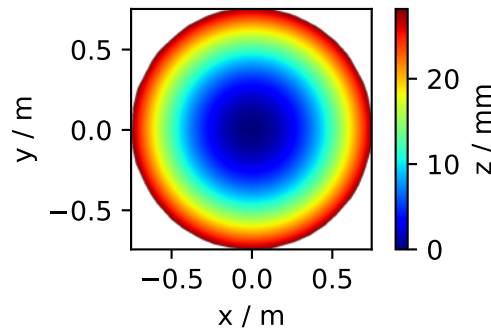


Figure 8.28: A reconstructed topography of the paraboloid specimen with $r_C = 10$ m for a systematic angle error of $0.1''$.

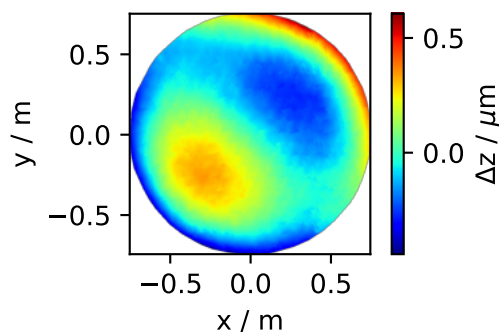


Figure 8.29: Topography from Fig. 8.28 with the input topography subtracted (systematic angle error $0.1''$).

the specimen with $r_C = 70$ m, the topography errors only change by a fraction of their value between the two different systematic angle errors. The only difference between the virtual experiments is the radius of curvature of the specimen. The maximum gradient of the specimen has increased by a factor of seven according to Eq. 11.5. The topography

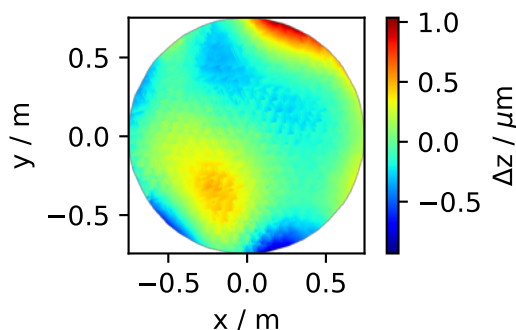


Figure 8.30: Topography with the input topography subtracted for a systematic angle error of $1''$ (PV).

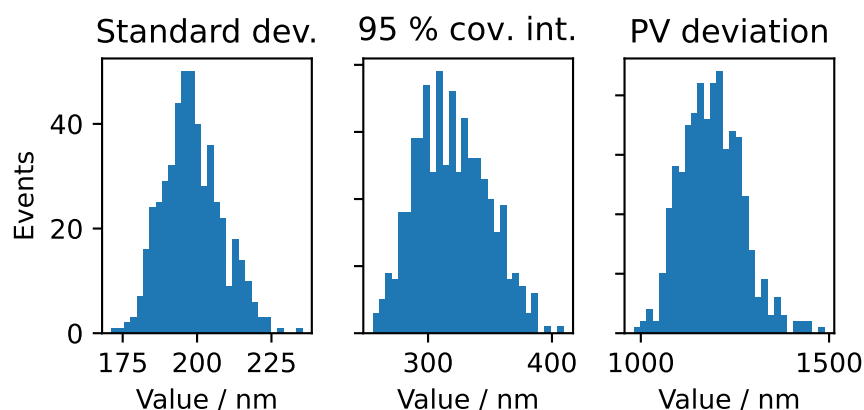


Figure 8.31: Statistics for the topography errors for $r_C = 10$ m and a systematic angle error of $0.1''$ PV.

errors due to the lateral position errors have reached the same order of magnitude as the ones due to the angle error. As expected, the lateral positioning accuracy is critical with large surface gradients. This means, that at the highest gradients of $r_C = 10$ m, a correction of the systematic angle errors alone would not be sufficient to achieve a small topography error, in contrast to the paraboloid with $r_C = 70$ m. Therefore, achieving a measurement uncertainty much smaller than the wavelength appears to be beyond the capabilities of the system for the given specifications.

