DOI: 10.7795/810.20260825A

Evaluation for linking the bilateral SIM.L-K1:2007.1 to the CCL-K1:2011

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Abstract

International measurement comparisons are essential to demonstrate equivalence of national calibration and measurement capabilities (CMCs) under the CIPM MRA. Linking enables regional or bilateral comparisons to be related to CIPM key comparisons through the determination of a linking invariant. This work presents the methodology applied to link the bilateral SIM.L-K1:2007.1 comparison to the CCL-K1:2011 key comparison for gauge block calibration by optical interferometry. The uncertainty estimation includes correlation effects, and the transferred Degrees of Equivalence are assessed using normalized errors, reduced chi-squared statistics and Birge ratios, confirming statistical consistency and robustness of the linking procedure.

1 Introduction

International measurement comparisons are widely recognized as a fundamental tool for demonstrating the reliability and equivalence of national calibration and measurement capabilities (CMCs). Within the framework of the CIPM Mutual Recognition Arrangement (MRA), these comparisons provide the experimental evidence required to support the mutual recognition of national measurement standards and calibration certificates [1]. To ensure the scientific validity and consistency of the results, the CIPM through its Consultative Committees (CCs) defines the measurands, artefacts, and measurement protocols to be applied by National Metrology Institutes (NMIs).

To enable the participation of NMIs with different levels of technical capabilities, several types of international comparisons are conducted. CIPM key comparisons typically involve a limited number of NMIs and lead to the establishment of the Key Comparison Reference Value (KCRV). In parallel, Regional Metrology Organization (RMO) key comparisons and bilateral comparisons are organized to promote wider regional participation, since not all NMIs can participate in a single CIPM key comparison. In the case of RMO comparisons, participation of at least one laboratory that has taken part in the corresponding CIPM key comparison is required to ensure traceability to the KCRV and consistency within the CIPM MRA framework [2].

The results of RMO or bilateral comparisons can be related to those of the corresponding CIPM key comparison through a process known as *linking*, and the laboratories that take part in both are referred to as *linking laboratories*. According to the guidelines of the Consultative Committee for Length (CCL), linking is based on the difference between the measurand from the CIPM comparison and the identical nominal measurement from the RMO or bilateral comparison. Although this difference constitutes a measurement, it must be clearly distinguished from the primary measurement results of the comparison being linked. To avoid ambiguity and following the notation proposed by Decker *et al.* [3], this quantity is referred to as *the linking invariant*.

From a technical perspective, the correct determination of the linking invariant and its associated uncertainty is critical for the reliable evaluation of equivalence between comparisons, as it directly affects the consistency of Degree of Equivalence (DoE) and the transfer of the KCRV to regional or bilateral results. An inadequate treatment of this invariant may lead to biased equivalence statements or underestimated uncertainties [3].

The present work focuses on the practical methodology used to determine the linking invariant and its associated uncertainty for the SIM.L-K1:2007.1 bilateral comparison [4], enabling its linkage to the

CCL-K1:2011 key comparison [5] in the field of gauge block calibration by optical interferometry. The analysis follows the general principles established for linking within the CIPM MRA framework [2] and the methodological considerations discussed in [3]. Figure 1 shows the comparison scheme within the framework of the CIPM MRA, highlighting the connection between the SIM.L-K1:2007.1 and the CCL-K1:2011. In this case, CENAM acts as the unique linking laboratory, providing the metrological link between the two key comparisons.

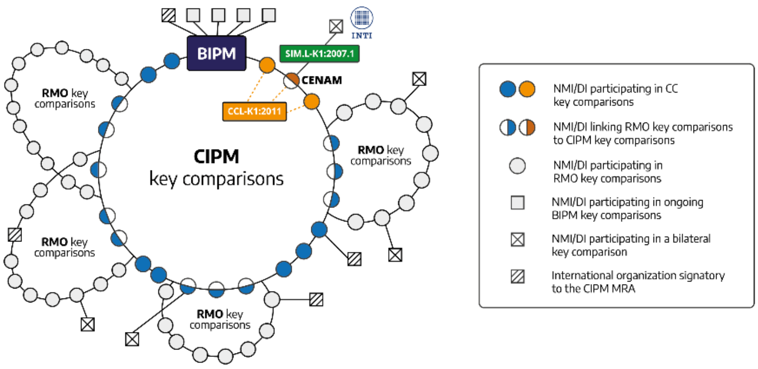

Figure 1: Comparison links, showing the connection between SIM.L-K1:2007.1 and CCL-K1:2011 (illustration adapted from CIPM MRA-G-11).

