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**Guideline**
**DKD-R 5-6**

# Determination of thermometer characteristics

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## Deutscher Kalibrierdienst (DKD) – German Calibration Service

Since its foundation in 1977, the DKD encompasses calibration laboratories of industrial enterprises, research institutes, technical authorities, surveillance and testing institutions. On 3rd May 2011, the German Calibration Service was reestablished as a *technical body* of PTB and accredited laboratories.

This body is known as *Deutscher Kalibrierdienst* (DKD for short) and is under the direction of PTB. The Guidelines and Guides developed by DKD represent the state of the art in the respective areas of technical expertise and can be used by the *Deutsche Akkreditierungsstelle GmbH* (the German accreditation body – DAkkS) for the accreditation of calibration laboratories.

The accredited calibration laboratories are now accredited and supervised by DAkkS as legal successor to the DKD. They carry out calibrations of measuring instruments and measuring standards for the measurands and measuring ranges defined during accreditation. The calibration certificates issued by these laboratories prove the traceability to national standards as required by the family of standards DIN EN ISO 9000 and DIN EN ISO/IEC 17025.

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# Foreword

DKD Guidelines are application documents that meet the requirements of DIN EN ISO/IEC 17025. The Guidelines contain a description of technical, process-related and organizational procedures used by accredited calibration laboratories as a model for defining internal processes and regulations. DKD Guidelines may become an essential component of the quality management manuals of calibration laboratories. The implementation of the Guidelines promotes equal treatment of the equipment to be calibrated in the various calibration laboratories and improves the continuity and verifiability of the work of the calibration laboratories.

The DKD guidelines should not impede the further development of calibration procedures and processes. Deviations from Guidelines as well as new procedures are permitted in agreement with the accreditation body if there are technical reasons to support this action.

The present Guideline was prepared in 2008 by the Technical Committee *Temperature and Humidity* in cooperation with PTB and accredited calibration laboratories.
The revised new edition only contains an updated imprint.
It is identical in content with DAkkS-DKD-R 5-6 (Edition 2010). DAkkS will withdraw the document DAkkS-DKD-R 5-6 by 01.01.2021 at the latest.

Edition: 2003, published by DKD

1. New edition 05/2008, by DKD
2. New edition: 2010, by DAkkS
3. New edition: 2018, by DKD, identical in content with the 2nd new edition

In the current revision 1, printing errors in table 6.13 have been corrected.

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# 1 Purpose and scope of application

The purpose of this guideline is to define generally valid procedures for the approximation of the characteristic curves of industrially used thermometers in order to create a uniform and industry-wide basis for the surveillance of test equipment. Likewise, the user of this guideline – i.e. the calibration laboratory as well as the user of the thermometer – shall be given instructions regarding the handling of approximation equations as well as the performance of the actual approximation.

Basically, this guideline is valid for all thermometers. However, it is especially tailored to the requirements of platinum resistance thermometers (especially Pt-100), thermocouples and thermistors. As these instruments have very different measurement uncertainties, depending on the type of sensor and temperature range, this guideline has been designed for different requirements related to the measurement uncertainty.
For some thermometer types (e.g. thermometers with electronic display or liquid-in-glass thermometers) the determination of a characteristic curve involves fundamental problems. The present guideline is not applicable for these types of thermometer.

This guideline is not intended to develop or prescribe newer or better approximation methods or characteristic curve types than those currently in use.
It rather seeks to recommend the optimum type(s) of characteristic for certain boundary conditions – such as temperature range and required measurement uncertainty – which are state of the art today. These recommendations are compatible with existing software and measuring instruments and can be easily entered or integrated.
Possibly, there are other types of characteristics which are equally suited or may even be better than those described here. However, in the case of limited dissemination or practicability, other mathematical descriptions of characteristic curves should only be used in justified cases.

# 2 Fundamentals

Thermometers for industrial applications (e.g. platinum resistance thermometers / Pt-100, thermocouples, thermistors) are usually calibrated at several temperature points in a temperature range desired by the user. Usually, the thermometer is not used exactly at the calibration points, but also between them. Therefore, the user often requires a continuous description of the relation between temperature and resistance or temperature and thermoelectric voltage over the entire temperature range used.
In most cases, this is achieved by specifying a mathematical equation which is determined by approximation of the measured temperature points.
This guideline describes in which way and under which boundary conditions such an approximation calculation should be carried out.

## 2.1 Types of characteristics (general)

Generally, the aim of determining a thermometer characteristic is to deduce a mathematically (mostly analytically) formulated relation $Y = f(T)$ from a small number of calibration points $(T_i ; Y_i = f(T_i))$ which can then be used in the whole covered measurement/calibration range to determine the temperature $T$ from the measured quantity $Y$.
Especially in the field of thermometer characteristics, there is the need to determine non-linear sensor characteristics from an often very small number of measuring points in such a way that the uncertainty in the entire range of interest is not significantly greater than the uncertainty of the measurement points (measured values).
The decision as to which approach should be selected for the functions of the characteristic depends on a series of factors:

- Existence of physically founded characteristic functions
- Technically, historically or normatively defined approaches
- Approaches supported by the software and hardware used
- Number of available calibration points
- Required measurement uncertainties
- Temperature range

When adapting model functions, the compatibility between data and model must always be checked, e.g. by means of the $\chi^2$ criterion. However, based on decades of experience with thermometer characteristics, especially with characteristic curves of industrial resistance thermometers, it can generally be assumed that the following model functions are compatible with the measurement data within the scope of uncertainties stated in Section 6, provided that no measurement error has occurred.

In principle, the range of available and commonly used functions for describing the individual behaviour of temperature sensors can be divided into:

### 1) *Individual characteristics* $Y(T)$ *with a thermometer-typical approach*

**Example:** Callendar-Van Dusen equation for Pt resistance thermometers (DIN EN 60751)

$$Y(T) = R(T) = R_0 \cdot \left[ 1 + A \cdot T + B \cdot T^2 + C \cdot T^3 \cdot (T - 100\,°\mathrm{C}) \right] \quad \text{with} \quad C = 0 \quad \text{for} \quad T > 0\,°\mathrm{C}$$

### 2) *Individual characteristics* $Y(T)$ *with a general approach*

**Example:** Polynomial for the characteristic of a thermocouple

$$Y(T) = U_{\mathrm{Th}}(T) = \sum_{i=0}^{m} a_i \cdot T^i$$

### 3) *Deviation functions from the reference functions* $Y_{\mathrm{N}}(T)$ *as a function of temperature*

**Example:** Deviation of the thermoelectric voltage $U_i(T_i)$ of a thermocouple from the reference values $U_{\mathrm{N}}(T_i)$ as a function of temperature. For Pt-PtRh thermocouples, 2nd order polynomials are commonly used in a wide temperature range.

$$\Delta U(T) = U_i(T) - U_{\mathrm{N}}(T) = a_0 + a_1 \cdot T + a_2 \cdot T^2$$

### *4) Deviation functions from the reference functions $Y_N(T)$ as a function of the displayed quantity*

**Example:**
The calibration certificate of standard Pt resistance thermometers at fixed points according to ITS-90 indicates for the resistance ratio $W(T_{90})$ deviation functions $\Delta W = \Delta W(T_{90}) = W(T_{90}) - W_r(T_{90})$ with respect to the reference characteristic $W_r(T_{90})$. Depending on the temperature range, different approaches – $\Delta W = f(W)$ or $\Delta W(T_{90}) - f(W(T_{90}))$ – are defined for $\Delta W(T_{90})$ in the form of polynomials (up to the 3rd order as maximum) in $W$ (for –38.8344 °C ≤ $T$ ≤ 961.78 °C) or with an additional term in ln($W$) (for –189.3442 °C ≤ $T$ ≤ 0.01 °C) [18].

Almost all commonly used characteristic curve approaches do not allow directly interpretable statements about special thermometer characteristics, their changes or drifts. An exception is the Callendar-Van Dusen equation (DIN EN 60751), where $R_0$ directly indicates the resistance at the ice point 0 °C and *A* the increase at the ice point as a measure of the quality of the Pt resistance wire used.

Characteristics of the form $Y(T) = f(T)$ contain the complete individual sensor description and must therefore describe (usually with only few coefficients) both the sensor type-specific non-linearity and the individual deviation of the sensor behaviour from the typical or standardized curve. By contrast, deviation functions just need to describe the individual deviation from the reference. Consequently, they allow a clearly better description of the sensor behaviour with simpler approaches and thus with a smaller number of coefficients to be determined (this is very much in line with the frequent demand for a small number of calibration points). The amount and importance of the values calculated with such deviation functions correspond to the corrections of the measurement values which are furnished by the calibration item and are usually stated in the calibration results.

Characteristics which – as calibration result – do not indicate the values of a reference characteristic $Y_R(T)$ or the assigned deviations of the display of the calibration object from a reference characteristic $\Delta Y = Y_i - Y_R$ as a function of the temperature, but as a function of the display value itself $\Delta Y = f(Y)$ or $Y_R = f(Y)$ are particularly simple and universally applicable. In this case, only quantities of identical physical dimensions are linked. In this case, only quantities of identical physical dimensions are linked. The sensor-typical behaviour does not need to be taken into account (or even known), since it has already been completely determined with the determination of $Y_R(T)$.

A special case of this approach are characteristics for directly indicating thermometers which describe the correction or the “correct temperature” as a function of the displayed temperature.

## 2.2 Approximation and interpolation procedures (general)

A selected or specified characteristic approach can be adapted to the respective calibration results as an approximation function or as an interpolation function. Hereby, the fundamental differences between the function types have to be taken into account:

### 2.2.1 Interpolation

An interpolation function directly reproduces all calibration values used as reference points. Uncertainties of the individual calibration points enter directly and completely into the determined characteristic, both globally (i.e. the characteristic curve is influenced in the entire range by each calibration point) and locally (i.e. each calibration point particularly influences the course of the characteristic curve in the directly adjacent value ranges). This behaviour is particularly problematic with an increasing non-linearity of the characteristic curve approach in case of interpolation functions with a global approach, i.e. an approach which encompasses the entire calibration range.

Interpolation with global approaches is mostly performed with polynomials which can ultimately be traced back to interpolation polynomials of the Lagrangian type.

$$Y(T) = \sum_{i=0}^{N} Y_i(T_i) \cdot L_i(T)$$

with

$$L_k(T) = \prod_{i=0;i\neq k}^{N} \frac{T - T_i}{T_k - T_i}$$

$$= \frac{(T-T_0)\cdot(T-T_1)\cdot \ldots \cdot(T-T_{k-1})\cdot(T-T_{k+1})\cdot \ldots \cdot(T-T_{N-1})\cdot(T-T_N)}{(T_k-T_0)\cdot(T_k-T_1)\cdot \ldots \cdot(T_k-T_{k-1})\cdot(T_k-T_{k+1})\cdot \ldots \cdot(T_k-T_{N-1})\cdot(T_k-T_N)}$$

In the case of a larger number of calibration points, polynomials of a high degree are thus obtained. The coefficients determined are directly influenced by the selection of the calibration points. It is therefore not possible to compare characteristics with each other (not even those of the same sensor with characteristics that have been determined at an earlier time). With increasing polynomial degree (large number of calibration points), the instability of the course of the function increases between the reference points compared to smaller deviations at the reference points. Therefore, an upper reasonable limit for such global interpolation polynomials amounts to approximately 5 calibration points.

Interpolation functions are suitable if the calibration points are well-defined, or if the points have a very small uncertainty (e.g. in the case of fixed-point calibrations) and/or if the characteristics have a very small non-linearity and thus only require very few calibration points (in the case of ITS-90 deviation functions or in the case of directly indicating thermometers). The characteristics of Pt resistance thermometers according to ITS-90 fulfil all these requirements at the same time and therefore are suitable for a simple description of the individuality of thermometers with smallest uncertainties.

Interpolation procedures are also suitable for the piecewise determination of characteristics for the intervals between adjacent calibration points. Here, the problems of global approaches (tendency to oscillate due to the high order of the functions) can be avoided. The effects of a single calibration point on the shape of the curve are limited to adjacent reference point intervals (only local effects). For practical application, especially the piecewise linear interpolation (open polygon) and the interpolation with cubic spline functions (which is, however, no purely local procedure) are of special interest.

**Polygon interpolation:**

From $N$ calibration points $Y_i = Y_i(T_i)$ with $i = 0 \ldots N-1$, the $N-1$ piecewise linear functions

$$Y_k(T) = \frac{(T_{k+1} - T)\cdot Y_k + (T - T_k)\cdot Y_{k+1}}{T_{k+1} - T_k} \quad \text{for} \quad T_k \le T \le T_{k+1} \quad \text{with} \quad k = 0 \ldots N-2$$

are calculated.

Note: This is not a recursive representation. According to the nomenclature selected in the introduction, $Y(T)$ is the function for the whole range $T_0 \leq T \leq T_{N\text{-}1}$. $Y_\text{k}(T)$ are the piecewise functions, and $Y_\text{k}$ or $Y_\text{k}(T_\text{k})$ are the reference points (measurement values).

**Spline interpolation:**

From $N$ calibration points $Y_i = Y_i(T_i)$ with $i = 0 \ldots N-1$, the $N-1$ piecewise defined cubic functions are calculated as follows

$$Y_k(T) = a_{0k} + a_{1k} \cdot (T - T_k) + a_{2k} \cdot (T - T_k)^2 + a_{3k} \cdot (T - T_k)^3 \quad \text{for} \quad T_k \leq T \leq T_{k+1} \quad \text{with} \quad k = 0 \ldots N-2$$

The coefficients $a_{0k}; a_{1k}; a_{2k}; a_{3k}$ $(k = 0 \ldots N-2)$ are then calculated from a system of equations resulting from the consistency requirements of the functions including the 1st and 2nd derivations at the reference points.

**Note:** Under mathematical aspects, the spline interpolation is rather a local than a global procedure. Depending on the severity of the selected consistency conditions to be met, only the second and, maximally, the third adjacent point still have an influence on the coefficients in the respective interval. All other points lying further away do not have any influence.

