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A Data-Driven Reinterpretation of Angular Calibration Based on Circular Subdivision within a Digital Metrology Framework

Marcos M. Souza[1*], Luiz H. B Vieira[1], Wellington B. S. Jesus[1] and Thiago L. Fernandes[1]

[1]*National Institute of Metrology, Quality and Technology (Inmetro)*
Av. Nossa Senhora das Graças, 50, CEP: 25250-020, Xerém campus, Rio de Janeiro, Brazil

[*] *Corresponding author: mmsouza@inmetro.gov.br*

Abstract

This work revisits angular calibration based on circular subdivision and presents a data-oriented interpretation within a digital metrology context. The approach does not modify the experimental procedure, but introduces calibration as an inverse parameter estimation problem using redundant observations and least-squares adjustment. Experimental datasets are used to illustrate the identification of systematic effects and the interpretation of measurement uncertainty in a task-specific manner. The results indicate that typical calibration data can be evaluated within a model-based framework, highlighting the roles of redundancy, data structure, and inference in dimensional metrology.

1 Introduction

Accurate angular measurement plays a relevant role in a wide range of industrial applications, including geodetic surveying, precision engineering, large-scale dimensional metrology, and industrial alignment. Instruments such as theodolites and total stations remain useful in these contexts [1,2], and their metrological performance impacts the accuracy of spatial measurements. Consequently, the calibration of angular measuring systems continues to be a critical task to ensure both metrological traceability and quality of dimensional measurements.

Traditional approaches to angular calibration are typically based on comparison methods involving reference artefacts [3,4], such as precision indexing tables [5] or angular standards. Among these, techniques based on circular subdivision redundancy, i.e. commonly referred to as rosette methods [6], have been employed due to their capability to distribute systematic errors and improve measurement consistency. These methods rely on performing multiple measurements at different relative angular positions between the reference system and the instrument under calibration. Despite their performance, such methods are often presented and interpreted within a predominantly procedural framework. As a result, the underlying structure of the measurement problem, characterized by a large set of interdependent observations and a relatively small number of unknown parameters, is not always explicitly explored. In particular, the potential of these methods to be formulated as problems of parameter estimation based on redundant data can be discussed in the context of dimensional metrology.

In parallel, the emergence of digitalization of metrology has introduced new perspectives on measurement processes [7, 8, 9], emphasizing the integration of physical measurement systems with data-driven models, computational analysis, and measurement traceability frameworks. Within this paradigm, measurements are increasingly interpreted as structured datasets, and metrological tasks are formulated in terms of inference problems, where unknown quantities are estimated from redundant and often indirect observations. This shift could provide more robust uncertainty evaluation methods, improve design of experiments, and enhance integration with digital infrastructures encompassed by recent discussions and policies on quality infrastructure [10,11].

In this context, the present work revisits a typical angular calibration approach based on circular subdivision redundancy and reinterprets it as a problem of inverse parameter estimation. The measurement approach is formulated such that the observed angular differences correspond to linear combinations of error components associated with the indexing table, the instrument under calibration, and their relative alignment. By exploiting the redundancy inherent to the measurement scheme, the resulting system of equations is evaluated using least-squares adjustment, allowing the simultaneous estimation of these error sources. The main contribution of this work lies not in modifying the experimental procedure itself, but in proposing a revised interpretation within a contemporary metrological perspective. By explicitly addressing data redundancy, parameter estimation, and metrological traceability, this formulation establishes a conceptual bridge between classical angular calibration and emerging digital metrology approaches. It also provides a basis for discussing uncertainty in a task-specific manner, considering the interplay between measurement configuration, model assumptions, and data quality.

This paper is organized as follows. Section 2 presents background on angular calibration and digital metrology. Section 3 introduces the proposed interpretation within a data-driven framework. Section 4 presents examples of experimental results and their analysis. Section 5 provides the final discussions and outlook.

2 Background

Angular calibration methods based on circular subdivision are widely used in dimensional metrology for instruments such as theodolites and rotary tables. These methods rely on performing measurements at multiple relative angular positions between a reference system and the instrument under calibration, generating observation sets in which each measured quantity reflects the combined influence of different error sources. Rosette-based approaches are particularly effective in distributing systematic effects and improving measurement consistency.