The published final KCRVs for the CCL-K1:2011 key comparison [5] are considered as reference values and not re-evaluated in this work. The analysis is therefore restricted to the determination of the linking invariant and the transfer of the reference values to the SIM.L-K1:2007.1 results [4]. To assess the consistency and robustness of the linked results, statistical indicators are applied, including the classical Degree of Equivalence (DoE) for each participant and global consistency tests based on the chi-square statistic and the corresponding Birge ratio. These statistical tools provide quantitative criteria to evaluate the mutual compatibility of the results and the adequacy of the associated uncertainty estimates within the linking framework.

2 Linking Invariant Model

Linking between laboratories, as defined by the CCL, is based on the difference between corresponding measurand in CIPM and regional or bilateral comparisons, which is defined as the linking invariant, $\overline{l_\iota}$, according to equation (1),

$$\overline{l_\iota} = l_{CCL,i} - l_{SIM,i}, \tag{1}$$

where $l_{CCL,i}$ and $l_{SIM,i}$ are the measurement reported by the linking laboratory for gauge block i in the respective comparisons, namely CCL-K1:2011 [5] and SIM.L-K1:2007.1 [4].

The associated standard uncertainty, $u(\overline{l_\iota})$ is evaluated from the standard uncertainties reported by the linking laboratory in both comparisons, including correlation terms [2,3,6]:

$$u^2(\overline{l_\iota}) = u^2(l_{CCL,i}) + u^2(l_{SIM,i}) + 2\,cov(l_{CCL,i}, l_{SIM,i}), \tag{2}$$

where the covariance term considers specific conditions to the measurements. In gauge block calibration by optical interferometry, potential sources of correlation include wavefront errors, thermal expansion coefficients, the use of Edlén equation for refractive index correction, etc.

The Degree of Equivalence (DoE) of an RMO laboratory (INTI in this case) relative to the CCL-KCRV, for each gauge block i, is then calculated as:

$$d_{INTI,i} = l_{SIM,i} + \overline{l_\iota} - l_{KCRV,i}, \tag{3}$$

where $l_{KCRV,i}$ is the final Key Comparison Reference Value reported for CCL-K1:2011 for the gauge block i [5]. The corresponding standard uncertainty, $u(d_{INTI,i})$ is given by:

$$u(d_{INTI,i}) = \sqrt{u^2(l_{SIM,i}) + u^2(l_{KCRV,i}) + u^2(\overline{l_i}) - 2\,cov(l_{KCRV,i}, \overline{l_i})}\,. \tag{4}$$

The application of this model relies on the assumption that the linking invariant is independent of the participating RMO laboratories and can therefore be applied uniformly to all RMO results at the corresponding nominal length [2, 6].

2.1 Correlation in the Uncertainty Evaluation

The inclusion of the correlation term in equations (2) and (4) is justified by the fact that the same measurement system was used by the linking laboratory (CENAM) in both CCL-K1:2011 and SIM.L-K1:2007.1. Since common systematic effects may contribute to both results, full statistical independence cannot be assumed.

The covariance between the results reported in both comparisons was evaluated as:

$$cov(l_{CCL,i}, l_{SIM,i}) = r_i\, u(l_{CCL,i})\, u(l_{SIM,i})$$

where r_i is the correlation coefficient associated with the shared uncertainty components [6]. This coefficient was estimated by identifying the uncertainty components common to both comparisons, such as wavelength realization, optical configuration, environmental corrections based on Edlén equation and thermal expansion coefficients, depending on the nominal length considered. In this case, only components associated to the method were assumed as potential sources of correlation since no identical artefacts were used in CCL-K1:2011 and SIM.L-K1:2007.1.