**Attention:** Some standard software for spline interpolation prerequires equidistant reference points!

### 2.2.2 Approximation

In the case of approximation procedures and characteristics determined by using the corresponding approaches, the number $m$ of free parameters is always smaller than the number $N$ of calibration points used for the determination of the characteristic. For that reason, the approximation function usually does not reproduce the calibration values exactly, i.e. a deviation other than zero remains between the function values at the calibration temperatures and at the measured calibration values. In many practical cases, the course of the function is, on the other hand, no longer considerably influenced by small uncertainties of individual reference points. This is increasingly so, the larger the number of reference points $N$ is in relation to the number of free parameters $m$, and results from the statistical properties of the determination procedure.

The approximation is principally based on the fact that – on the basis of a selected measure – the deviations between the measured values $Y_i(T_i)$ and a parameter-dependent function approach $f(T_i, a_1, a_2, \ldots, a_m)$ are minimized by a suitable variation of the parameters $a_1, a_2, \ldots, a_m$. In most cases which are relevant to practice, the deviance $S$ is used as measure to be minimized according to Gauss:

$$S = \sum_{i=0}^{N-1} g_i \cdot \left[\, Y_i(T_i) \; - f(T_i, a_1, a_2, \ldots, a_m) \,\right]^2 \rightarrow \mathit{Minimum} \tag{2-1}$$

The procedures based thereon are also referred to as “regression procedure” or “least square fit” procedure. The parameters of the regression function $f(T, a_1, a_2, \ldots, a_m)$ are determined by solving the non-linear equation system:

$$\frac{\partial S}{\partial a_1} = 0\,; \quad \frac{\partial S}{\partial a_2} = 0\,; \quad \ldots \quad ; \quad \frac{\partial S}{\partial a_m} = 0$$

**Note:** In conventional calculations, the uncertainty of the $T_\mathrm{i}$ is included in the uncertainty of the $Y_\mathrm{i}(T_\mathrm{i})$ and therefore, does not need to be considered in many cases.

The following function approaches are commonly used:

- Linear functions:

$$f\left(T, a_1, a_2\right) = a_1 + a_2 \cdot T$$

- $(m - 1)$ order polynomials:

$$f\left(T, a_1, a_2, \ldots, a_m\right) = \sum_{i=1}^{m} a_i \cdot T^{i-1} \quad \text{with} \quad m < N$$

- Sensor-specific functions, e.g. in particular the Callendar-Van Dusen equation:

$$f\left(T, R_0, A, B, C\right) = R_0 \cdot \left[\, 1 + A \cdot T + B \cdot T^2 + C \cdot T^3 \cdot \left(T - 100\ {}^\circ\mathrm{C}\right) \,\right] \quad \text{with } C = 0 \text{ for } T > 0\ {}^\circ\mathrm{C} \text{ and } N > 4$$

The normal equation system can be established and solved using specially developed programmes. However, a constantly increasing number of standard and special software packages are available for this purpose.

Under theoretical aspects, the equation should be overdetermined with 10 additional measurement values in order to be able to judge the suitability of the selected mathematical model. For the (usual) case – where the characteristics are determined for known sensor types for which agreement with the mathematical model has already been checked – the use of – at least – two additional measurement values (degree of freedom $\nu \geq 2$) is sufficient in order to reduce the measurement effort as far as possible. Investigations have shown that – under such conditions – the occurring additional error of the characteristic is small.

Consequently, the following applies to the number of calibration points:

- $N \geq m + 2$ as minimum requirement for checked models
- $N \geq m + 10$ as a data set with good statistical certainty (for unchecked models)

In the case of approximation procedures, the selection of the $g_\mathrm{i}$ as weighting factors in the calculation of the sum $S$ of the deviances allows the single calibration points to be weighted. A different measurement uncertainty $u_\mathrm{i}$ which can be assigned to the single calibration points can thus easily be taken into account by selection of $g_\mathrm{i}$.

$$g_i = \frac{1}{u_i^2} \tag{2-2}$$

The multitude of standard software packages available for approximation is, however, considerably limited if weighting of the measurement values is required.

## 2.3 Uncertainties of the characteristic (general)

A user does not use the thermometer exactly at a calibrated temperature, but – with the aid of an interrelation determined between temperature and input quantity (e.g. resistance, thermoelectric voltage, etc.) – at any temperature within the calibrated temperature range. The properties of the mathematical models usually applied in industry cause additional errors because these models cannot adequately reproduce the properties of the real thermometer characteristic. This can especially be seen in the case of precision measurements, considering that the function determined (on the basis of the selected mathematical model) often runs outside the uncertainty range (95 %) of the measuring points $\pm U[Y_i(T_i)]$.

The uncertainty of the characteristic of the thermometer is composed of the uncertainty of the calibration points used for calculation, of the uncertainty of the agreement of the characteristic curve approach and the real thermometer behaviour, and of possible numerical uncertainty contributions (rounding errors).

### 2.3.1 Uncertainty in the case of interpolation functions

- The uncertainty at the individual calibration points is equal to the calibration uncertainty of the respective point.
- The uncertainty between the calibration points depends only on the uncertainty of the adjacent reference points (piecewise interpolation) or on the uncertainties of all calibration points (global interpolation function) – depending on the approach applied. Due to the unequivocal functional interrelation between the interpolation function $Y(T)$ and the reference points $Y_i(T_i)$, it can be calculated with the instruments of the Gaussian law of propagation on uncertainty if the individual contributions are not correlated:

$$u^2\left(y(T)\right)=\sum_{i=0}^{N}\left[\left(\frac{\partial y}{\partial T_i}\right)^2\cdot u^2\left(T_i\right)\right]=\sum_{i=0}^{N}\left[C_i^2\cdot u^2\left(T_i\right)\right]$$

With only a few reference points, still manageable sensitivity coefficients $C_{n,i}$ can be obtained for polygonial interpolation or Lagrange polynomials; with many reference points or spline interpolation, the sensitivity coefficients $C_{n,i}$ obtained are no longer manageable. In these cases, the $C_{n,i}$ can easily be determined by numerical calculation of the respective partial derivations at the reference points.

### 2.3.2 Uncertainty in the case of approximation functions

The determination of the uncertainty in approximation functions consists of several steps for the estimation of the individual contribution:

***1) Uncertainty of the approximation function $f(T_i,a_1,a_2,\ldots,a_m)$ at the calibration points***

The mean uncertainty of an approximation function with *m* coefficients (or free parameters of the approximation equation) to be determined and $N$ reference points can be calculated from the sum of the deviances remaining after approximation (residual squares sum):

$$u^2=\frac{1}{N-m}\cdot\sum_{i=1}^{N}\left[\,y_i(T_i)\;-f(T_i,a_1,a_2,\ldots,a_m)\,\right]^2 \qquad \text{with} \quad \nu=N-m$$

$N–m$ is the effective degree of freedom $\nu$ of the system. If this value assumes values smaller than 10 (this is almost always the case in temperature calibrations), a corresponding coverage factor (→ Table G.2 from [1]) must – analogously to the determination of the type-A uncertainty of a measurement series of smaller extent (→ equation (4) from [1] or equation (5) from [24]) – be taken into account.

## 2) *Estimation of the uncertainty of the approximation function $f(T, a_1, a_2, \ldots, a_m)$ between the calibration points*

The uncertainty contribution of the approximation function $Y(T) = f(T, a_1, a_2, \ldots, a_m)$ is generally obtained as follows:

$$u^2(y(T)) = \sum_{i=1}^{m}\left[\left(\frac{\partial f}{\partial a_i}\right)^2 \cdot u^2(a_i)\right] + 2\cdot\sum_{i=1}^{m-1}\sum_{l=i+1}^{m}\left[\frac{\partial f}{\partial a_i}\cdot\frac{\partial f}{\partial a_l}\cdot u(a_i, a_l)\right]$$

The first sum contains the uncertainty contributions of the individual coefficients $a_1, a_2, \ldots, a_m$ and the second sum expression contains the contributions which are due to the correlation of the coefficients. The required derivations can easily be calculated analytically or numerically from the approach for $f(T, a_1, a_2, \ldots, a_m)$ and the determined coefficients $a_1, a_2, \ldots, a_m$. They represent the sensitivity coefficients for the contributions of the individual uncertainties. However, the uncertainties $u(a_i)$ and the covariances $u(a_i, a_l)$ are not always available (if the coefficient matrix of the normal equation system for the approximation is available, they can be taken from this matrix).

This results in a combined uncertainty which is not constant over the entire calibration range and which usually increases towards the edges of the calibration interval.

If there are no covariances $u(a_i, a_l)$ or uncertainties $u(a_i)$, the residual square sum can be used as a good estimate. However, it does not consider the dependence on the model and – in unfavourable cases – can be falsifying.

$$u^2(Y(T)) \approx \frac{1}{N-m}\cdot\sum_{i=1}^{N}\left[Y_i(T_i) - f(T_i, a_1, a_2, \ldots, a_m)\right]^2$$

This, however, yields a fixed value for the entire calibration range while ignoring the variation of the approximation uncertainty over the calibration range.

When using numerically determined values for the covariances $u(a_i, a_l)$ or the uncertainties $u(a_i)$ from the coefficient matrix of the normal equation system, potential numerical uncertainty contributions of the software, caused by instability, are already partially considered.

The uncertainty components caused by the deviation of the functional approach from the real thermometer behaviour are not covered by the uncertainties stated so far. This deviation is only recorded with a higher number of measuring points ($N \geq m+10$) in the case of approximation functions. With a small number of calibration points or interpolation functions, additional uncertainty contributions must be taken into account for which empirical values are given in the sections dealing with the individual thermometers; these values depend on the type of thermometer and characteristic as well as on the temperature range.

The uncertainty $u_{\text{characteristic}}(T)$ of the temperature which is to be assigned can be estimated from the uncertainty $u(Y(T))$ of the characteristic curve by means of the increase of the characteristic curve.

$$u_{\text{characteristic}}\left(T\right) \approx \frac{1}{\frac{\partial f}{\partial T}} \cdot u\left(Y\left(T\right)\right)$$

The uncertainty of the calibration points to be stated in the calibration result (temperatures and corrections calculated from the displayed values with the aid of the individual characteristic curve) is then made up of the uncertainty of the individual calibration point $u(T)$ (taken from the measurement uncertainty budget) and the fraction $u_{\text{characteristic}}(T)$.

## 2.4 Selection of the reference points and of the type of characteristic (general)

In the case of global interpolation characteristics, the number of reference points should be as small as possible – depending on the non-linearity of the sensor – and the reference points should be distributed as uniformly as possible over the calibration range. For piecewise interpolation, this requirement is valid correspondingly for each interpolation interval.

For approximation functions, the number of calibration points must be greater than the number of free characteristic parameters. The difference between the number of calibration points and the number of free parameters corresponds to the number of degrees of freedom of the system. There should be at least 2 but – if possible – the number should be greater than 5 (→ 2.2.2).

When determining the minimum number of calibration points, it must be considered that maybe not all coefficients of all measuring points are determined which means that additional conditions have to be taken into account.

A widely spread method for approximating the characteristic of Pt-resistance thermometers with the Callendar-Van Dusen equation (DIN EN 60751) is, for example, the following:

First, the coefficients $R_0$, $A$ and $B$ are determined using all calibration points $T_i \geq 0$ °C. In a second step, the coefficient $C$ is determined by using the calibration points $T_i < 0$ °C.

Hence:

| | | |
|---|---|---|
| $T_{\min} \geq 0$ °C | $m = 3$ | $(R_0, A, B)$ |
| $T_{\max} \leq 0$ °C | $m = 4$ | $(R_0, A, B, C)$ |
| $T_{\min} < 0$ °C $< T_{\max}$ | $m = 4$ | $(R_0, A, B, C)$ |

This results in the following requirement for the number of points $N$ in the two temperature ranges:

| | | |
|---|---|---|
| $T_{\min} \geq 0$ °C | $N \geq 5$ | |
| $T_{\max} \leq 0$ °C | $N \geq 6$ | |
| $T_{\min} < 0$ °C $< T_{\max}$ | $N \geq 8$ | ($N \geq 5$ for step 1 and $N \geq 3$ for step 2) |

The latter requirement, that is to say $N \geq 8$, is a theoretical maximum estimate. In practice, it can probably be reduced. The extent to which this is possible in individual cases must be examined.

This example shows the problems of “divided functions”, i.e. a larger number of calibration points is required.

In this case, the problem can be avoided by using the algorithm from chapter 6.3.2. Here, the approximation is carried out in one step, and the requirement regarding the number of reference points is obtained with $N \geq 6$.
In addition, the boundary conditions from chapter 3.4.4 must also be observed.
As the uncertainty of the characteristic increases towards the ends of the calibration interval – taking the correlation between the coefficients into account which may never be completely neglected – extrapolations must – as far as possible – be avoided as they may very quickly lead to very great errors. Therefore, a least one point must be selected at the ends of the required calibration interval.

Extrapolations are only possible by taking into account a significantly greater uncertainty under the following conditions (simultaneous compliance with all conditions required):

- The characteristic represents a deviation function with **little non-linearity**.
- An approximation with **low order functions** is performed ($m \leq 4$), i.e. there are only a few free parameters.
- The **number of calibration points is significantly larger** than the number of free parameters ($N \geq m + 5$).

In the case of Pt-PtRh thermocouples (type S), these requirements can, for example, be met at the upper temperature limit up to approx. 150 K extrapolation.

In the case of very small uncertainties of the individual calibration points *and* a small number of points distributed approximately equidistantly, interpolation procedures should be preferred to approximation approaches. However, this requires an exactly known mathematical model of the desired characteristic curve.

If a sufficient number of calibration points is available, approximation procedures can – due to their principle – partly compensate the effect of the uncertainty of the individual points on the characteristic.

A physically justifiable approach with a corresponding physical importance of the single characteristic coefficients is, in any case, to be preferred to a general approach, e.g. a polynomial approach.

Types of characteristics which have already been realized in the software and/or in the hardware of the calibration object are to be preferred.

## 2.5 Software election / validation

A variety of commercial software is available to numerically determine the coefficients of the selected equation with given value pairs (here: measured values. As to the software selection, no general specifications can be made. In addition to the use of standard software, it is also possible for the laboratory to create its own evaluation software or to use software supplied by the customer.

In any case, the numerical method should be validated for each type of characteristic used. The use of test data sets with known characteristic curve results is recommended. For this purpose, the test and example data sets indicated in the individual sections of this Guideline for the different thermometer types (→ 0) may be used.

It is also possible to compare the results of different programmes or calculation methods. In doing so, it is important not to compare the coefficients but only the resulting functions. Due to different algorithms and different calculation accuracies, there may be different coefficients. However, these coefficients compensate each other and thus lead to an almost identical curve.