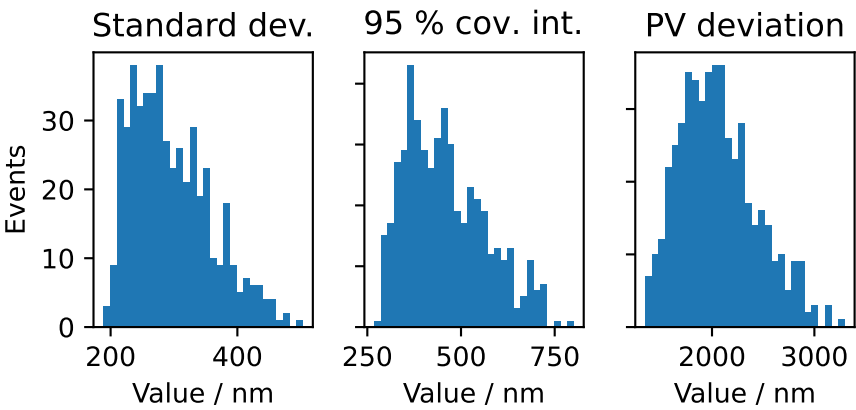


Figure 8.32: Statistics for the topography errors for $r_C = 10\text{ m}$ and a systematic angle error of $1''$ PV.

Table 8.17: Comparison of the reconstructed topography errors for the paraboloid with $r_C = 10\text{ m}$ and 1.5 m in diameter.

Deviation	Systematic angle error (PV)	
	0.1''	1''
Standard deviation	(199 ± 10) nm	(296 ± 63) nm
95% coverage interval	(320 ± 29) nm	(461 ± 108) nm
PV deviation	(1189 ± 80) nm	(2060 ± 376) nm

9 Summary and discussion

Within this work, a stitching interferometry measurement system has been set up that is capable of measuring true freeform optical surfaces for optics with a lateral size of up to 200 mm x 1000 mm. The lowest radii of curvature measured were about 2 m. The strength of the developed setup is the absolute form measurement because absolute distance interferometry and high accuracy angle measurements are applied whereas the tilts of the subapertures are not fitted. Several specimens were measured to demonstrate different features of the measurement system. The flat specimen was a proof of principle for the stitching interferometry with the tiltmeters as flatness reference for the linear stages. The spherical specimen was used to demonstrate the feasibility of the form measurement of curved specimen with an external high accuracy angle measurement. Measuring the chirp structured specimen, the ITF of the system was examined. The ITF decreases for smaller spatial wavelengths due to the lateral resolution of the interferometer optics. The resolution limit was not reached with the chirp structure specimen used in the experiments. In the region of overlap between the subapertures, the lateral position accuracy of the data points can also become a limiting factor when the system is poorly aligned. The measurement of the car side window demonstrated the capability of the setup to measure freeform surfaces. The comparison with a CMM measurement agreed well with the interferometric measurement. This measurement also demonstrated the possibility to measure and process a high number of subapertures (> 3000).

In comparison to the *traceable multiple sensor method* - from which the measurement method is derived, among others -, the new method provides an areal measurement instead of a profile measurement. The maximum slope has been greatly extended, as the TMS can only measure nearly flat specimen. The self-referencing of the reference flat of the TMS was not adapted yet, as other error contributions, especially the angle errors of the stage, are larger uncertainty contributions.

An uncertainty budget has been set up for the topography measurement according to GUM [20]. A model equation for a profile through the centre of a stitched topography was found, based on which an uncertainty analysis was carried out. It works ideally for measuring systems and specimens with symmetrical properties. With non-symmetrical properties, the uncertainty can be investigated for different profiles of the specimen. In comparison to the full simulation, the budget shows an overestimation of the standard uncertainty and the expanded uncertainty ($k = 2$) by a factor of approximately two.

The height residuals between stitched neighbouring subapertures are ambiguous. The residuals contain other uncertainty contributions that cause the subapertures to not fit well. Therefore, they can in principle also be a measure for other errors, but this has to be proved. For the large freeform surface specimen, the uncertainty was approximated with the stitching residuals. It remains to be shown that this error is mainly equivalent to the

integrated angular error. The analytic uncertainty budget overestimates the topography uncertainty by an approximate factor of 2 in comparison to the simulation. This is the same behaviour as for the analytic uncertainty budgets for the other specimens measured. It indicates that an estimation of the uncertainty on basis of the stitching residuals alone is possible, as the residuals result from the errors of the measurands contributing to the stitched topography.