Similarly, correlation between the linking invariant and the CCL-KCRV was considered in equation (4). Since the linking laboratory contributed to the determination of the KCRV in CCL-K1:2011 [5], statistical dependence exists between $\overline{l_i}$ and $l_{KCRV,i}$. Consequently, non-zero covariance term was introduced when evaluating $u(d_{INTI,i})$. This covariance was derived following the approach described in [3]. The addition of this correlation term reduced the combined standard uncertainty by less than 4 nm at 0.5 mm and by up to 12 nm for nominal lengths greater than100 mm. In all the cases, the reductions were small compared with the combined standard uncertainty.

This treatment prevents underestimation of uncertainty that could arise from neglecting shared systematic effects and ensures a conservative evaluation of equivalence.

2.2 Considerations in Linking SIM.L-K1:2007 to CCL-K1:2011

The SIM.L-K1:2007.1 bilateral comparison between CENAM (México) and INTI (Argentina) was conducted in 2019 and piloted by INTI for the range from 0.5 mm to 300 mm [4]. The CCL-K1:2011 key comparison was piloted by CENAM for short gauge blocks and by NRC (Canada) for long gauge blocks [5]. The following considerations were applied in this linking analysis. The linking process was restricted to steel gauge blocks only. The results were grouped according to nominal length range, distinguishing between short and long gauge blocks, following the structure used in CCL-K1:2011 [5]. Gauge blocks for which linking laboratory exhibited an $E_n > 1$ in CCL-K1:2011 comparison were excluded from the linking analysis, thereby avoiding propagation of statistically inconsistent results. The final KCRVs used correspond to the iterations that achieved consistency in CCL-K1:2011, as reported.

Linking invariants were determined for each nominal length range using the results of the linking laboratory in both comparisons. When identical nominal lengths were not available, the applicability of scaling was evaluated based on the uncertainty model and its dependence on the nominal length. For all nominal lengths where the ratio between the gauge block length was less than two, the measurement uncertainties were considered comparable and no *ad hoc* scaling was applied. An exception was the 50 mm nominal length, which was not measure in CCL-K1:2011 comparison. In this case, both the measurand and its standard uncertainty were scaled to the 100 mm CCL-K1:2011 nominal length by using the ratio of the corresponding standard uncertainties. The linking invariant was then determined at 100 mm, taking into account the correlation between results.

In the long length range case, the CCL nominal length of 254 mm is compared with the corresponding 300 mm results from SIM.L-K1:2007.1. Although the nominal lengths are not identical, the ratio between nominal lengths is below a factor of two, and the associated uncertainty model does not indicate significant length-dependent systematic effects. Therefore, no scaling was required in this case. Two 300 mm nominal length gauge blocks from different manufacturers were considered in order to assess consistency at the upper limit of the comparison range. Table 1 summarizes the linking invariants determined for the nominal length considered in CCL-K1:2011 comparison [5], together with the corresponding nominal lengths measured in SIM.L-K1:2007.1 [4].

Table 1: Linking invariants, $\overline{l_i}$, determined for each nominal length considered in both comparisons.

Range	L_{nCCL}/mm	$\overline{l_i}$/nm	$u(\overline{l_i})$/nm	L_{nSIM}/mm
Short	0.5	-54.0	9.3	0.5
	3	-55	12.5	2.5
	100	19.5	20.3	50†
		88.0	20.3	100
Long	152.4	457.0	29.3	150
	254	230.0	51.7	300A
		782.0		300B

†scaled

3 Results and Analysis

Table 2 presents the Degrees of Equivalence (DoE) of INTI relative to the CCL-K1:2011 KCRVs for the gauge blocks considered in the linking analysis. The corresponding expanded uncertainties, $U(d_{INTI})$ (k=2), and normalized errors, E_n, are also reported. All normalized errors satisfy $|E_n| < 1$, demonstrating consistency of the linked INTI results with the CCL-K1:2011 reference values within the stated uncertainties.