For that reason, the uncertainty of the coefficients is of lesser importance for the application. However, the uncertainty contribution of the resulting thermometer characteristic must be taken into account.

The numerical stability is to be investigated with respect to the number of calibration points (addition or omission of points as reference points) and with respect to the effects of small uncertainties of the calibration points (defined slight change of individual points and investigation of the resulting effects on the specific characteristic curve).

**As part of the routine use of the software, it is strongly recommended to re-enter the original data (measured values) into the determined function after each approximation in order to check the correctness of the determined coefficients.**

## 2.6 Calibration certificate

The document DAkkS-DKD-5 “Instructions on how to compile a calibration certificate” serves as basis for the creation of a calibration certificate. When specifying the measurement uncertainty, the document DAkkS-DKD-3 “Indication of the measurement uncertainty in calibrations” and here, in particular, Chapter 6 must be observed.

When specifying a characteristic curve in the calibration certificate, which is normally the case, it is stated in the section “Measurement results”. It is indicated after the individual calibration values and their measurement uncertainties.

If several sensors are listed in a calibration certificate, e.g. for multi-channel measuring instruments, the characteristic curves can be summarized according to the calibration values of all sensors or measuring channels, **or** they can be summarized with the respective set of measured values of a sensor or measuring channel. a clear assignment of the sensor, the measurement channel, the calibration values, including their measurement uncertainties and the associated characteristic must be ensured.

In some cases, e.g. in the case of thermocouples, it may be necessary to indicate several partial characteristics. The valid temperature ranges for the respective parts of the characteristic curve must be clearly assigned. Ambiguous definitions, e.g. by overlapping partial characteristic curves, are to be avoided. They are permissible if they are stated in valid standards or guidelines, e.g. ITS-90.

In special cases, it may be necessary to specify several characteristic curve types for one set of calibration data. This may be necessary, for example, if a thermometer is operated on different devices which differ in their internal data preparation. However, this procedure should be limited to exceptions. Here, too, a clear assignment must be ensured.

For each characteristic or partial characteristic, the valid temperature range must be indicated, and it must be possible to assign an uncertainty to it. The measurement uncertainty stated must contain all uncertainty components, including the components of the calibration values (influences of the standard, the calibration object and the calibration procedure) and the influence of the limited mathematical descriptiveness (approximability) of the thermometer behaviour by the corresponding characteristic function (mathematical model).

If different measurement uncertainties are stated as a function of the temperature, the limits of the sections must not stringently coincide with the limits of possibly existing partial characteristics.

If an unambiguous assignment is possible, the measurement uncertainty for the characteristic can also be stated in the section “Measurement uncertainty”.

It is not necessary to indicate the uncertainty of coefficients; however, it may serve as an indication to limit the number of digits of the indicated coefficients to a reasonable level.

An example for drawing up the sections “Measurement results” and “Measurement uncertainty” in a calibration certificate can be found in Appendix 6.4.

# 3 Platinum resistance thermometers

Basically. this chapter applies to all platinum resistance thermometers. However, it is specially tailored to the requirements of Pt-100 thermometers. As these show very different measurement uncertainties – depending on the sensor type, the instrument model and the temperature range – different requirements for the measurement uncertainty are taken into account.

## 3.1 Types of approximation equations

Equations of the form $t = f(R)$ have not become generally accepted in practical applications, because they describe the behaviour of a platinum resistance thermometer less effectively. Only with higher-order functions (and their known disadvantages) can characteristics for platinum resistance thermometers be well described.

In the following, some types of characteristics will be analysed which have proved their worth in practical applications, because – on the one hand – the behaviour of the platinum resistance thermometers is described with sufficient accuracy for most cases and – on the other hand – the mathematical effort required for the understanding and application of these equations is still justifiable. It is for this reason that the following types of characteristics have found widespread use.

The mathematical representation of the equations described is given in Table 3.1. Appendix 6.1.1 lists the uncertainty influences that are to be expected.

### 3.1.1 Callendar-Van Dusen equation

The Callendar-Van Dusen equation, which is also used in DIN EN IEC 60 751 to represent the so-called DIN characteristic curve, is widely used. (See equation No. 1 in Table 3.1)

It is appropriate for moderate requirements on measurement uncertainty over a wide temperature range when using common Pt-100 types.

### 3.1.2 Standard polynomial

For applications from –40 °C to up to 600 °C, standard polynomials (equations Nos. 2 to 4 in Table 3.) are well suited for mean to high uncertainty requirements. This uncertainty depends on the degree of the polynomial (see Appendix 0).

Before the approximation is performed it must be determined whether the polynomial function shall exactly pass through the value $R_0$ at 0 °C. If this is not necessary, $R_0$ can be approximated equitably with the other coefficients (→ 0).

### 3.1.3 Deviation polynomial

For large temperature ranges which shall be covered by approximation equations with small uncertainties, it is recommended to use polynomials to describe the deviation from the ITS-90 reference function[1]. (See equations Nos. 5 to 7 in Table 3.1)

Although calibration in accordance with ITS-90 is only permitted for platinum resistance thermometers made of high-purity platinum with a $W_{\text{Ga}} \geq$ 1.11807 ([2]), the ITS-90 reference function also describes the basic behaviour of industrial platinum resistance thermometers quite accurately. The deviation functions from this reference can be of very low order (typically of the first or second order). This – in addition to the small uncertainties – leads to a reduction in the measurement (time) effort.

This type of approximation is recommendable only for thermally stable types of thermometers, whose characteristic has been investigated with additional measurement points (degrees of freedom[3] $\nu \geq 5$) for the suitability of this equation type (or of the ITS-90 reference function as the basic function).

## 3.2 Use of the measurement values

In many cases, the original data for the approximation will be measurement values as indicated in the calibration certificate. The way the data are fed into the approximation mathematics depends on the thermal stability of the respective thermometer, the temperature range used and the required measurement uncertainty.

The following subsections deal with various aspects of data preparation; however, these aspects may not always be independent of each other and are therefore mentioned at several points.

### 3.2.1 $R_0$ value

In almost each calibration, also the resistance of the platinum thermometer at the ice point ($R_0$) or at the water triple point ($R_{0.01}$) is determined (often even several times – for example at the beginning and at the end of the calibration.

For less precise requirements, it is sufficient to approximate all data – including the measured values at the zero point – on an equal basis. Due to the measurements which are (in most) cases performed several times at the zero point, a higher weight is automatically obtained for this temperature (→ chapter 0 and equation (2-2)) during the approximation. This is reasonable, as the temperature of the zero point is generally represented with the largest accuracy and as the calibration object is – due to the multiple measurements (at different moments) – also very well known at this temperature.

In the case indicated in the test data set (→ 6.2.1), the $R_0$ value results from the approximation, i.e. it is the result of a calculation.

Alternatively, it is possible to use the arithmetic mean value of all $R_0$ or $R_{0.01}$ measurements or the value of the last zero-point measurement for the zero point. (This depends on whether a → "$W$ value formation" is carried out; Section 3.2.2)

The number of the required measurement values (→ 0) is independent of whether the zero point is specified by a measurement value (or by a mean value of measurement values) *or*

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[1] Determination of the assignment of temperature and resistance of a platinum resistance thermometer with ideally pure platinum. [18]

[2] Resistance ratio of the thermometer: $W_{\text{Ga}} = R(29{,}7646\ °\text{C}) / R(0.01\ °\text{C})$

[3] Degree of freedom $\nu = n - p$ with $n$: number of measurement points $p$: number of the free parameters of the approximation equation $f(t)$

whether it is approximated. Although the equation has one free parameter less if the zero point is specified, this is equalled out by the measurement value required for specifying $R_0$. The measurement effort thus remains unaffected.
In the equations, the zero point is usually represented by $R_0$.

If required by the customer, $R_0$ can be replaced by $R_{0.01}$. This must be taken into account before calculating the coefficients.

$R_{0.01}$ can be converted into $R_0$ by means of the following relation:

$$R_0 \cong R_{0.01} \cdot 0.999\,961 \quad (3\text{-}1)$$

### 3.2.2 Formation of the $W$ value

When a sensor shows drift, in most cases, mainly the resistance measurement values – e.g. at the water triple point – change. However, the characteristic in relation to the water triple point remains largely unchanged. An advantage of the ratio formation is that it allows a better characterization of the thermometer. As a result, it is largely independent of a change in the thermometer resistance after thermal or mechanical load, as long as a current water triple point value is known.

For drift-affected sensors or in case of very precise approximation requirements, a $W$ value must be formed before calculating the characteristic curve. This means that for each resistance measurement value $R_x$ at a temperature $t_x$, the ratio to the resistance value $R_{0.01}$ at the water triple point (0.01 °C) is formed via

$$W(t_x) = R_x/R_{0.01}. \quad (3\text{-}2)$$

Between the measurements at $t_x$ and 0.01 °C, the sensor of the thermometer must not considerably drift. These two measurements should be carried out as closely as possible in time. A measurement at the water triple point, directly after the measurement at $t_x$, is optimal. The more often the water triple point is measured (in the ideal case: after *each* temperature $t_x$), the more precise the approximation becomes. If – under certain conditions (e.g. in specific temperature ranges) – a drift of the sensor can be ruled out, a multiple measurement of the $R_{0.01}$ value is not required.

The equations 1 to 4 indicated below can also be used with $W$ values. The multiplication of the $W$ values with the current water triple point value results in resistance values equivalent to those used in the equations mentioned.

The form of the equations 1 to 4 indicated here allows the use of a current measurement value at the zero point without having to redetermine the coefficients. For the equations 5 to 7, this applies anyway because of their principle.

Similarly, this procedure can also be used with ice point measurements of $R_0$ at 0 °C. The conversion from $R_{0.01}$ into $R_0$ is described in section 3.2.1.

### 3.2.3 Weighting factors

If the measurement values used are affected by different measurement uncertainties, this can be taken into account during approximation by introducing weighting factors. However, depending on the arrangement of the measurement values and on the differences in the measurement uncertainties, there is the risk of polynomial overshoots. This risk is reduced by a large number of measurement values. Before weighting, an unfavourable or unrealistic approximation of the characteristic must be excluded. Especially the measurement uncertainties of the individual calibration points must be determined correctly.

In particular the marginal conditions in sections 0 and 0 must be taken into account here.

The mathematical realisation of the weighting is shown in section 2.2.2.

## 3.3 Arrangement of the reference points

- The reference points (measured values) must be distributed as uniformly as possible over the entire temperature range.
- When considering the deviation function between real characteristic and mathematical model, the reference points should ideally be located at the zero passages ("knots") and extreme values (maxima / minima) of this deviation function.
- There must be reference points at the limits of the temperature range.
- An extrapolation beyond these limits is not permissible and would lead to rapidly increasing deviations compared to the real thermometer.

## 3.4 Number of reference points

Practical and numerical investigations show that from approx. two degrees of freedom on ($\nu \geq 2$), the influence of the number of reference points is only of secondary importance for the approximation function of a typical Pt-100 thermometer.

In special cases, approximations with one degree of freedom ($\nu = 1$) or interpolations ($\nu = 0$) are also possible. However, in such cases the uncertainty of the resulting characteristic function is significantly increased. (cf. Table 6.1 and Table 6.2). It is only under certain conditions (e.g. limited temperature range) that such approximations yield useful results. ($\rightarrow$ 3.4.2)

### 3.4.1 Usually known thermometer types (normal case)

Correspondingly, the following table shows the **number of required measurement values** for $\nu = 2$ – for a Pt 100 type that does not meet the requirements of Section 3.4.2 (normal case):

| No. | Equation | Minimum number of measured values |
|---|---|---|
| 1 | Callendar-Van Dusen (CvD)<br>$R = R_0 \cdot (1 + At + Bt^2 + C(t - 100\ °\mathrm{C})t^3)$ \| $C = 0$ for $t > 0$ °C | 6[4]<br>5 for $t_{\min} \geq 0$ °C |
| 2 | 2nd order: $R = R_0 \cdot (1 + at + bt^2)$ | 5 |
| 3 | 3rd order: $R = R_0 \cdot (1 + at + bt^2 + ct^3)$ | 6 |
| 4 | 4th order: $R = R_0 \cdot (1 + at + bt^2 + ct^3 + dt^4)$ | 7 |
| 5 | ITS + 1st order: $W(t) = W_r(t) + a(W(t) - 1)$ | 4 |
| 6 | ITS + 2nd order: $W(t) = W_r(t) + a(W(t) - 1) + b(W(t) - 1)^2$ | 5 |
| 7 | ITS + 3rd order: $W(t) = W_r(t) + a(W(t) - 1) + b(W(t) - 1)^2 + c(W(t) - 1)^3$ | 6 |

**Table 3.1**

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[4] When special attention is paid to chapters 2.4 and 3.4.4.

### 3.4.2 Closely investigated thermometer types (special case)

Approximations with a lesser degree of freedom should, as a matter of principle, be performed only in exceptional, justified cases.

The following preconditions must *all* be fulfilled with less than two degrees of freedom ($\nu < 2$) when characteristics are to be approximated or interpolated:

- The thermometer type (same type designation and instrument type) is known from earlier investigations with additional measurement points (degrees of freedom $\nu \geq 5$) with respect to the scatter of the characteristics of several (≥ 10) thermometer specimens; in the case of a maximum temperature range from 0 °C to 100 °C , degrees of freedom of $\nu \geq 3$ are sufficient for the investigation.
- The interpolation function has been proved to be suitable for this type of thermometer and temperature range, i.e. the approximability of the characteristic of this type of thermometer has been examined in earlier investigations with additional measuring points and with respect to the deviation between the characteristic function and the measured values.
- The temperature range should be limited (usually –40 °C to 200 °C – or part thereof) and must be within the thoroughly investigated temperature range.
- The reference points (measured values) are evenly distributed over the entire temperature range (see also Section 3.3).
- The influence of the number of measuring points on the uncertainty of the approximation of the characteristic must be determined and taken into account.
- By this, the measurement uncertainty is increased compared to a calibration with additional measuring points. (This is to be taken into account in the uncertainty budget!)

According to the degree of freedom, the number of required measurement values is reduced (see Table 3.1).
If *one* of the above conditions is not fulfilled, two degrees of freedom are the absolute minimum. Further information regarding the selection of reference points (→ 2.4) and the resulting uncertainties (→ 2.3) are given in the chapter “Fundamentals”.