From a broader perspective, such measurement schemes can be interpreted as redundant systems, where the number of observations exceeds the number of unknown parameters. In this context, each observation can be expressed as a function of multiple error components, and the calibration problem can be formulated as an inverse parameter estimation problem. By exploiting data redundancy, the unknown parameters can be estimated using least-squares adjustment, enabling improved robustness and consistency assessment, as explored in previous research [12]. Figure 1 shows the experimental approach employed in this study.

Figure 1: Experimental approach: on the left, the system for calibration of the horizontal scale of theodolites and total stations; on the right, the coordinate measuring machine adopted to support the design of experiments [12].

Recent developments in digital metrology have emphasized the interpretation of measurements as data-driven processes, where measurement results are obtained through the combination of experimental data and explicit mathematical models. Within this framework, measurement systems are described in terms of structured datasets and inference procedures, allowing a more systematic treatment of metrological traceability and measurement uncertainty. Measurement uncertainty evaluation can become task-dependent, reflecting the interaction between the measurement configuration, the adopted model, and the available data. In this work, a typical angular calibration method based on circular subdivision is revisited under this perspective, by formulating the measurement process as a redundant inverse problem and interpreting the results within a digital metrology framework.

3 Interpretation within Digital Metrology

The angular calibration procedure is based on a circular subdivision scheme, in which a series of angular measurements is acquired at different relative positions between a reference indexing system and the instrument under calibration. Each acquisition corresponds to an angular difference that is influenced by multiple error sources. Typically, the measurement process can be described (or designed) by a linear model in which each observation is expressed as a combination of unknown parameters associated.

Within the proposed framework, the calibration procedure is interpreted as a data-driven inference problem, in which the measurement results emerge from the interaction between experimental data and an explicit mathematical model. This formulation can enable a task-specific interpretation of measurement uncertainty, as the estimated parameter uncertainties are related to the structure of the design matrix and the redundancy of the dataset. Accordingly, the performance of calibration is linked to the measurement configuration and the quality of data generated.

Figure 2 illustrates the angular calibration scheme and its formulation as a parameter estimation problem. Multiple observations are acquired at different angular positions, each representing a combination of contributions from the reference indexing system, the instrument under calibration, and alignment effects. This measurement configuration leads to a redundant system, in which the set of observed angular differences is expressed through a linear model and solved via least-squares adjustment, enabling the simultaneous estimation of error components and their associated uncertainty contributions.