For the short gauge range, the DoE values vary between 3 nm to 21 nm, while for the long range the DoE ranges from -21 nm to 36 nm. No monotonic trend with nominal length is observed in either range. The scaled 50 mm result, evaluated at 100 mm, remains fully consistent with other short range results. Its normalized error is 0.32 well below unity, confirming the applied scaling approach does not compromise statistical compatibility.

Figure 2 to Figure 4 show the DoE of INTI relative to the CCL-K1:2011 KCRV for selected nominal lengths: 0.5 mm, 100 mm and 254 mm. The figures include all participants contributing to the final CCL-K1:2011 KCRVs and standard uncertainty bars are shown. For the 100 mm case, the scaled 50 mm result from SIM.L-K1:2007.1 is also included. For the long range, the CCL nominal length of 254 mm is compared with the corresponding 300 mm results from SIM.L-K1:2007.1. Although the nominal lengths are not identical, no scaling was required, as justified in section 2.3. In all the cases, the INTI DoE lies well within the dispersion of the CCL-K1:2011 results.

Table 2: INTI Degrees of Equivalence relative to the CCL-K1:2011 KCRV.

Range	L_{nom}/mm	d_{INTI}/nm	$U(d_{INTI})$/nm	E_n
Short	0.5	3	27	0.10
	2.5	4	32	0.15
	50†	21	65	0.32
	100	16	67	0.23
Long	150	-21	96	-0.22
	300A	10	182	0.06
	300B	36	182	0.20

†scaled

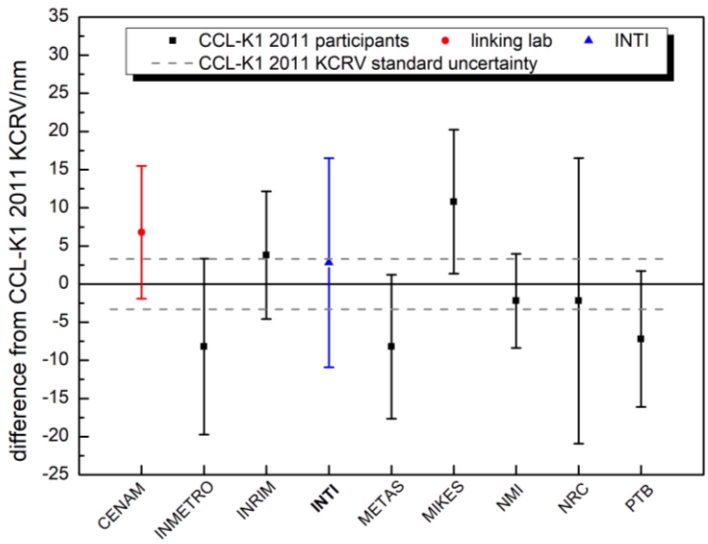

Figure 2: Degree of Equivalence of INTI relative to the CCL-K1:2011 KCRV for 0.5 mm, including all participants contributing to the final CCL-K1:2011 KCRVs. All results are shown with their standard uncertainty bars.