### 3.4.3 Known thermometer specimens (recalibration)

If an individual thermometer is recalibrated, and if that thermometer is known from at least one previous calibration with additional measurement points, the number of required measurement values for subsequent calibration(s) may be reduced. The requirements are comparable to those in Section 3.4.2.

To approximate or interpolate characteristics with less than two degrees of freedom ($\nu < 2$), the following prerequisites must *all* be met:

- The thermometer specimen is known from previous calibrations with additional measurement points (degrees of freedom $\nu \geq 3$); in the case of a maximum temperature range from 0 °C to 100 °C, additional measurement points for degrees of freedom $\nu \geq 2$ are sufficient.
- The suitability of the characteristic function for this thermometer specimen has been confirmed by calibration(s) with additional measuring points.
- The temperature range should be limited (usually –40 °C to 200 °C – or part thereof) and must lie inside the temperature range for which the calibration(s) with additional measurement points has/have been performed.
- The reference points (measured values) are uniformly distributed over the entire temperature range (see also Section 3.3).

- The influence of the number of measuring points on the uncertainty of the approximation of the characteristic must be determined and taken into account.
- Thus, the measurement uncertainty is increased compared to a calibration with additional measuring points. (This is to be taken into account in the uncertainty budget!)

### 3.4.4 Temperatures below 0 °C

Due to the properties of platinum resistance thermometer sensors, the determination of the characteristic in the negative temperature range is sometimes problematic or affected by clearly larger uncertainties.

When calibrating a platinum resistance thermometer in the negative temperature range, the following points must be observed:

- For measuring ranges starting below –40 °C, at least two reference points have to be located in the negative range. If the measuring range starts below –80 °C, at least three points are required in the negative range. The number is valid without counting the measurement values at 0 °C.
- For measuring ranges starting below –40 °C and ending above 200 °C, the value for the minimum number of reference points from Table 3.1 must be increased by at least one (degrees of freedom $\nu \geq 3$). This may also be the case with lower upper range limits to ensure that there are enough reference points in the positive range of the characteristic.
- For a good approximation in the negative temperature range below –80 °C, it is advisable to approximate the deviations of the measured values from the ITS-90 reference function (→ 3.1.3).

  Alternatively, a function divided at 0 °C can be used. However, this procedure requires a corresponding number of reference points for each partial function (→ Table 3.1). This in turn requires an increased measuring effort.

When using the Callendar-Van Dusen equation for a temperature range of $T_{\mathrm{min}}$ < 0 °C < $T_{\mathrm{max}}$, the algorithm from Chapter 6.3.2 should be used to be able to use the number of required measuring points according to Table 3.1. Otherwise, special attention must be paid to Chapter 2.4.

If – according to section 3.4.2 – the type of thermometer is very well known and if, for example, a deviation function is used for this specific type of thermometer, these points may be ignored. This may, however, result in – sometimes drastically – increased uncertainties of the approximation.

### 3.4.5 High temperatures

Above approx. 500 °C, some thermometer types show a behaviour which is untypical for Pt-100. This is reflected by a flattened characteristic of these thermometers.

The actual shape of the characteristic in this temperature range must then be determined by means of additional measuring points.

In this case, it must be clarified whether

- the mathematical model selected for the desired application (temperature range, measurement uncertainty) in accordance with Appendix 6.1.1 is nonetheless applicable or if increased approximation errors must be expected, and
- the number of reference points according to Table 3.1 is sufficient.

This investigation is only required for some representative specimens (≥ 5) of a thermometer type, but for each desired temperature range.

## 3.5 Determination of an appropriate description of the characteristic

The decision as to which approximation equation should be used depends primarily on the desired measurement uncertainty and the required temperature range. Given that – apart from exceptions (→ 3.4.2, 3.4.3) – the condition regarding the number of degrees of freedom $\nu \geq 2$, must be met, these considerations should always be made before starting the calibration. That is the only way to select suitable calibration points (number and arrangement) for the desired approximation equation.

Details regarding the selection of a suitable equation can be found in Section 3.1. Appropriate approximation equations and indications of their uncertainties are dealt with in Section 6.1. The mathematical bases are given in Section 2.3.

Appendix 6.1.1 offers some standard approximation equations for different cases. Suitable equations can also be determined using commercially available mathematical or special approximation software. (→ 2.5).

The uncertainty component to be expected, which is not only caused by the measurement but also by the properties of the characteristic curve equation used, is given in Appendix 6.1.1.

## 3.6 Calculation method

The software is selected according to Section 2.5.

All approximation equations listed in Appendix 6.1.1 can be traced back to a Gaussian polynomial which can be solved using a matrix equation. As an example, the equation system has been established for the second-order polynomial in Appendix 6.3.1.

The Callendar-Van Dusen equation constitutes an exception. Although it is a divided function, a closed solution exists here, too, in the case of which all coefficients are determined in one step. The advantage is that the part of the characteristic curve below 0 °C is *also* used to determine the coefficients $a$ and $b$. The equation system for the Callendar-Van Dusen equation is set out in Appendix 6.3.2.

### 3.6.1 Testing of the calculation method

After each approximation, the measured values should be entered into the resulting function and compared with the original data in order to rule out or detect calculation, measurement and transmission errors.

In any case, new calculation methods or software must undergo a fundamental test. General information on this can be found in Chapters 2.5 and 6.2, special information on platinum resistance thermometers in 6.2.1.

# 4 Thermocouples

## 4.1 Types of approximation equations

Subsequently, we will look at two possibilities which, in practice, have proved to be useful for describing the individual thermoelectric properties of thermocouples sufficiently well, and which keep the mathematical effort for understanding and handling the associated equations at a reasonable level.

### 4.1.1 Standard polynomial

Standard polynomials are generally well suited to describe the relationship between temperature and thermoelectric voltage when calibrating thermocouples with ignoble thermo wires by means of the comparison method. Depending on the type of thermocouple and the required measurement uncertainties, 4th to 6th order polynomials are usually sufficient. For lower requirements on the measurement uncertainty and limited temperature ranges, it is often possible to use polynomials from the 2nd order on.

### 4.1.2 Deviation polynomial

To calibrate thermocouples with thermo wires of noble metals by the comparison method or at fixed points, it is recommended to use deviation polynomials of the 2nd or 3rd order for the respective reference characteristics. Especially for fixed-point calibrations with only a limited number of calibration points, it is necessary to establish a relationship to the respective reference characteristic.

## 4.2 Use of the measurement values

### 4.2.1 Weighting factors

If the measured values have different measurement uncertainties, this can be taken into account during approximation by introducing weighting factors. However, depending on the arrangement of the measured values and the differences in the measurement uncertainties, there is the risk of polynomial overshoots. This risk can be reduced by using a large number of measurement values. Before weighting, it must be ensured that an unfavourable or unrealistic approximation of the characteristic can be ruled out.

The general conditions specified in Sections 4.3 and 4.4 are to be particularly observed.

The mathematical realisation of the weighting is shown in Section 2.2.2.

## 4.3 Arrangement of the reference points

- When using the comparison method, the reference points (measured values) must be distributed as evenly as possible over the entire temperature range.
- Reference points must be provided at the limits of the temperature range.
- Extrapolation beyond these limits is permissible but leads to increased measurement uncertainties in the extrapolated temperature ranges (→ 4.3.1).

### 4.3.1 Extrapolation of the measurement results

Basically, an extrapolation of measured values is only necessary when calibrating thermocouples made of noble metals (types S, R and platinum/palladium thermocouples) at fixed points due to a lack of calibration points above the solidification temperature of copper. For calibrations using the comparison procedure, the calibration points can generally be selected without restriction, on the basis of the required temperature range.

Estimates of the additional uncertainty in the case of an extrapolation of the measured thermoelectric voltages to 1200 °C can be derived from statistical investigations of the calibration results of 33 type S thermocouples which were calibrated at fixed points at PTB's “Temperature” department. Deviation functions of the 1st and/or 2nd to 3rd order from the reference characteristic were determined for all thermocouples. In a further step, the thermoelectric voltages at 1200 °C were calculated using the two or three approximation equations determined for each thermocouple, and the maximum difference between these voltages was determined. At 1200 °C, the mean voltage difference of the 33 thermocouples amounted to 2.3 µV and the maximum thermoelectric voltage difference (one thermocouple) amounted to 3.8 µV. If the last value is regarded as a limiting value, a rectangular distribution can be assumed so that an uncertainty contribution of 2.2 µV must be considered in the uncertainty budget. If the mean thermoelectric voltage difference is used (normal distribution), a comparable value of 2.3 µV is obtained as uncertainty contribution.

These estimates are in conformity with the published data [30] for the extrapolation of thermoelectric voltages of thermocouples types S and R up to 1300 °C (mean deviation: 1.4 µV, maximum deviation: 3 µV).

## 4.4 Number of reference points

An interpolation (→ 2.2.1) of the measured values, i.e. the use of only as many measuring points at different temperatures as the equation used has free parameters, only makes sense under limited conditions. The following prerequisites must be fulfilled:

- the thermocouple is known from previous calibrations with additional measuring points (degrees of freedom $\nu \geq 5$) with respect to the scattering of the measured values;
- the number of calculated coefficients of the selected interpolation equation (no deviation function) corresponds to the number of coefficients of the respective reference characteristic for this type of thermocouple and temperature range
- *or* a deviation function from reference characteristics is determined;
- the thermocouple is calibrated only in a very limited temperature range;
- compared to a calibration with additional measurement points, an increased measurement uncertainty is stated,
- this component must be determined and taken into account in the uncertainty analysis of the characteristic.

In all other cases, the number of known reference points must exceed the number of free parameters searched for. For calibrations using the comparison method, the number of degrees of freedom must – in the normal case (i.e. the above conditions are not met) – be set to $\nu \geq 2$.

Further information on the selection of the reference points (→ 2.4) and the resulting uncertainties (→ 2.3) is given in the chapter “Fundamentals”.

## 4.5 Determination of a suitable description of the characteristic

The decision as to which approximation equation should be used depends primarily on the desired measurement uncertainty and the required temperature range. As the requirement for the number of the degrees of freedom $\nu \geq 2$ must, apart from some exceptions (→ 0), be complied with, the number of the free parameters or the order of the approximation equation is limited due to a limited number of measurement values (reference points). In the case of a fixed-point calibration, this is of particular relevance.

When calibrating according to the comparison procedure, the desired measurement uncertainty should always be considered before starting the calibration. This is the only way to select suitable calibration points (number and arrangement) for the desired approximation equation.

Details on how to select an appropriate equation can be found in Section 4.1. Suitable approximation equations are dealt with in Section 6.1. The mathematical principles are given in Section 2.3.

Appendix 6.1.2 proposes common approximation equations for various cases. Suitable equations can also be determined by means of commercially available mathematical or special approximation software (→ 2.5).

## 4.6 Calculation method

The software is selected according to Section 2.5.

All approximation equations listed in Appendix 6.1.2 can be traced back to a Gaussian polynomial which can be solved using a matrix equation. Appendix 0 shows the equation system for the 2nd order polynomial.

### 4.6.1 Testing of the calculation method

After each approximation, the measured values should be entered into the resulting function and compared with the original data in order to rule out or detect calculation, measurement and transmission errors.

Independent of this, new calculation methods or software must be subjected to a fundamental test. Corresponding general information can be found in Chapters 2.5 and 6.2, and special information on thermocouples in 6.2.2.

# 5 Other thermometers

## 5.1 Thermistors

Thermistors are ceramic semiconductor resistors. For thermometric measurement tasks, above all the types with a negative temperature coefficient (thermistors, NTC resistors) are appropriate. These thermistor types are mainly used in the temperature range between –100 °C and maximally 300 °C.

The approximated exponential dependence of the resistance on the temperature is indicated by characteristic curves for which the following applies:

$$R_T = R_{T0} \exp\left(\frac{\beta(T_0 - T)}{T \cdot T_0}\right) \qquad (5\text{-}1)$$

with

$T$ temperature in K
$T_0$ reference temperature in K
$R_\mathrm{T}$ resistance at temperature $T$
$R_{\mathrm{T}0}$ resistance at reference temperature $T_0$
$\beta$ constant (form- and material-dependent) in K

The same is valid for the dependency of the temperature on the resistance $T = f(R)$:

$$T = \left[\frac{1}{\beta}\ln\left(\frac{R_T}{R_{T0}}\right) + \frac{1}{T_0}\right]^{-1} \qquad (5\text{-}2)$$

Within limited temperature ranges (see Table 5.1), the characteristic of a thermistor can be approximated in principle by means of a linear correlation between $\ln R_\mathrm{T}$ and $1/T$.

| **Temperature margin** ($t \geq 0$ °C) | **Uncertainty** |
|---|---|
| 10 K | 0.01 K |
| 20 K | 0.04 K |
| 30 K | 0.10 K |
| 40 K | 0.20 K |
| 50 K | 0.30 K |

**Table 5.1:** Uncertainties when using equations 5-1 and 5-2 [34]

Over larger temperature ranges, the uncertainties associated with such a linear approximation are often no longer tolerable. For this reason, the dependence $\ln R_\mathrm{T} = f(1/T)$ should be approximated with higher-order polynomials. Very good results were achieved by using a third-degree polynomial (Steinhart and Hart equation):

$$\frac{1}{T} = a_0 + a_1(\ln R_T) + a_2(\ln R_T)^2 + a_3(\ln R_T)^3 \qquad (5\text{-}3)$$

or

$$\ln R_T = A_0 + \frac{A_1}{T} + \frac{A_2}{T^2} + \frac{A_3}{T^3} \qquad (5\text{-}4)$$

Extensive investigations with 17 different metal oxide ceramics in the temperature range between –80 °C and 260 °C and with resistances between 10 Ω and 2 MΩ at 25 °C were carried out by Sapoff et al. [35]. When using 3rd degree polynomials (equations 5-3 and 5-4), the results can be summarized as follows:

When using equations 5-3 and 5-4, interpolation errors occur; these errors lie in the order of the measurement uncertainty during the calibration of the thermistors if the following conditions are fulfilled:

- temperature margin of 100 K in the temperature range –80 °C $\leq t \leq$ 260 °C
- temperature margin of 150 K in the temperature range –60 °C $\leq t \leq$ 260 °C
- temperature margin von 150 K to 200 K in the temperature range 0 °C $\leq t \leq$ 260 °C

As the equations 5-3 and 5-4 have four unknown parameters, at least four measurement points are required for a calibration to determine the values of the parameters. When the thermistor types are not exactly known (normal case), at least six measurement points are required to avoid an unnecessary increase in the uncertainty of the resulting characteristic function and to check the mathematic model used.