The uncertainty of radius of curvature was derived from the topography uncertainty with a simple 2D Monte-Carlo simulation. The topography uncertainty error was chosen as the standard deviation of a Gaussian distribution from which the PV value of a random walk is drawn. The random walk represented the integration of errors. This method retrieves realistic values that, however, are not normally distributed. The results are nearly independent of the spatial resolution and number of data points, as long as they are chosen in a reasonable range. This shows the suitability to model integrated errors. Statistical errors that are not integrated (e.g. pixel noise) can also be added in the future. Here, it was not modelled as they play a minor role.

In comparison with the 3D Monte Carlo simulations (which are summarized in the next paragraph afterwards), the simplified radius of curvature uncertainty model showed to be a reasonable 2D approximation. It leads to very close expanded uncertainty intervals ($k = 2$). However, the resulting radius values have a different underlying probability density distribution than the results from the 3D simulation. The different distributions result from different modelling of the systematic errors. In the large simulation, the manufacturer tolerances were realized as a constant PV value for error signals generated by the random walk. In the simplified 2D determination of the radius uncertainty, the tolerances were converted into a standard deviation according to the *GUM* and combined quadratically. From the resulting normal distribution, the PV value of the random walk was drawn, leading to error signals with different PV heights.

The Monte Carlo simulation according to the GUM supplement S1 is a more detailed modelling of the process in comparison of the analytic uncertainty budget. The uncertainty contributions were considered in the virtual subaperture measurement data. From there on, the same evaluation as in the experiments was carried out.

The uncertainties determined from the uncertainty budget and the simulation lie close together as can be seen in Sec. 8.3. However, the uncertainty budget tends to overestimate the uncertainty by a factor two. This makes sense as the uncertainty budget is an upper limit estimated.

The deviations observed in the experiments between different types of topography measurements were always smaller than, but close to, the uncertainties obtained from the simulation. This shows that the simulation represents the measurement system and the measurement process well.

Finally, the measurement system under development for specimens up to 1.5 m in diameter was presented in Sec. 8.4 and simulated. As the simulations and further considerations show, the developed stitching method can be transferred to a larger mea-

surement system. The most important result of the simulations is that the remaining systematic errors must be smaller than $0.1''$. Since this is a very challenging requirement at the current state of the art, it makes sense to include the unknown systematic tip/tilt angle errors of the angle measuring devices as independent variables in the stitching to further minimize the height offsets between the subapertures. Also, the alignments of the tiltmeters and the autocollimator to the coordinate system can be included as independent variables. As shown in the simulations and observed in experiments, systematic angles errors very likely lead to form errors that correspond to oblique astigmatisms. This phenomenon will also be worthwhile to further investigate in.

For the flat surfaces and specimens with $r_C = 70$ m, measurements with an accuracy well below a wavelength in the UV/VIS/IR range seem possible with the given system specifications, if the stitching algorithm can be further optimized to reduce the remaining angle errors to below $0.1''$. This remains to be demonstrated. For specimens with a radius of curvature of $r_C = 10$ m, the accuracy of the lateral positions (due to stage straightness, cavity length measurement, interferometer optics) has a greater importance. According to the simulation of the large system, a measurement error well below a wavelength will not be achievable.

For a diameter of 1.5 m, the change of the direction of gravity leads to a topography error of 177 nm according to Eq. 4.38 which has to be corrected. Furthermore, the retrace error of the Fizeau interferometer must be investigated and corrected if necessary. The simulations can be further refined to investigate separate relevant error components depending on the input quantity errors. For example, the input-output deviations for different Zernike polynomials can be investigated.

10 Outlook

To conclude, some possibilities are presented to further develop the realized form measurement system and the large system for specimen of up to 1.5 m under development.

10.1 Stitching

To improve the stitching, a self-referencing method for the reference wave and other parameters describing errors as in Ref. [58] can be implemented. This algorithm can be implemented pixelwise or based on Zernike or other polynomials fitted to the original data points. The latter gives much smaller systems of equations than pixel-by-pixel calculations. It must be noted that certain parameters in stitching algorithms can be ambiguous: For a spherical specimen, a lateral position error has the same effect as a tilt error. Also, statistical and systematic errors must be treated correctly. A propagation of systematic errors misinterpreted as statistical errors must be prevented or at least included in the uncertainty budget. A restriction to global fit parameters (like the reference flat form error and position dependent tip/tilt errors of the actuators) would make sense to prevent error misinterpretation. A specimen could also be measured at several positions and orientations relative to the measurement system, to determine systematic errors of the measurement system.