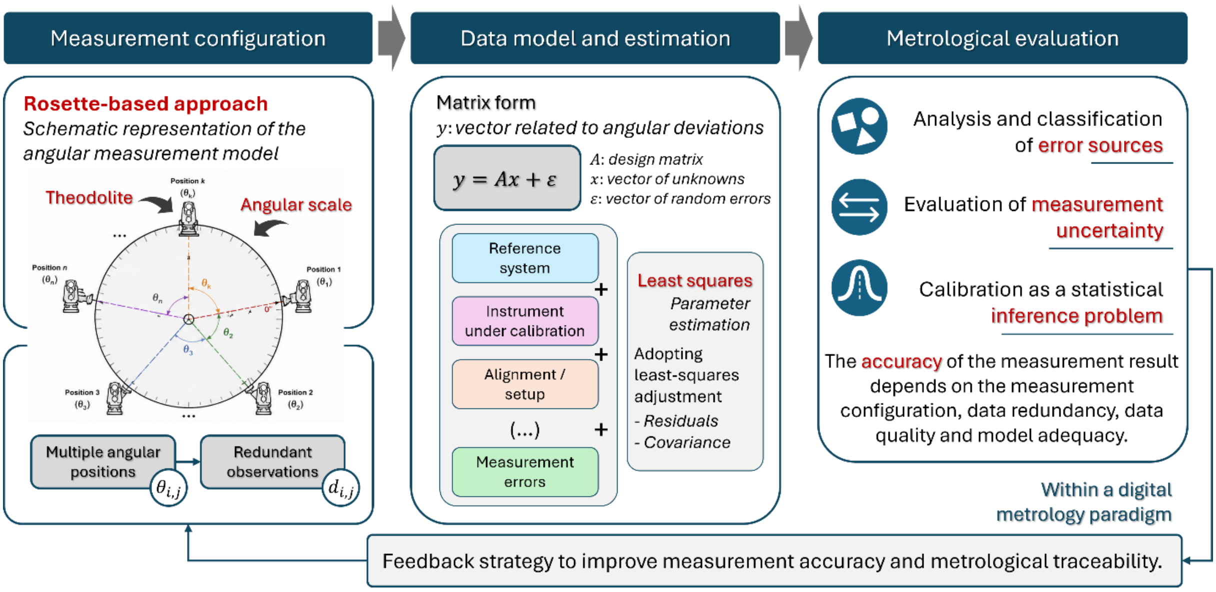

Figure 2: Angular calibration approach within a digital metrology framework.

In other words, redundant measurements at multiple angular positions can enable simultaneous estimation of error components with improved measurement accuracy and metrological traceability within a digital metrology framework. The formulation presented in this framework highlights that the calibration problem can be represented as a structured system of linear equations, in which each observation contributes to the estimation of multiple error components. In this context, the least-squares adjustment provides an approach to combine the observations, including information about residuals and covariance, which are relevant for assessing the quality of measurements. By framing the calibration procedure as an inference problem, the proposed approach can contribute to enabling a more transparent analysis of how measurement uncertainty propagates through the system and how different sources of error can be separated. This perspective is aligned with current developments in digital metrology, in which measurement processes are increasingly understood as data-driven and model-based.

4 Experimental Setup and Results

The experimental results presented in this section are based on previously acquired datasets obtained from angular calibration using a circular subdivision scheme. The objective is not to provide a detailed description of the experimental procedure, but to illustrate how the proposed data-driven formulation enables interpretation of the calibration process as a parameter estimation problem.

The measured angular differences were organized according to the approach described in Section 3, in which each observation is expressed as a combination of error components associated with the reference indexing system, the instrument under calibration, and alignment effects. The resulting redundant system was evaluated using least-squares adjustment, providing estimates of the error parameters as well as residuals and covariance information.

Figure 3 (on the left) presents an example of an estimated error profile obtained for a theodolite calibration. The observed behavior reveals a smooth and structured variation along the angular positions, suggesting the presence of systematic effects. The consistency of the profile indicates that the adopted model adequately captures the dominant error contributions. To assess the stability of the estimation over time, repeated calibrations of the same instrument were analyzed. Figure 3 (on the right) shows the estimated error profiles obtained in two independent calibration campaigns separated in time. The results exhibit similar bias, with only minor deviations at specific angular positions, indicating that the method is able of detecting both stable and time-dependent effects in the instrument’s behavior.

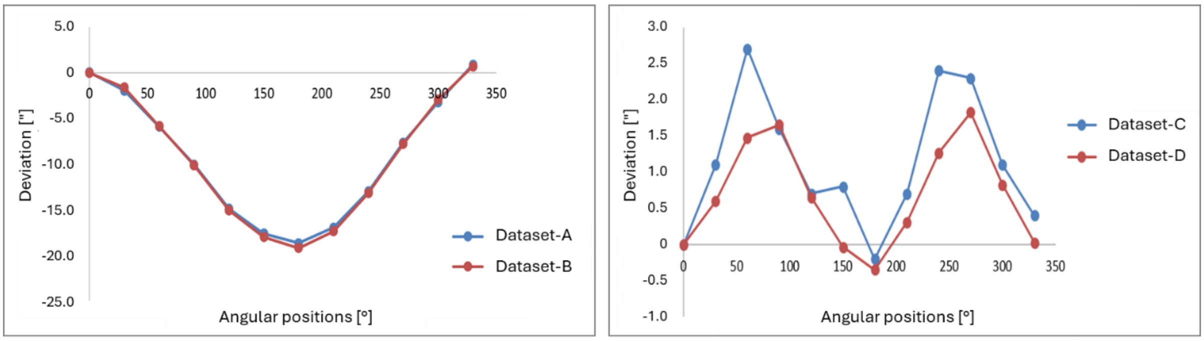

Figure 3: Comparison of angular deviation profiles for different datasets. Left: datasets acquired within the same measurement campaign. Right: datasets acquired in distinct measurement campaigns, separated by approximately three years.