With a small temperature range (max. 50 K) the approximability of the thermistor characteristic improves; in this case, five measuring points may be sufficient.

In order to approximate or interpolate characteristics (of exactly known thermistor types) with less than two degrees of freedom ($\nu < 2$), *all* of the following points should be observed:

- the type of thermometer (same designation and type) is known from previous investigations with additional measuring points (degrees of freedom $\nu \geq 5$) with respect to the scattering of the characteristic of several (≥ 10) thermometer specimens; at a temperature margin of max. 60 K, degrees of freedom of $\nu \geq 3$ are sufficient for the investigation
- the interpolation function has been proved to be suitable for this type of thermometer and temperature range, i.e. the possibility of approximating the characteristics of this thermometer type has been investigated in previous investigations with additional measurement points with respect to the deviation between the characteristic function and the measurement values
- the temperature margin should not exceed 100 K and must lie within the thoroughly investigated temperature range
- the reference points (measurement values) are uniformly distributed over the entire temperature range
- the influence of the number of measuring points on the uncertainty of the characteristic approximation must be determined and taken into account
- the measurement uncertainty is thus increased compared to a calibration with additional measuring points. (This is to be taken into account in the uncertainty budget!)

Similar requirements apply when recalibrating an individual thermistor specimen with $\nu < 2$. Again, *all* requirements must be met:

- the thermometer specimen is known from previous from previous calibrations with additional measurement points (degrees of freedom $\nu \geq 3$); at a temperature margin of max. 60 K, additional measurement points for degrees of freedom $\nu \geq 2$ are sufficient
- the suitability of the characteristic function for this thermometer specimen has been confirmed by calibration(s) with additional measuring points
- the temperature margin should not exceed 100 K and must lie within the temperature range in which the calibration(s) with additional measuring points has (have) been performed
- the reference points (measured values) are equally distributed over the entire temperature range
- the influence of the number of measuring points on the uncertainty of the characteristic approximation must be determined and taken into account
- the uncertainty of measurement is thus increased compared to a calibration with additional measuring points. (This is to be taken into account in the uncertainty budget!)

Approximations with less than two degrees of freedom ($\nu < 2$) should only be performed in justified exceptional cases.

However, the calibration effort can also be reduced for less well-known thermistor types by omitting the quadratic terms in the above equations. Sapoff und Siwek [36] investigated the accuracies of the characteristic which can be achieved by using the simplified equations 5-5 and 5-6 for the description of the relationship between temperature and resistance.

$$\frac{1}{T} = b_0 + b_1[\ln R_T] + b_3[\ln R_T]^3 \qquad (5\text{-}5)$$

$$\ln R_T = B_0 + \frac{B_1}{T} + \frac{B_3}{T^3} \qquad (5\text{-}6)$$

The uncertainty contribution caused by the approximation of the characteristic when using equations 5-5 and 5-6 is estimated as summarized in Table 5.2 [34].

| Temperature margin | Temperature range | Uncertainty |
|---|---|---|
| 50 K | 0 °C $\leq t \leq$ 260 °C | 0.001 K to 0.003 K |
| 50 K | –80 °C $\leq t \leq$ 0 °C | 0.01 K to 0.02 K |
| 100 K | 0 °C $\leq t \leq$ 260 °C | 0.01 K |
| 100 K | –80 °C $\leq t \leq$ 25 °C | 0.02 K to 0.03 K |
| 150 K | 50 °C $\leq t \leq$ 200 °C | 0.015 K |
| 150 K | 0 °C $\leq t \leq$ 150 °C | 0.045 K |
| 150 K | –60 °C $\leq t \leq$ 90 °C | 0.1 K |
| 200 K | 0 °C $\leq t \leq$ 200 °C | 0.08 K |

**Table 5.2:** Uncertainties when using equations 5-5 and 5-6

As the non-linearity of the relation $\ln R_\mathrm{T} = f(1/T)$ depends – in addition to the temperature range used – also on the material properties of the thermistor, the stated uncertainties are only estimates of the uncertainty contribution caused by the approximation of the characteristic.

The software or calculation method used to calculate the coefficients of the above equations (5-3 and 5-5) should be thoroughly tested before release. For this purpose, fictitious measurements are given in Appendix 6.2.3. After approximation of these values, the result can be compared with the results given in the same section.

After each approximation, the measured values should be entered into the resulting function and compared with the original data in order to rule out or detect calculation, measurement and transmission errors.

## 5.2 Liquid-in-glass thermometers

The linearity of the reading of a liquid-in-glass thermometer largely depends on the cross-sectional area of the thermometer capillaries and their change over the length of the thermometer. A non-linear expansion of the thermometer liquid can also contribute to the non-linearity of the thermometer reading; however, this should already be taken into account in the scale division since the behaviour of the usual thermometer liquids (mercury and alcohol) is well known.

The change of the cross-sectional area of the thermometer capillaries does not follow any principles which can be approximated. An interpolation between the measurement points is, therefore, normally not performed.
A possible approach, however, is the gradual linear interpolation between the measuring points. Also see Chapter 2.2.1.

Normally, however, characteristics are not approximated for liquid-in-glass thermometers.

Further details regarding the handling of liquid-in-glass thermometers can be found in the respective PTB Testing Instruction [13].

## 5.3 Direct-reading electronic thermometers

Direct-reading thermometers with electronic (generally digital) display consist of a sensor element (e.g. Pt-100, thermocouple, thermistor, diode sensor, transistor sensor, band gap sensor, quartz oscillator (QuaT) sensor) and a measuring and indicating electronics. The electronics linearizes the characteristic of the corresponding sensor and leads to a direct indication of the sensor temperature. In a first approximation, an (additional) linearization of the sensor characteristic by the user is not required.

However, there are errors caused by an insufficient description of the characteristic by the electronic system. As the characteristic used by the electronic system is in most cases not known, it is not possible for the user to apply a sensor-based characteristic curve, because a backward calculation to the actual measurement values of the sensor is not possible.

Instead, the attempt can be made to correct the indicated values – section by section – by a linear interpolation between the measurement points (→2.2.1, polygon interpolation). For direct-reading electronic thermometers, however, characteristics are normally not approximated.

# 6 Appendix

## 6.1 Recommended approximation equations

The following subsections 6.1.1 to 6.1.2 offer common approximation equations for widely spread thermometer types. The indicated measurement uncertainties to be expected only refer to the properties of the resulting function – which describes the thermometer behaviour– as compared to the real characteristic of the respective thermometer. The actual measurement errors (i.e. measurement deviations) must be considered separately.

In the case of measurement errors that are assumed to be approximately random, the effects on the resulting function become smaller with the number of measurement points; i.e., the more measurement points there are, the smaller the effects on the resulting function. In this case, the approximated function has a compensating effect (→ 2.2.2). The limits of the calibration range form an exception; here, the influence of measurement errors increases.

The component of the uncertainty of the characteristic function which is caused by measurement errors (deviation-affected reference points) can be estimated with the aid of “worst-case simulations” (variation of the measurement values in the range of the measurement uncertainty) when the calculated coefficients are checked (→ 0). This component thus describes the influence of the measurement errors between the reference points. This is, among others, caused by the propagation of (and a possible increase in) the measurement uncertainty of single measurement points to the complete characteristic function. The component has not been (and cannot be) considered in the uncertainty tables in the following subsections.

In general, this uncertainty component is negligible compared to the actual measurement uncertainties of the calibration points – given that a reasonable approximation equation is used, and the arrangement and number of reference points have been selected in accordance with this Guideline.

In summary, there are three uncertainty components which contribute to the overall uncertainty of the approximated characteristic of a thermometer:

- the actual measurement uncertainties of the calibration points,
- the error in the characteristic curve caused by the propagation of the measurement errors,
- and the error of the mathematical model – i.e. the deviation between calculated and actual thermometer characteristic, in the case of an ideal measurement.

Only the error of the mathematical model is discussed and estimated in Chapter 6.1.1.

All three influences are partially correlated with each other, so the overall uncertainty can only be determined in a first approximation by formation of a quadratic sum. Further information can be found in Chapter 2.3.2.

### 6.1.1 Recommended approximation equations for Pt-100

The following tables list the recommended approximation equations of the Pt-100 characteristic curve for different temperature ranges along with the error intervals estimated as typical. The half-width of the respective error interval is given. The probability distribution is rectangular. The error interval only refers to the approximation, i.e. the quality of the respective mathematical model, and does not include any other uncertainty components.

The indicated limits were obtained both from practical measurements and from mathematically theoretical considerations. For thermometers behaving untypically for a Pt-100 or being very unstable, an approximation might show much larger deviations.

In addition, the general conditions specified in Sections 3.2 to 3.4 apply.

The indications for the equations 6.1.1-5 to 6.1.1-7 are approximate values, as they could be determined only from practical measurements which are always also affected by a measurement uncertainty and whose thermometer types used must not necessarily be representative.

The grey-coloured cells in the Tables are approximation equations which are not suitable for the respective temperature range as the number of parameters (or the order) would be unnecessarily high. The measurement uncertainty of the calibration which could usually be achieved would oppose a further reduction in the overall uncertainty when the respective equations were used.

Especially for calculations without degree of freedom ($\nu = 0$, Table 6.2) there is – in the case of these temperature ranges and equations – also the risk of polynomial overshoots and disproportionately large deviations from the real characteristic of the thermometer. As basically no additional measurement values are available here for the checking of the characteristic function, the mentioned deviations would usually not be recognized.

Degrees of freedom $\nu \geq 2$

| **Temperature range** | | **–190 to 600 °C** | **–80 to 200 °C** | **–80 to 600 °C** | **–40 to 0 °C** | **–40 to 200 °C** | **–40 to 400 °C** | **0 to 50 °C** | **0 to 100 °C** | **0 to 200 °C** | **0 to 400 °C** | **0 to 600 °C** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Eq. 6.1.1-1** | **CvD** | 40 mK | 6 mK | 40 mK | < 1 mK | 5 mK | 25 mK | < 1 mK | < 1 mK | 4 mK | 25 mK | 40 mK |
| **Eq. 6.1.1-2** | **2nd order** | 1.3 K | 40 mK | 120 mK | < 1 mK | 10 mK | 40 mK | | | | | |
| **Eq. 6.1.1-3** | **3rd order** | 0.8 K | 20 mK | 70 mK | ▨ | 1 mK | 5 mK | ▨ | ▨ | < 1 mK | 4 mK | 40 mK |
| **Eq. 6.1.1-4** | **4th order** | 0.4 K | 10 mK | 15 mK | ▨ | ▨ | 2 mK | ▨ | ▨ | ▨ | 1 mK | 5 mK |
| **Eq. 6.1.1-5** | **ITS+1st ord.** | [5] | (≈ 60 mK)[5] | (≈100 mK)[5] | ▨ | < 5 mK | < 10 mK | ▨ | ▨ | ▨ | < 10 mK | < 60 mK |
| **Eq. 6.1.1-6** | **ITS+2nd ord.** | [5] | (≈ 15 mK)[5] | (≈ 20 mK)[5] | ▨ | ▨ | < 5 mK | ▨ | ▨ | ▨ | < 5 mK | < 10 mK |
| **Eq. 6.1.1-7** | **ITS+3rd ord.** | [5] | ▨ | (≈ 15 mK)[5] | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ |

**Table 6.1:** Typical error intervals of the uncertainty contribution of the characteristic approximation of Pt-100 ($\nu = 2$)

Degrees of freedom $\nu = 0$

| **Temperature range** | | **–190 to 600 °C** | **–80 to 200 °C** | **–80 to 600 °C** | **–40 to 0 °C** | **–40 to 200 °C** | **–40 to 400 °C** | **0 to 50 °C** | **0 to 100 °C** | **0 to 200 °C** | **0 to 400 °C** | **0 to 600 °C** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Eq. 6.1.1-1** | **CvD** | ▨ | ▨ | ▨ | < 1 mK | < 10 mK | ▨ | < 1 mK | < 2 mK | < 8 mK | ▨ | ▨ |
| **Eq. 6.1.1-2** | **2nd order** | ▨ | ▨ | ▨ | < 1 mK | < 20 mK | ▨ | | | | ▨ | ▨ |
| **Eq. 6.1.1-3** | **3rd order** | ▨ | ▨ | ▨ | ▨ | < 2 mK | ▨ | ▨ | ▨ | < 1 mK | ▨ | ▨ |
| **Eq. 6.1.1-4** | **4th order** | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ | ▨ |

**Table 6.2:** Typical error intervals of the uncertainty contribution of the characteristic approximation of Pt-100 ($\nu = 0$)

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[5] Estimation or statement currently not possible due to lack of sufficiently available measurement data.
Grey fields: Corresponding equations are not reasonable for these temperature ranges.

**Eq. 6.1.1-1** **Callendar-Van Dusen (CvD):** $R = R_0 \cdot (1 + At + Bt^2 + C(t - 100\ °\mathrm{C})t^3) \mid C = 0$ **für** $t > 0\ °\mathrm{C}$

**Eq. 6.1.1-2** **2nd order:** $R = R_0 \cdot (1 + at + bt^2)$

**Eq. 6.1.1-3** **3rd order:** $R = R_0 \cdot (1 + at + bt^2 + ct^3)$

**Eq. 6.1.1-4** **4th order:** $R = R_0 \cdot (1 + at + bt^2 + ct^3 + dt^4)$

**Eq. 6.1.1-5** **ITS + 1st order:** $W(t) = W_r(t) + a(W(t) - 1)$

**Eq. 6.1.1-6** **ITS + 2nd order:** $W(t) = W_r(t) + a(W(t) - 1) + b(W(t) - 1)^2$

**Eq. 6.1.1-7** **ITS + 3rd order:** $W(t) = W_r(t) + a(W(t) - 1) + b(W(t) - 1)^2 + c(W(t) - 1)^3$

Required number of calibration points, see Table 3.1 in Section 3.4.1.