10.2 Uncertainty budget

One well justifiable improvement can be made to make the uncertainty estimation more accurate instead of it being an upper limit estimate: Instead of evaluating the longest path through the specimen (from edge to edge), the topography can be evaluated relative to its lateral centre and the mean height. The centre can be defined as the mean values $\langle x \rangle_{\text{topo}}$ and $\langle y \rangle_{\text{topo}}$. The model equation for the topography would be

$$z(x, y) = z_k(x, y) - \langle z \rangle_{\text{topo}}(\langle x \rangle_{\text{topo}}, \langle y \rangle_{\text{topo}}) + \sum_{s=1}^{N_{\text{path from centre}}} \Delta z_s \quad (10.1)$$

where $\langle z \rangle_{\text{topo}}$ is the mean value of the topography and $N_{\text{path from centre}}$ is the number of subapertures from the centre. The uncertainty of the mean values would be small as it is an average of many points, and systematic constant offsets are irrelevant. This would yield a smaller uncertainty for contributions integrated over the specimen, so that they are closer to the simulation results. One example would be the integrated systematic angle error. For the uncertainty contributions that are not integrated, it could be appropriate to also calculate the mean values of the position dependent sensitivity coeffi-

cients. Here, the maximum values were used, giving an upper limit. One example would be the averaging of the surface gradients which are used to calculate the propagation of the straightness error of the linear stages. Also, these values would only be counted once (topography defined relative to mean height) instead of twice (topography defined relative to a single reference point).

The most appropriate way to determine an uncertainty for the whole topography is to start from the uncertainty for each single point as $u(z(x,y))$, derived from model equation 10.1. To get a value describing the whole measurement, the RMS topography uncertainty must be calculated. It is given by

$$\langle u(z) \rangle_{\text{RMS}} = \sqrt{\frac{\int_A u(z(x,y))^2 dA}{A}} \quad (10.2)$$

where A is the area of the specimen in x and y . It is useful as the RMS topography uncertainty is directly linked to the RMS wavefront error of an optical element which determines the imaging quality.

A further improvement would be the separation of uncertainty contributions for different spatial wavelengths of the specimen surface or for different Zernike polynomials.

For the more empirical uncertainty based on the stitching residual as for the car side window, it would make sense to look at the height and gradient mismatch of neighbouring subapertures.

For the uncertainty of the radius estimation in 2D, it might be useful to normalize the topography standard deviation (of the single topography value) to the given RMS topography uncertainty. This would replace the method of drawing the PV value from a normal distribution. In any case, this must be done if an RMS uncertainty is calculated according to the previous suggestions.

10.3 Positioning system

To optimize the positioning system, an obvious possibility is to use more accurate systems and encoders and to calibrate the system with a minimum remaining uncertainty. Thus, sufficient accuracies for X-ray mirrors as presented in Sec. 2.1.2 should be achievable with the small system. The limitations hereby lie within the calibrations achievable for the system as well as the stability for the given environment conditions and measurement times. The 3D optical rotation sensor that was presented in Sec. 5.5.2 can be used for an angle (and even position) measurement when the necessary accuracies have been achieved. Alternatively, a multiple beam interferometer with, for example, retroreflectors can replace the autocollimator - provided that the geometry allows for it. For the lateral position accuracy, it is advantageous to have the interferometer pivot point as close as possible to the measured surface. However, the design of a positioning system sets limits to the minimal distance of pivot point and surface. For example, larger rotation tables are generally more accurate than smaller ones, but the distance to the specimen

increases. Also, the effects of a higher weight must be considered. Using a goniometer to put the pivot point onto the surfaces could minimize the lateral position error due to angle errors. However, it cannot prevent tilt errors of the subapertures.

Using an interferometer with a variable and remotely adjustable focal length can replace the z -stage of the system. For the positioning system it is also important that stage vibrations, e.g. due to air-bearing stages, are not in the same frequency range as the interference signal from the wavelength sweeps and do not cause positional drifts (compare Ref. [122]).