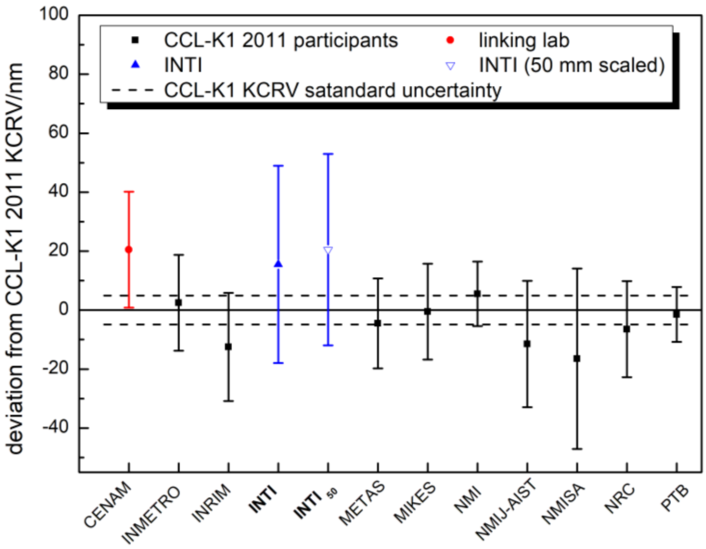

Figure 3: Degree of Equivalence of INTI relative to the CCL-K1:2011 KCRV at 100 mm, including all participants contributing to the final CCL-K1:2011 KCRVs. The scaled 50 mm result is also included. All results are shown with their standard uncertainty bars.

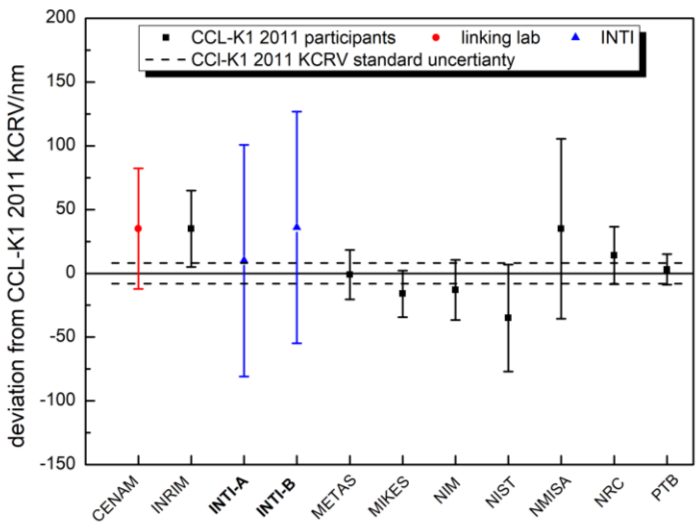

Figure 4: Degree of Equivalence of INTI relative to the CCL-K1:2011 KCRV for 254 mm, (300 mm in SIM.L-K1:2007.1; no scaling required). Both 300 mm lengths considered in the linking analysis are shown. All results are shown with their standard uncertainty bars.

The global statistical consistency of the linked results was evaluated using the reduced chi-squared statistic calculated from the Degrees of Equivalence. For the short range, a value of $\chi^2_{\nu S} = 0.08$ was obtained, while for the long range $\chi^2_{\nu L} = 0.12$. These values are significantly lower than unity, indicating no evidence of over-dispersion, i.e. the observed dispersion of the DoEs is smaller than expected from the stated uncertainties. The observed dispersion of the linked degrees of equivalence is therefore compatible with the assumed uncertainty model. No systematic deviation between the linked INTI results and the CCL-K1:2011 KCRV is observed within the evaluated nominal lengths. The corresponding Birge ratios are also below unity for both ranges ($R_{B,short} = 0.28$ and $R_{B,long} = 0.35$), confirming the absence of over-dispersion and supporting the internal consistency of the linked results.

4 Conclusion

The application of the proposed methodology for linking key comparisons has proved its practical validity and robustness. Although the mathematical formulation for the determination of the linking invariant is conceptually straightforward, careful evaluations of correlation terms and the establishment of well-defined criteria for data analysis were required to ensure reliable uncertainty evaluation. The outcomes show good agreement between the linked DoE of SIM.L_K1:2007.1 and the reference values of CCL-K1:2011, demonstrating statistical consistency across the evaluated nominal lengths. The reduced chi-squared statistics and Birge ratios confirm global consistency of the transfer results, with no indicators of systematic deviation. These findings support the validity of the linking procedure and the robustness of the uncertainty model adopted.

The impact of the assumed correlation was limited: in all cases, the reductions in combined standard uncertainty due to correlation terms were small compared with the combined uncertainty, confirming that the linking results are not significantly affected by the adopted correlation assumptions. This behaviour reinforces the stability and confidence in the linking approach applied in this work. The methodology presented provides a reproducible framework that can be applied to similar linking exercises in dimensional metrology.

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