### 6.1.2 Recommended approximation equations for thermocouples

Standard polynomial of $m$-th order:

$$E(t) = \sum_{i=0}^{m} a_i \cdot t^i \quad \text{with} \quad m \in \{2...6\}$$

for calibrations including ice point measurement (0 °C), e.g. to consider thermoelectric inhomogeneity at room temperature.
Required number of calibration points $N$ (including ice point measurement): $N \geq m + 3$

Standard polynomial of $m$-th order:

$$E(t) = \sum_{i=1}^{m} a_i \cdot t^i \quad \text{with} \quad m \in \{2...6\}$$

for calibrations without ice point measurement (0 C).
Required number of calibration points $N \geq m + 2$

Deviation polynomial of $m$-th order:

$$E(t) = E_\mathrm{r}(t) + \sum_{i=0}^{m} a_i \cdot t^i \quad \text{with} \quad m \in \{1...3\}$$

for calibrations including ice point measurement (0 °C), e.g. to consider thermoelectric inhomogeneity at room temperature.
With $E_\mathrm{r}(t)$ being the reference thermoelectric voltage of the respective thermocouple type at temperature $t$.
Required number of calibration points $N$ (including ice point measurement): $N \geq m + 3$

Deviation polynomial of $m$-th order:

$$E(t) = E_\mathrm{r}(t) + \sum_{i=1}^{m} a_i \cdot t^i \quad \text{with} \quad m \in \{1...3\}$$

for calibrations without ice point measurement (0 °C).
With $E_\mathrm{r}(t)$ being the reference thermoelectric voltage of the respective thermocouple type at temperature $t$.
Required number of calibration points $N \geq m + 2$

When standard polynomials are used for the determination of the characteristic, the uncertainty of the approximation can at the same time be regarded as the uncertainty of the characteristic determined. When polynomials are used which deviate from the respective reference characteristics, the uncertainty of the reference characteristics with respect to the ITS-90 must be considered in addition for the determination of the individual characteristic for the thermocouple to be calibrated. This uncertainty contribution can be estimated from the uncertainty of the respective measurements (or calculations) performed for its determination. Up to approximately 1000 °C it lies, for type S and R thermocouples, in the range of a few mK. In the case of higher temperatures and for type B thermocouples, it corresponds approximately to the temperature differences between the temperature scales IPTS-68 and ITS-90.

## 6.2 Test data sets

Before a self-established calculation method or a software (regardless of whether the same has been self-established or has been bought as standard software) is released for the concrete case of the calibration of thermometers (Pt-100, thermocouples, NTCs etc.) and/or the subsequent approximation of the characteristics, this software or procedure must be tested for its functionality.

If different programs or calculation methods are used, this can be done by a comparison of the results. It is important not to compare the coefficients of the approximated equation but only the resulting functions. Due to different algorithms and different calculation accuracies, different coefficients may occur which, however, compensate each other and thus lead to approximately identical curve shapes.

For the testing of the software or of the approximation procedure, test data sets are shown in the subsections 6.2.1 to 6.2.3. An approximation based on these data should lead to a function curve comparable to the characteristic curve on the basis of the coefficients which are also indicated there.

An agreement within 10 % of the error intervals or uncertainties indicated in Appendix 6.1 can be regarded as sufficient for practical applications.

### 6.2.1 Test data set for Pt-100

To check the software or the approximation procedure, a test data set (fictitious calibration of a Pt-100) is given in Table 6.3. An approximation based on these data with equations 6.1.1-1 to 6.1.1-7 should lead to a comparable function curve of the characteristic based on the coefficients given in Table 6.4 to Table 6.7, Table 6.9 and Table 6.10.

If already the digits of the coefficients shown in large print in the tables in chapters 6.2.1.1 to 6.2.1.3 are identical with the calculations of software to be tested, the two results agree to a few 0.1 mK or better, and a comparison of the curve shapes can in most cases be dispensed with.

| **Temperature** | **Resistance** | **Measurement uncertainty** $(k = 2)$ | | **Weighting factor** | |
|---|---|---|---|---|---|
| in °C | in Ω | in mΩ | corresponds in mK to | (in $m\Omega^{-2}$) | normalized |
| *0.010* | *100.0054* | *4.0* | *10* | 0.0625 | 1.591981 |
| 417.765 | 253.3138 | 10.0 | 25 | 0.01 | 0.254717 |
| 303.988 | 213.5463 | 10.0 | 25 | 0.01 | 0.254717 |
| 200.000 | 175.9075 | 6.0 | 15 | 0.0277778 | 0.707547 |
| 80.008 | 130.9283 | 4.0 | 10 | 0.0625 | 1.591981 |
| *0.010* | *100.0047* | *4.0* | *10* | 0.0625 | 1.591981 |
| –51.275 | 79.7726 | 6.0 | 15 | 0.0277778 | 0.707547 |
| –25.714 | 89.8977 | 6.0 | 15 | 0.0277778 | 0.707547 |
| *0.010* | *100.0045* | *4.0* | *10* | 0.0625 | 1.591981 |

**Table 6.3:** Test data set for Pt-100

The weighting factors are calculated according to equation (2-2). The *normalized* weighting factors are calculated such that their sum corresponds to the number of data pairs. Thus, it is easier to statistically compare the results with the unweighted results (e.g. number of effective degrees of freedom, standard uncertainty, etc.). Neither the actual coefficients nor the course of the curve is affected by the standardisation (normalization) of the weighting factors.

### 6.2.1.1 Unweighted measurement values

All measured values (including the $R_{0.01}$ values[6]) were approximated for the standard polynomials (equations 6.1.1-2 to 6.1.1-4) using the least squares method. A $W$ value formation was not performed. All values were equally weighted.
The following coefficients are obtained:

| | **2nd order polynomial** *(Equation 6.1.1-2)* | **3rd order polynomial** *(Equation 6.1.1-3)* | **4th order polynomial** *(Equation 6.1.1-4)* | **Unit** |
|---|---|---|---|---|
| $R_0$ | 99.995516119 | 99.999381848 | 100.000751561 | Ω |
| $a$ | 3.9117908987E-03 | 3.9135250878E-03 | 3.9136086594E-03 | $°C^{-1}$ |
| $b$ | –5.7880716086E-07 | –5.9617124471E-07 | –6.0537905213E-07 | $°C^{-2}$ |
| $c$ | | 3.1212478437E-11 | 8.3204212690E-11 | $°C^{-3}$ |
| $d$ | | | –7.4250861722E-14 | $°C^{-4}$ |

**Table 6.4:** Coefficients for standard polynomials from the test data (unweighted)

Below you will find the coefficients for the Callendar-Van Dusen equation which have been approximated according to the method described above. Positive, negative and $R_{0.01}$values

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[6] Attention: The $R_{0.01}$ values (from Table 6.3) are measured values, the $R_0$ values (from Table 6.4 ff.), however, are coefficients (calculated values).

have been equally considered and approximated in one step. There is no difference in the weighting of the values.

| | **Callendar-VanDusen** *(Equation 6.1.1-1)* | **Unit** |
|---|---|---|
| $R_0$ | 100.000443911 | Ω |
| $A$ | 3.9107762809E-03 | °C$^{-1}$ |
| $B$ | –5.7693522900E-07 | °C$^{-2}$ |
| $C$ | –1.1855984344E-11 | °C$^{-4}$ |

**Table 6.5:** Coefficients of the Callendar-Van Dusen equation from the test data (unweighted)

### 6.2.1.2 Weighted measurement values

As opposed to the previous chapter, the measurement uncertainty of the values is taken into account and the measured values are weighted according to Section 2.2.2 / Equation (2-2). Apart from that, the calculations are carried out according to the procedure described in Chapter 6.2.1.1.

When using weighted measurement values, the following coefficients are obtained for the standard polynomials (equations 6.1.1-2 to 6.1.1-4):

| | **2nd order polynomial** *(Equation 6.1.1-2)* | **3rd order polynomial** *(Equation 6.1.1-3)* | **4th order polynomial** *(Equation 6.1.1-4)* | **Unit** |
|---|---|---|---|---|
| $R_0$ | 99.998008247 | 99.999914018 | 100.000805602 | Ω |
| $a$ | 3.9120434880E-03 | 3.9134693887E-03 | 3.9135562623E-03 | °C$^{-1}$ |
| $b$ | –5.8008948879E-07 | –5.9704124207E-07 | –6.0599486783E-07 | °C$^{-2}$ |
| $c$ | | 3.3545043917E-11 | 8.9273621082E-11 | °C$^{-3}$ |
| $d$ | | | –8.4635350764E-14 | °C$^{-4}$ |

**Table 6.6:** Coefficients for standard polynomials from the test data (weighted)

For the Callendar-Van Dusen equation (equation 6.1.1-1), the following coefficients are obtained for weighted measurement values:

| | **Callendar-Van Dusen** *(Equation 6.1.1-1)* | **Unit** |
|---|---|---|
| $R_0$ | 100.000923150 | Ω |
| $A$ | 3.9111868772E-03 | °C$^{-1}$ |
| $B$ | –5.7817938946E-07 | °C$^{-2}$ |
| $C$ | –1.0825644340E-11 | °C$^{-4}$ |

**Table 6.7:** Coefficients of the Callendar-Van Dusen equation from the test data (weighted)

The digits in small print have an influence of less than 0.1 mK of the temperature equivalent when calculating the resistance. Normally, they do not need to be indicated, but are listed here for better comparability. The required number of digits also depends on the temperature range of the thermometer and the exponent of the coefficient which means that it is not necessarily identical for different thermometers. The data here refer to the temperature range of the test data set.

### 6.2.1.3 Deviation function with $W$ values

According to the mathematics of the ITS-90, the deviation from the ITS-90 reference function can also be approximated. However, this is *not* an ITS-90 calibration [18], given that industrial Pt100 sensors do not meet the criteria for $W_{\mathrm{Ga}}$ or $W_{\mathrm{Hg}}$ (see also Section 3.1.3 and [19]) and that the calibration is usually not performed at the temperatures of the ITS-90 fixed points.

In many cases, however, the mathematical method is also suitable for Pt-100 to describe the behaviour of thermometers over large temperature ranges with relatively small uncertainties using relatively few calibration points.

The coefficients of equations 6.1.1-5 to 6.1.1-7 are determined by transforming the equation

$$\Delta W(W(T_{90})) = W(T_{90}) - W_{\mathrm{r}}(T_{90}) = a\cdot\left[W(T_{90})-1\right] + b\cdot\left[W(T_{90})-1\right]^2 + c\cdot\left[W(T_{90})-1\right]^3 \tag{6.2.1-1}$$

with $W(T_{90})$ = $R(T_{90})/R(0.01\ {}^\circ\mathrm{C})$
$R(T_{90})$: Thermometer resistance at temperature $T_{90}$
$R(0.01\ {}^\circ\mathrm{C})$: Thermometer resistance at temperature 0.01 °C
$W_{\mathrm{r}}(T_{90})$: $W$ value of the ITS-90 reference function at temperature $T_{90}$ (→ [18])

into the following form

$$y = a\cdot x + b\cdot x^2 + c\cdot x^3 . \tag{6.2.1-2}$$

Depending on the desired equation (6.1.1-5 to 6.1.1-7), the coefficients $b$ and/or $c$ can be set to zero.

The coefficients of the transformed form can thus be calculated using any standard approximation software.

Before the actual approximation, the resistance values are converted into $W$ values. Each resistance value is divided by the corresponding resistance value at the triple point of the water. The more often the water triple point is measured – ideally after every other calibration point – the more accurate the method. As can be expected, this will rarely be the case in industrial practice; therefore, the test data set is designed accordingly (→Table 6.3).

In this example, the next water triple point measurement is used for calculating the $W$ value. The formulas for the calculation of $W_{\mathrm{r}}$ of the ITS-90 can be found in [18].

| $T_{90}$ in °C | $W(T_{90})$ | $W(T_{90}) - 1$ ($x$) | $W_{\mathrm{r}}(T_{90})$ | $W(T_{90}) - W_{\mathrm{r}}(T_{90})$ ($y$) |
|---|---|---|---|---|
| 417.765 | 2.5330189481 | 1.5330189481 | 2.5627566505 | –0.0297377024 |
| 303.988 | 2.1353626380 | 1.1353626380 | 2.1573226787 | –0.0219600407 |
| 200.000 | 1.7589923274 | 0.7589923274 | 1.7736633068 | –0.0146709794 |
| 80.008 | 1.3092214666 | 0.3092214666 | 1.3152020735 | –0.0059806069 |
| –51.275 | 0.7976901039 | –0.2023098961 | 0.7938191079 | 0.0038709960 |
| –25.714 | 0.8989365479 | –0.1010634521 | 0.8969957508 | 0.0019407970 |

**Table 6.8:** Transformed test data set with $W$ values for Pt-100

The measurement points at 0.01 °C are omitted as here, the characteristic must, as a matter of principle, pass through the point ($T_{90}$ = 0.01 °C; $W$ = 1).

Without weighting, the following coefficients are obtained for equations 6.1.1-5 to 6.1.1-7:

| | **ITS + 1st order** *(Equation 6.1.1-5)* | **ITS + 2nd order** *(Equation 6.1.1-6)* | **ITS + 3rd order** *(Equation 6.1.1-7)* | **Unit** |
|---|---|---|---|---|
| $a$ | –1.9368259648E-02 | –1.9241321496E-02 | –1.9240372859E-02 | — |
| $b$ | | –1.0023952445E-04 | –1.0254256788E-04 | — |
| $c$ | | | 1.1402911401E-06 | — |

**Table 6.9:** Coefficients for the ITS deviation function (unweighted)

If the transformed test data from Table 6.8 are approximated using the weighting factors from Table 6.3, the following coefficients are obtained for equations 6.1.1-5 to 6.1.1-7:

| | **ITS + 1st order** *(Equation 6.1.1-5)* | **ITS + 2nd order** *(Equation 6.1.1-6)* | **ITS + 3rd order** *(Equation 6.1.1-7)* | **Unit** |
|---|---|---|---|---|
| $a$ | –1.9355977513E-02 | –1.9266959203E-02 | –1.9257654779E-02 | — |
| $b$ | | –8.2649269755E-05 | –1.1036006940E-04 | — |
| $c$ | | | 1.5035448679E-05 | — |

**Table 6.10:** Coefficients for the ITS deviation function (weighted)

In order to calculate the temperature from a $W$ value by means of the equation 6.2.1-1 and the determined coefficients, the value of the ITS-90 reference function $W_{\mathrm{r}}(T_{90})$ must be calculated by using the following equation:

$$W_{\mathrm{r}}(T_{90}) = W(T_{90}) - a \cdot \left[W(T_{90}) - 1\right] - b \cdot \left[W(T_{90}) - 1\right]^2 - c \cdot \left[W(T_{90}) - 1\right]^3 \tag{6.2.1-3}$$

By using the *inverse* reference function from [18] the temperature $T_{90}$ searched for can be determined.