10.4 Speeding up the measurement

To decrease the measurement time, the duration of the wavelength sweeping can be decreased. The image sensor of the camera must still be able to temporally resolve the temporal interference signal during the wavelength sweep. Also, better environmental conditions reduce drifts during the measurement so that small topography and cavity length uncertainties can be achieved with a smaller laser tuning range. The angular alignment of the Fizeau interferometer to the surface gradient at each position can be extrapolated from the last measurement position, given the optical surface has no sudden lateral changes in the surface gradients. Also the absolute distance can be predicted this way.

To deal with strong curvatures of the specimen, one possibility is to use a small fraction of the interferometer aperture, in which the fringes are resolved. Another way can be to improve the absolute distance interferometry and circumvent the necessity to resolve the fringes. For this, it is necessary to have a relative accuracy better than $\frac{\lambda}{2}$ for the cavity length measurement of neighbouring pixels, so that their absolute phase difference can be calculated. This requires very stable environmental conditions regarding the refractive index of the surrounding air and the mechanical stability. In addition, the retrace error has to be corrected.

10.5 Simulation

In the simulation, the stage errors are modelled as "jagged" whereas they are rather continuous in the measurements. The reason is that the simulated angular errors of the stages (rotational and linear) are also jagged due to the random walk generating the error function. In the real measurement system, that stage errors have mostly mid- and low-spatial frequencies. This could be taken into account by removing the high frequency components from the error signals.

The simulation can be made more detailed at many points. However, if the simplifications in the simulation are well chosen, the results will hardly be more meaningful. A reasonable improvement would be the inclusion of the retrace error and the alignment of the stages for large and curved specimens.

10.6 General outlook

The presented form measurement system can be further optimized for traceable form measurements with high accuracy. The measurement of large optics with an even lower uncertainty is feasible. For the existing system, especially the rotation stages need to be calibrated more accurately.

An implementation of the presented methods enables the realization of a versatile form measurement system for optics with and without freeform components of up to 1.5 m in diameter at a high accuracy. The closer the specimens gets to the limits of the curvature specifications, the more demanding the calibration of the system becomes. As a final step, a round robin test can validate the measurement system and measurement process.

11 Appendix

11.1 Measurements with a coordinate measurement machine using a chromatic confocal sensor

The chromatic confocal sensor (CCS) [19] is a point wise measuring distance sensor. It uses a lens with an intentional chromatic aberration to focus white light of a lamp or a LED onto a surface. Due to the chromatic dispersion of the lens, the focal length is dependent on the wavelength. The light is reflected back from the surface and imaged into a plane with a pinhole. Only the wavelength component of the light for which the focal length is matched passes the pinhole with the complete intensity. With a spectrum analyser, the wavelength with the highest intensity and thereby the distance to surface can be determined.

In general, the distance measurement needs to be calibrated for the slopes of the SUT [116]. For small angles this effect is negligible [116]. This also applies to the specimens measured in this work. For topography measurements, the CCS is mounted to a coordinate measurement machine to scan the SUT. During a scanning measurement with a stage, the distance measured by the chromatic confocal sensor contains the topography of the specimen as well as the straightness errors of the stage. The sensor can also be moved in z -direction to increase the measurement range but this was not necessary for the measurements in this work. Then, the position errors of the z -stage also influence the measured distance.

The straightness errors of the stage or stages carrying the chromatic confocal sensor can be measured with a flatness standard. The remaining height uncertainty depends on the flatness of the normal and on the repeatability of the height error of the stage. For the measurement of the 50 mm diameter sphere segment (see Fig. 6.9), the Fizeau interferometer reference flat from Fig. 6.3 was measured as reference. In the measurement in Sec. 6.25 from Ref. [116], the stage errors were not corrected.

11.1.1 Uncertainty of the topography measurement with the coordinate measurement machine

The measurement was performed with the x -stage of the positioning system presented in Chapter 5. The chromatic confocal sensor was a *Precitec Chrocodile 2S* with a measurement range of 300 μm and a resolution of 10 nm. During the measurement, the sensor was moved from position to position and the measurement was performed when the sensor was stationary.