To further illustrate the robustness of the proposed formulation, multiple calibration datasets obtained under different measurement conditions were analyzed. Figure 4 (left) shows deviation profiles derived from repeated calibrations conducted within the same measurement campaign, highlighting the consistency of the results despite variations in operator and measurement configuration. Figure 4 (right) presents error profiles obtained from independent calibration campaigns carried out under distinct conditions, where increased dispersion and localized discrepancies can be observed. These results indicate that, while the model remains stable, variations in measurement context may lead to deviations that approach or exceed the expected uncertainty range.

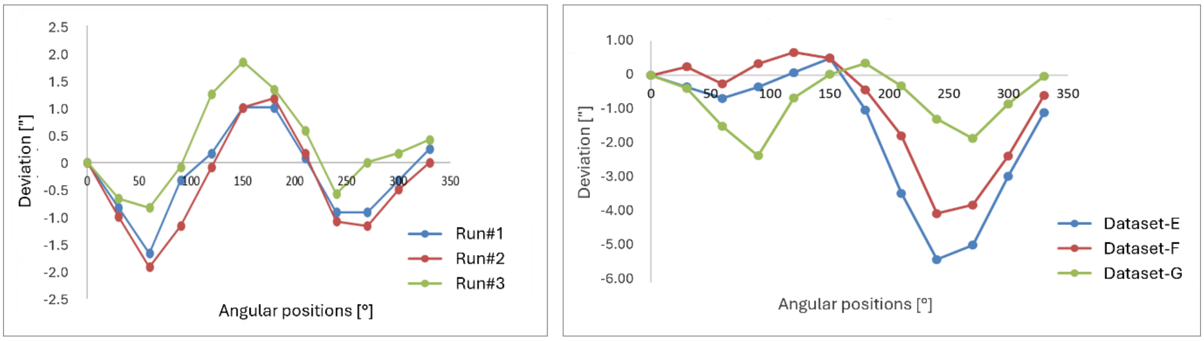

Figure 4: Angular deviation profiles from independent calibration datasets. Left: repeated calibrations within the same campaign, and different operators. Right: calibrations performed at different time periods.

The least-squares adjustment can also provide access to covariance information, enabling a task-specific interpretation of measurement uncertainty. The results indicate that the redundancy of the dataset contributes to the robustness of the parameter estimation and supports metrological traceability within the proposed framework. Overall, these results demonstrate that classical angular calibration data can be interpreted within a data-driven framework, in which measurement results emerge from the interaction between experimental observations and an explicit inference model. The approach highlights the role of redundancy, model structure, and data quality in determining both measurement accuracy and uncertainty.

5 Final Discussions and Outlook

This work supports the interpretation of angular calibration as a structured inference problem, in which measurement results arise from the adjustment of a model to redundant observational data. The results show that typical calibration datasets already contain sufficient structure to identify systematic effects, assess stability, and ensure consistent behavior under varying conditions. This suggests that improvements in metrology can be achieved not only through instrumentation, but also through more effective use of existing data, e.g. including contemporary tools involving data mining and machine learning.

This perspective is aligned with current initiatives for digital metrology, where calibration is increasingly understood as a data-centric and model-based process. Within this context, metrological traceability can be supported by transparent models, reproducible estimation procedures and predictive models. The approach is limited by the assumed linear model and by the adequacy of the chosen parameterization. Future work may address nonlinear effects, real-time implementation, and integration with digital environments, including advanced computational tools. In summary, the proposed framework provides a first reinterpretation of angular calibration, motivating future investigations and highlighting the role of inference, redundancy, and data structure in modern dimensional metrology.

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