To determine the value $W(T_{90})$, a rapidly converging approximation solution can be used:

1) $W^*(T_{90}) := W_{\mathrm{r}}(T_{90})$
2) $W^*(T_{90}) := W_{\mathrm{r}}(T_{90}) + a \cdot \left[W^*(T_{90}) - 1\right] + b \cdot \left[W^*(T_{90}) - 1\right]^2 + c \cdot \left[W^*(T_{90}) - 1\right]^3$
3) Step 2 is to be repeated until the change of $W^*(T_{90})$ can be neglected.
4) $W(T_{90}) \approx W^*(T_{90})$

Typically, no more than 5 iteration steps are required.

### 6.2.1.4 Tabular values of the approximated test data

To check the calculation methods from temperature to resistance (or $W$ value) and vice versa, the results obtained using the coefficients determined above are shown below.

Using the temperature values directly on the original test data (→ Table 6.3 or Table 6.8), it is also possible to check the suitability of the mathematical models for this specific case (test data set).

In the following table, the resistance values were determined from given temperatures using equations 6.1.1-1 to 6.1.1-4 and the coefficients from Table 6.4 and Table 6.5 (unweighted approximation.

| | **2nd order polynomial** | **3rd order polynomial** | **4th order polynomial** | **CvD** |
|---|---|---|---|---|
| **Temperature** in °C | **Resistance** in Ω | **Resistance** in Ω | **Resistance** in Ω | **Resistance** in Ω |
| –50 | 80.292 74 | 80.282 45 | 80.280 13 | 80.280 01 |
| 0 | 99.995 52 | 99.999 38 | 100.000 75 | 100.000 44 |
| 50 | 119.408 90 | 119.418 23 | 119.418 59 | 119.410 18 |
| 100 | 138.532 89 | 138.541 34 | 138.539 33 | 138.531 44 |
| 150 | 157.367 49 | 157.371 05 | 157.367 53 | 157.364 24 |
| 200 | 175.912 70 | 175.909 70 | 175.906 66 | 175.908 57 |
| 250 | 194.168 52 | 194.159 63 | 194.159 06 | 194.164 42 |
| 300 | 212.134 95 | 212.123 17 | 212.125 95 | 212.131 81 |
| 350 | 229.811 99 | 229.802 68 | 229.807 45 | 229.810 73 |
| 400 | 247.199 64 | 247.200 50 | 247.202 56 | 247.201 18 |
| –51.275 | 79.786 54 | 79.775 75 | 79.773 24 | 79.771 99 |
| –25.714 | 89.898 92 | 89.896 73 | 89.897 05 | 89.903 55 |
| 0.010 | 99.999 43 | 100.003 30 | 100.004 67 | 100.004 35 |
| 80.008 | 130.921 08 | 130.930 49 | 130.929 42 | 130.920 61 |
| 200.000 | 175.912 70 | 175.909 70 | 175.906 66 | 175.908 57 |
| 303.988 | 213.555 49 | 213.543 68 | 213.546 71 | 213.552 47 |
| 417.765 | 253.307 79 | 253.314 56 | 253.313 73 | 253.310 56 |

**Table 6.11:** Tabular values for standard polynomials and CvD (unweighted)

In the following table, the resistance values were determined from given temperatures using equations 6.1.1-1 to 6.1.1-4 and the coefficients from Table 6.6 and Table 6.7 (weighted approximation).

| | **2$^{nd}$ order polynomial** | **3$^{rd}$ order polynomial** | **4$^{th}$ order polynomial** | **CvD** |
|---|---|---|---|---|
| **Temperature** in °C | **Resistance** in Ω | **Resistance** in Ω | **Resistance** in Ω | **Resistance** in Ω |
| –50 | 80.293 16 | 80.282 90 | 80.280 20 | 80.279 96 |
| 0 | 99.998 01 | 99.999 91 | 100.000 81 | 100.000 92 |
| 50 | 119.412 82 | 119.418 40 | 119.418 31 | 119.412 49 |
| 100 | 138.537 59 | 138.540 89 | 138.538 76 | 138.534 97 |
| 150 | 157.372 32 | 157.369 88 | 157.366 97 | 157.368 35 |
| 200 | 175.917 01 | 175.907 91 | 175.906 44 | 175.912 64 |
| 250 | 194.171 66 | 194.157 47 | 194.159 43 | 194.167 84 |
| 300 | 212.136 27 | 212.121 10 | 212.126 93 | 212.133 95 |
| 350 | 229.810 85 | 229.801 30 | 229.808 64 | 229.810 96 |
| 400 | 247.195 38 | 247.200 59 | 247.203 01 | 247.198 89 |
| –51.275 | 79.786 90 | 79.776 20 | 79.773 30 | 79.772 04 |
| –25.714 | 89.900 42 | 89.897 29 | 89.897 18 | 89.903 06 |
| 0.01 | 100.001 92 | 100.003 83 | 100.004 72 | 100.004 83 |
| 80.008 | 130.925 54 | 130.930 31 | 130.928 95 | 130.923 72 |
| 200 | 175.917 01 | 175.907 91 | 175.906 44 | 175.912 64 |
| 303.988 | 213.556 64 | 213.541 64 | 213.547 72 | 213.554 48 |
| 417.765 | 253.302 28 | 253.315 40 | 253.313 55 | 253.307 21 |

**Table 6.12:** Tabular values for standard polynomials and CvD (weighted)

In the following table, the $W$ values were determined from given temperatures using equations 6.1.1-5 to 6.1.1-7 and the coefficients from Table 6.9 (unweighted approximation). The values $W_r(T_{90})$ of the ITS-90 reference function are also listed for comparison.

| | **ITS + 1st order** | **ITS + 2nd order** | **ITS + 3rd order** | $W_r(T_{90})$ |
|---|---|---|---|---|
| **Temperature in °C** | **$W$ value** | **$W$ value** | **$W$ value** | |
| –50 | 0.802 805 47 | 0.802 777 09 | 0.802 776 81 | 0.798 986 16 |
| 0 | 0.999 960 87 | 0.999 960 86 | 0.999 960 86 | 0.999 960 11 |
| 50 | 1.194 110 95 | 1.194 131 42 | 1.194 131 52 | 1.197 870 54 |
| 100 | 1.385 310 03 | 1.385 343 41 | 1.385 343 50 | 1.392 772 81 |
| 150 | 1.573 585 73 | 1.573 624 81 | 1.573 624 81 | 1.584 695 09 |
| 200 | 1.758 963 50 | 1.759 001 37 | 1.759 001 27 | 1.773 663 31 |
| 250 | 1.941 469 36 | 1.941 499 43 | 1.941 499 24 | 1.959 703 98 |
| 300 | 2.121 126 03 | 2.121 142 04 | 2.121 141 82 | 2.142 840 29 |
| 350 | 2.297 947 13 | 2.297 943 10 | 2.297 942 95 | 2.323 086 11 |
| 400 | 2.471 932 34 | 2.471 902 58 | 2.471 902 63 | 2.500 441 10 |
| | | | | |
| –51.275 | 0.797 736 60 | 0.797 707 38 | 0.797 707 09 | 0.793 819 11 |
| –25.714 | 0.898 952 86 | 0.898 939 27 | 0.898 939 15 | 0.896 995 75 |
| 0.010 | 1.000 000 00 | 1.000 000 00 | 1.000 000 00 | 1.000 000 00 |
| 80.008 | 1.309 213 15 | 1.309 242 26 | 1.309 242 36 | 1.315 202 07 |
| 200.000 | 1.758 963 50 | 1.759 001 37 | 1.759 001 27 | 1.773 663 31 |
| 303.988 | 2.135 333 25 | 2.135 347 87 | 2.135 347 66 | 2.157 322 68 |
| 417.765 | 2.533 063 87 | 2.533 023 67 | 2.533 023 82 | 2.562 756 65 |

**Table 6.13:** Tabular Values for ITS deviation function (unweighted)

To obtain resistance values, the corresponding $W$ values must be multiplied by the last (current) resistance value at the water triple point. In this case:

$$R(T_{90}) = W(T_{90}) \cdot 100.0045\,\Omega.$$

In the following table, the $W$ values were determined from given temperatures using equations 6.1.1-5 to 6.1.1-7 and the coefficients from Table 6.9 (weighted approximation). The values $W_r(T_{90})$ of the ITS-90 reference function are also listed for comparison.

| | **ITS + 1st order** | **ITS + 2nd order** | **ITS + 3rd order** | $W_r(T_{90})$ |
|---|---|---|---|---|
| **Temperature** in °C | $W$ **value** | $W$ **value** | $W$ **value** | |
| –50 | 0.802 803 10 | 0.802 782 72 | 0.802 779 75 | 0.798 986 16 |
| 0 | 0.999 960 87 | 0.999 960 86 | 0.999 960 86 | 0.999 960 11 |
| 50 | 1.194 113 29 | 1.194 127 19 | 1.194 128 04 | 1.197 870 54 |
| 100 | 1.385 314 67 | 1.385 336 28 | 1.385 336 61 | 1.392 772 81 |
| 150 | 1.573 592 64 | 1.573 616 06 | 1.573 615 13 | 1.584 695 09 |
| 200 | 1.758 972 65 | 1.758 992 22 | 1.758 989 94 | 1.773 663 31 |
| 250 | 1.941 480 70 | 1.941 491 05 | 1.941 487 86 | 1.959 703 98 |
| 300 | 2.121 139 54 | 2.121 135 53 | 2.121 132 38 | 2.142 840 29 |
| 350 | 2.297 962 77 | 2.297 939 53 | 2.297 937 83 | 2.323 086 11 |
| 400 | 2.471 950 07 | 2.471 902 95 | 2.471 904 52 | 2.500 441 10 |
| | | | | |
| –51.275 | 0.797 734 16 | 0.797 713 18 | 0.797 710 10 | 0.793 819 11 |
| –25.714 | 0.898 951 64 | 0.898 941 99 | 0.898 940 77 | 0.896 995 75 |
| 0.010 | 1.000 000 00 | 1.000 000 00 | 1.000 000 00 | 1.000 000 00 |
| 80.008 | 1.309 216 88 | 1.309 236 13 | 1.309 236 79 | 1.315 202 07 |
| 200.000 | 1.758 972 65 | 1.758 992 22 | 1.758 989 94 | 1.773 663 31 |
| 303.988 | 2.135 346 93 | 2.135 341 56 | 2.135 338 47 | 2.157 322 68 |
| 417.765 | 2.533 082 34 | 2.533 025 67 | 2.533 028 91 | 2.562 756 65 |

**Table 6.14:** Tabular Values for ITS deviation function (weighted)

To obtain resistance values, the corresponding $W$ values must be multiplied by the last (current) resistance value at the water triple point. In this case:

$$R(T_{90}) = W(T_{90}) \cdot 100.0045\,\Omega.$$

### 6.2.2 Test data set for thermocouples

Table 6.15 provides a set of test data (calibration of a Type S thermocouple by means of the comparison procedure) for checking the software or the approximation procedure. An approximation on the basis of these data should lead to a comparable course of the function of the characteristic based on the coefficients shown in Table 6.16 and on the residual deviations of the measurement values from the respective characteristic in Table 6.17.

| **Temperature** $t$ / °C | **Thermoelectric voltage** $U$ / µV | **Reference voltage** $U_\mathrm{R}$ / µV | **Difference** $U - U_\mathrm{R}$ / µV |
|---|---|---|---|
| 0 | 0.1 | 0 | +0.1 |
| 200 | 1439.0 | 1440.8 | –1.8 |
| 350 | 2783.5 | 2785.8 | –2.3 |
| 500 | 4231.3 | 4233.3 | –2.0 |
| 650 | 5752.0 | 5753.0 | –1.0 |
| 800 | 7345.5 | 7345.0 | +0.5 |
| 900 | 8451.0 | 8449.2 | +1.8 |
| 1000 | 9589.7 | 9587.1 | +2.6 |
| 1100 | 10759.5 | 10756.5 | +3.0 |

**Table 6.15:** Test data set for thermocouples

For a 3rd order deviation polynomial and two standard polynomials of the 5th and 6th order, the coefficients shown in Table 6.16 are obtained.

| **Coefficients** | 3rd order **deviation polynomial** | 5th order **standard polynomial** | 6th order **standard polynomial** |
|---|---|---|---|
| $a_0$ / µV | 0.1979 | 0.0059 | 0.1006 |
| $a_1$ / µV·°C$^{-1}$ | –0.01697 | 5.57712 | 5.44478 |
| $a_2$ / µV·°C$^{-2}$ | 3.21919E-5 | 0.01049 | 0.01181 |
| $a_3$ / µV·°C$^{-3}$ | –1.29895E-8 | –1.38628E-5 | –1.86144E-5 |
| $a_4$ / µV·°C$^{-4}$ |  | 1.03401E-8 | 1.81586E-8 |
| $a_5$ / µV·°C$^{-5}$ |  | –2.95240E-12 | –8.96427E-12 |
| $a_6$ / µV·°C$^{-6}$ |  |  | 1.75112E-15 |

**Table 6.16:** Coefficients of the polynomials from the test data set for thermocouples

Table 6.17 shows the residual deviations of the calculated characteristics from the given measurement values.

| **Temperature** $t$ / °C | 3rd order **deviation polynomial** | 5th order **standard polynomial** | 6th order **standard polynomial** |
|---|---|---|---|
| 0 | –0.10 | 0.09 | –0.00 |
| 200 | 0.21 | –0.68 | 0.01 |
| 350 | 0.05 | 1.32 | –0.01 |
| 500 | –0.14 | –0.63 | 0.00 |
| 650 | –0.20 | –0.83 | 0.04 |
| 800 | –0.07 | 0.79 | –0.11 |
| 900 | 0.27 | 0.78 | 0.12 |
| 1000 | 0.17 | –1.24 | –0.06 |
| 1100 | –0.19 | 0.39 | 0.01 |

**Table 6.17:** Residual deviations of the measured thermoelectric voltages from the respective calculated characteristic ($U_{\text{measurement value}} - U_{\text{characteristic}}$) in µV

### 6.2.3 Test data set for thermistors

Table 6.18 provides a set of test data (fictitious calibration of a thermistor) to check the software or the approximation method. An approximation based on these data with equations 6.2.3-1 to 6.2.3-4 should lead to a comparable course of the function of the characteristic on the basis of the coefficients indicated in Table 6.19.