The uncertainty $u_{\text{CMM}}(z)$ of a topography with the coordinate measurement machine can be expressed in dependence of the uncertainty of the distance measurement $u_{\text{CCS}}(d)$, the z -position repeatability of the linear stage and the flatness of the reference (here the transmission flat) with $u_{\text{TF}}(z) = \frac{v(\text{PV})}{\sqrt{12}}$. The pointwise z -repeatability of the stage was estimated as $u_{\text{stage repeatability}}(z) = 100 \text{ nm}$ from four repeated measurements. The combined uncertainty is

$$u_{\text{CMM}}(z) = \sqrt{u_{\text{CCS}}(d)^2 + u_{\text{TF}}(z)^2 + u_{\text{stage repeatability}}(z)^2} \quad (11.1)$$

The repeatability of the stage is the dominant contribution. Therefore, the uncertainty for the CMM measurement is $u(z_{\text{CCS}}) \approx 100 \text{ nm}$. For the sphere of 50 mm in diameter and 10 m in radius of curvature, this height error is equivalent to a radius of curvature 95% coverage interval of $[r_c - 27 \text{ mm}, r_c + 27 \text{ mm}]$ with 95% coverage according to the method in Sec. 7.3.

11.1.2 Finding the centre y -coordinate of the spherical specimen

To find the centre of a specimen with a circular edge, a scan in y -direction was performed (see Fig. 11.1) with steps of $50 \mu\text{m}$. The two intersection points of this secant line with the specimen edge are determined. The point in the centre between them is on the centre line (blue) containing the centre of the circle since the x - and y -stage are perpendicular to each other.

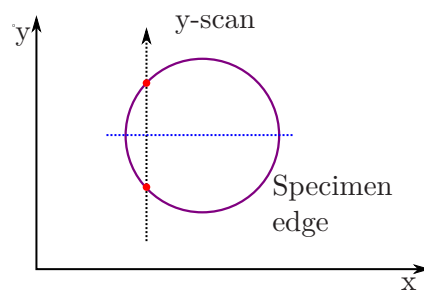


Figure 11.1: Illustration how the y -coordinate of the blue profile through the centre was sought after.

11.1.3 Calculating the standard uncertainty from a specified tolerance range

GUM [20] 4.3.7 provides guidance on how to treat Type B errors (given by other sources than repeated measurements) when only a tolerance range is specified. If a tolerance range a_{PV} is given for a quantity q , the following reasoning can be applied: The true value lies within the interval. Since no further informations are given, the probability

where the true value is located is the same everywhere. From an information theory perspective, the uniform probability density distribution maximizes the information entropy according to the principle of maximum entropy. Mathematically expressed, the probability distribution is a rectangular distribution. It can be transformed to a standard uncertainty according to

$$u(q) = \frac{a_{PV}}{\sqrt{12}} \quad (11.2)$$

This standard uncertainty has the same variance as the rectangular distribution. The aim is to have a standardized method to deal with distributions other than Gaussian distributions. The corresponding standard uncertainty can also be derived for other distributions.

11.1.4 Parabola as an approximation of a circular segment

The sagitta of a circular segment with radius R is given by

$$h(x) = R - \sqrt{R^2 - x^2} \quad (11.3)$$

This can be approximated for $x \ll R$ with the first term of the Taylor expansion at $x = 0$:

$$h(x) = \frac{x^2}{2R} + \mathcal{O}(x^3) \quad (11.4)$$

The gradient of the surface near $x = 0$ is given by

$$\frac{dh(x)}{dx} = \frac{x}{R} \quad (11.5)$$

In 3D, a spherical surface can be approximated with a paraboloid in the same way.

List of abbreviations

3D-ORS	three-dimensional optical rotation sensor
AC	autocollimator
CCS	chromatic confocal censor
CGH	computer-generated hologram
CMM	coordinate measurement machine
CMOS	complementary metal-oxide-semiconductor
ELT	Extremely Large Telescope
EM	electromagnetic
FFT	fast Fourier transform
GUM	Guide to the Expression of Uncertainty in Measurement
HMD	head-mounted display
HUD	head-up display
ITF	instrument transfer function
JWST	James Webb Space Telescope
LED	light-emitting diode
OPD	optical path difference
OPL	optical path length
PTB	Physikalisch-Technische Bundesanstalt
PV	peak-to-valley
RMS	root mean square
SCaLA	stitching interferometry with cavity length and angle measurement
SEM	standard error of the mean
SER	standard error of the regression
SHS	Shack-Hartmann sensor
SUT	surface under test

TM tiltmeter

TMS traceable multiple sensor

TWI tilted-wave interferometer

VIM International Vocabulary of Metrology

WFE wavefront error

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