If already the large-print digits of the coefficients indicated in the tables of this chapter are identical with the calculations of the software to be tested, the two results agree to a few 0.1 mK or better and a comparison of the curve shapes can in most cases be dispensed with.

| **Temperature** $t$ in °C | **Thermistor resistance** $R_T$ in Ω |
|---|---|
| –40 | 336 500 |
| –20 | 97 080 |
| 0 | 32 650 |
| 25 | 10 000 ($R_{T0}$) |
| 50 | 3 603.0 |
| 80 | 1 255.0 |

**Table 6.18:** Test data set for thermistors

The above data are entered into

$$\frac{1}{T} = a_0 + a_1[\ln R_T] + a_3[\ln R_T]^3 \qquad (6.2.3\text{-}1)$$

or

$$\frac{1}{T} = a_0 + a_1\left[\ln R_T\right] + a_2\left[\ln R_T\right]^2 + a_3\left[\ln R_T\right]^3 \tag{6.2.3-2}$$

with $T = t + 273.15$ K.

Likewise, the form

$$\frac{1}{T} = a_0 + a_1\left[\ln(R_T / R_{T0})\right] + a_3\left[\ln(R_T / R_{T0})\right]^3 \tag{6.2.3-3}$$

or

$$\frac{1}{T} = a_0 + a_1\left[\ln(R_T / R_{T0})\right] + a_2\left[\ln(R_T / R_{T0})\right]^2 + a_3\left[\ln(R_T / R_{T0})\right]^3 \tag{6.2.3-4}$$

is sometimes applied.

### *1) Approximation by means of transformation*

The approximation is usually performed by using standard polynomials; the data are transformed beforehand with $x = \ln(R_\mathrm{T})$ or $x = \ln(R_\mathrm{T} / R_\mathrm{T0})$ and $y = 1/T$.
An approximation then furnishes the coefficients given in Table 6.19 and Table 6.20 or an equivalent function curve.

| | **Equation 6.2.3-1** | **Equation 6.2.3-2** | **Unit** |
|---|---|---|---|
| $a_0$ | 1.1295343566E-03 | 1.1418856078E-03 | $\mathrm{K}^{-1}$ |
| $a_1$ | 2.3409324908E-04 | 2.3020424818E-04 | $(\ln \Omega)^{-1}\ \mathrm{K}^{-1}$ |
| $a_2$ | — | 3.9895899090E-07 | $(\ln \Omega)^{-2}\ \mathrm{K}^{-1}$ |
| $a_3$ | 8.7532010810E-08 | 7.4178797508E-08 | $(\ln \Omega)^{-3}\ \mathrm{K}^{-1}$ |

**Table 6.19:** Coefficients for thermistors from the test data with $x = \ln(R_T)$

| | **Equation 6.2.3-3** | **Equation 6.2.3-4** | **Unit** |
|---|---|---|---|
| $a_0$ | *3.359490268E-03* | 3.3539460517E-03 | $\mathrm{K}^{-1}$ |
| $a_1$ | *2.53930869E-04* | 2.5643118888E-04 | $\mathrm{K}^{-1}$ |
| $a_2$ | — | 2.4485949237E-06 | $\mathrm{K}^{-1}$ |
| $a_3$ | *8.396964E-07* | 7.4178791523E-08 | $\mathrm{K}^{-1}$ |

**Table 6.20:** Coefficients for thermocouples from the test data with $x = \ln(R_T/R_{T0})$

The equations 6.2.3-2 and 6.2.3-4 are equally well suited and, with the respective coefficients indicated, lead to an identical course of function.
However, equations 6.2.3-1 and 6.2.3-3show a different behaviour.
**When using this test data, equation 6.2.3-3 is much less suited!**

Other test data show a similar behaviour. The larger the temperature range of the measurement values, the worse suited seems equation 6.2.3-3 compared to equation 6.2.3-1.

If this equation is to be used in spite of all difficulties, the suitability must – as already explained in the section “Fundamentals” – be checked for the concrete case.

### *2) Approximation of the original function*

If the original function is directly approximated (without previous transformation of the measurement data) – e.g. by approximation methods – then different coefficients with a slightly changed course of the function are obtained. This method leads to slightly better results [see equation (2-1)] but is mathematically more difficult to solve.

An approximation then yields the coefficients given in the following tables or an equivalent course of the function.

| | **Equation 6.2.3-1** | **Equation 6.2.3-2** | **Unit** |
|---|---|---|---|
| $a_0$ | 1.1302024127E-03 | 1.1470200810E-03 | $\mathrm{K}^{-1}$ |
| $a_1$ | 2.3398622636E-04 | 2.2856809546E-04 | $(\ln \Omega)^{-1}\ \mathrm{K}^{-1}$ |
| $a_2$ | — | 5.6885010109E-07 | $(\ln \Omega)^{-2}\ \mathrm{K}^{-1}$ |
| $a_3$ | 8.7895497057E-08 | 6.8425603427E-08 | $(\ln \Omega)^{-3}\ \mathrm{K}^{-1}$ |

**Table 6.21:** Coefficients for thermistors from the untransformed test data (1 and 2)

| | **Equation 6.2.3-3** | **Equation 6.2.3-4** | **Unit** |
|---|---|---|---|
| $a_0$ | *3.359512929E-03* | 3.3539278611E-03 | $\mathrm{K}^{-1}$ |
| $a_1$ | *2.52740115E-04* | 2.5646040935E-04 | $\mathrm{K}^{-1}$ |
| $a_2$ | — | 2.4595194192E-06 | $\mathrm{K}^{-1}$ |
| $a_3$ | *8.897925E-07* | 6.8425589256E-08 | $\mathrm{K}^{-1}$ |

**Table 6.22:** Coefficients for thermistors from the untransformed test data (3 and 4)

The above-mentioned warnings also apply to equation 6.2.3-3!

The digits in small print have an influence of less than 0.1 mK on the calculation of the temperature. Normally, they do not need to be specified but are listed here for better comparability. The required number of digits also depends on the temperature range of the thermometer and the exponent of the coefficient, so it is not necessarily identical for different thermometers. The data here refer to the temperature range of the test data set.

By way of example, the following deviations from the calculated characteristic curves have been determined using the coefficients from Table 6.19 and the resulting functions for the measured values (reference points):

| **Temperature** in °C | **Deviation equation 6.2.3-1** in Ω | in °C | **Deviation equation 6.2.3-2** in Ω | in °C |
|---|---|---|---|---|
| –40 | 25.6 | 0.001 | –34.3 | –0.002 |
| –20 | 5.0 | 0.001 | 34.5 | 0.006 |
| 0 | –13.2 | –0.008 | –8.7 | –0.005 |
| 25 | –0.5 | –0.001 | –2.7 | –0.006 |
| 50 | 2.7 | 0.019 | 1.6 | 0.012 |
| 80 | –0.5 | –0.013 | –0.2 | –0.005 |

**Table 6.23:** Deviations of the measured thermistor resistance values from the respective calculated characteristic ($R_{\text{measurement value}} - K_{\text{characteristic}}$)

## 6.3 Mathematical tools

### 6.3.1 Algorithm for polynomial approximation

To determine, for example, the coefficients for a 2nd order polynomial by using the equation $R_\mathrm{i} = R_0 \cdot (1 + at_\mathrm{i} + bt_\mathrm{i}^2)$ *(corresponds to equation (6.1.1-2), Appendix 6.1.1)* with $n$ measurement values $R_\mathrm{i}$ ($t_\mathrm{i}$), the following matrix equation must be solved:

$$\begin{pmatrix} n & \sum t_i & \sum t_i^2 \\ \sum t_i & \sum t_i^2 & \sum t_i^3 \\ \sum t_i^2 & \sum t_i^3 & \sum t_i^4 \end{pmatrix} \bullet \begin{pmatrix} R_0 \\ R_0 \cdot a \\ R_0 \cdot b \end{pmatrix} = \begin{pmatrix} \sum R_i \\ \sum R_i \cdot t_i \\ \sum R_i \cdot t_i^2 \end{pmatrix} \qquad (6.3.1\text{-}1)$$

### 6.3.2 Algorithm for the Callendar-Van Dusen approximation

To determine the coefficients of the widely used Callendar-Van Dusen equation $R_\mathrm{i} = R_0\ (1 + At_i + Bt_i^2 + C(t_i - 100\ °\mathrm{C})t_\mathrm{i}^3)$ with $C = 0$ für $t_i > 0$ °C) (corrresponds to equation (6.1.1-1)), the matrix system expands as follows:

$$\begin{pmatrix} n & \sum t_i & \sum t_i^2 & \sum t_{\mathrm{neg},j}^3 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) \\ \sum t_i & \sum t_i^2 & \sum t_i^3 & \sum t_{\mathrm{neg},j}^4 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) \\ \sum t_i^2 & \sum t_i^3 & \sum t_i^4 & \sum t_{\mathrm{neg},j}^5 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) \\ \sum t_{\mathrm{neg},j}^3 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) & \sum t_{\mathrm{neg},j}^4 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) & \sum t_{\mathrm{neg},j}^5 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) & \sum t_{\mathrm{neg},j}^6 (t_{\mathrm{neg},j} - 100\ °\mathrm{C})^2 \end{pmatrix}$$

$$\bullet \begin{pmatrix} R_0 \\ R_0 \cdot A \\ R_0 \cdot B \\ R_0 \cdot C \end{pmatrix} = \begin{pmatrix} \sum R_i \\ \sum R_i \cdot t_i \\ \sum R_i \cdot t_i^2 \\ \sum R_{\mathrm{neg},j} \cdot t_{\mathrm{neg},j}^3 (t_{\mathrm{neg},j} - 100\ °\mathrm{C}) \end{pmatrix} \qquad (6.3.2\text{-}1)$$

with $t_{\mathrm{neg,j}}$ representing only the subset of all measured negative temperatures ($t_i < 0$ °C) and $R_{\mathrm{neg,j}}(t_{\mathrm{neg,j}})$ representing only the resistance measurement values at negative temperatures.
The advantage of this closed procedure is that the approximation is performed in one step and positive and negative values equally influence all coefficients. Thus, the $C$ coefficient is not only determined by the negative measurement values, and the other coefficients are not only determined by the positive measurement values.

## 6.4 Sample of a calibration certificate

The sections “Measurement results” and “Measurement uncertainty” in a calibration certificate can, for example, be drawn up as follows:

DEUTSCHE AKKREDITIERUNGSSTELLE **DAkkS**

**Measurement results**

Table 1 shows the measurement results in the order of calibration:

**Table 1:** Measurement values

| Temperature $t_{90}$ in °C | | Thermoelectric voltage in µV | Uncertainty in µV *) |
|---|---|---|---|
| Cu fixed point | 1084.62 | 13158.1 | 4.5 |
| Ag fixed point | 961.78 | 10708.0 | 3.7 |
| Al fixed point | 660.32 | 5710.4 | 2.1 |
| Zn fixed point | 419.53 | 2918.7 | 1.5 |
| Sn fixed point | 231.93 | ——— | —— |
| Ice point | 0.00 | ——— | —— |

*) The indicated uncertainty comprises the uncertainty components of the temperature measurement and realisation.

The characteristic function of the calibrated thermocouple is calculated from these measurement values and from the reference function for thermocouples used in the laboratory as subsequent polynomial with the coefficients stated in Table 2:

$$E\,/\,\mu\mathrm{V} = \sum_{i=0}^{8} a_i \bullet (t_{90}/°\mathrm{C})^i$$ for the temperature range: 0 °C to 660.323 °C

$$E\,/\,\mu\mathrm{V} = \sum_{i=0}^{6} b_i \bullet (t_{90}/°\mathrm{C})^i$$ for the temperature range: > 660.323 °C to 1100 °C

with $E$: Thermoelectric voltage in µV

and $t_{90}$: Temperature in °C (ITS-90).

**Table 2:** Coefficients

| | | | |
|---|---|---|---|
| $a_0$ | $-5.40580438 \cdot 10^{-02}$ | $b_0$ | $-4.97767758 \cdot 10^{+02}$ |
| $a_1$ | $5.18978480 \cdot 10^{+00}$ | $b_1$ | $1.00753718 \cdot 10^{+01}$ |
| $a_2$ | $4.60796668 \cdot 10^{-03}$ | $b_2$ | $-1.57960423 \cdot 10^{-02}$ |
| $a_3$ | $-9.60227100 \cdot 10^{-06}$ | $b_3$ | $3.63617000 \cdot 10^{-05}$ |
| $a_4$ | $2.99224300 \cdot 10^{-08}$ | $b_4$ | $-2.69015090 \cdot 10^{-08}$ |
| $a_5$ | $-2.01252300 \cdot 10^{-11}$ | $b_5$ | $9.56273660 \cdot 10^{-12}$ |
| $a_6$ | $-1.26851400 \cdot 10^{-14}$ | $b_6$ | $-1.35707370 \cdot 10^{-15}$ |
| $a_7$ | $2.25782300 \cdot 10^{-17}$ | | |
| $a_8$ | $-8.51006800 \cdot 10^{-21}$ | | |

The coefficients are only valid for the temperature range 0 °C to 1100 °C. An extrapolation leads to rapidly increasing deviations from the actual thermometer behaviour.

**Measurement uncertainty**

The uncertainty stated is the expanded measurement uncertainty resulting from the standard measurement uncertainty multiplied by the coverage factor $k = 2$. t has been determined according to DAkkS-DKD-3. With a probability of 95 %, the value of the measurand lies within the assigned value interval.

| Temperature range | | | Measurement uncertainty |
|---|---|---|---|
| 0 °C | to | 960 °C | 0.4 K |
| > 960 °C | to | 1100 °C | 0.5 K |

These measurement uncertainties comprise the uncertainties of the standards (fixed points), of the calibration procedure, of the mathematical approximation of the characteristics as well as of the properties of the calibrated thermocouple, in particular with regard to its short-term stability and its thermoelectric inhomogeneity. The long-term stability of the thermocouple is not included.

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