Physikalisch-Technische Bundesanstalt

DKD

**Guideline
DKD-R 3-10
Sheet 2**

**Dynamic calibration of force
transducers according to the
sinusoidal method**

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Deutscher Kalibrierdienst (DKD)

Since its foundation in 1977, the DKD has brought together calibration laboratories of industrial enterprises, research institutes, technical authorities, inspection and testing institutes. On 3 May 2011, the DKD was reestablished as a *technical body* of PTB and the accredited laboratories.

This body is known as *Deutscher Kalibrierdienst* (DKD – *German Calibration Service*) and is under the direction of PTB. The guidelines and guides elaborated by DKD represent the state of the art in the respective technical areas of expertise and can be used by the *Deutsche Akkreditierungsstelle GmbH* (the German accreditation body – DAkkS) for the accreditation of calibration laboratories.

The accredited calibration laboratories are now accredited and supervised by DAkkS as legal successor of the DKD. They carry out calibrations of measuring instruments and measuring standards for the measurands and measuring ranges defined during accreditation. The calibration certificates issued by these laboratories prove the traceability to national standards as required by the family of standards DIN EN ISO 9000 and DIN EN ISO/IEC 17025.

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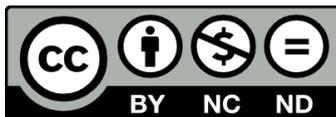
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Foreword

DKD guidelines are application documents that meet the requirements of DIN EN ISO/IEC 17025. The guidelines contain a description of technical, process-related and organizational procedures used by accredited calibration laboratories as a model for defining internal processes and regulations. DKD guidelines may become an essential component of the quality management manuals of calibration laboratories. The implementation of the guidelines promotes equal treatment of the equipment to be calibrated in the various calibration laboratories and improves the continuity and verifiability of the work of the calibration laboratories.

The DKD guidelines should not impede the further development of calibration procedures and processes. Deviations from guidelines as well as new procedures are permitted in agreement with the accreditation body if there are technical reasons to support this action.

Calibrations by accredited laboratories provide the user with the security of reliable measuring results, increase the confidence of customers, enhance competitiveness in the national and international markets, and serve as metrological basis for the monitoring of measuring and test equipment within the framework of quality assurance measures.

This guideline has been drawn up by the DKD Technical Committees *Force and Acceleration* and *Materials Testing Machines* and approved by the Board of DKD.

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1 Introduction

1.1 Definition of dynamic forces

A dynamic force is the product of mass and acceleration with the mass or acceleration varying over time. Through the calibration of dynamic forces, the frequency-dependent (dynamic) properties of the force measurement-related application can be determined.

1.2 Purpose and scope of application

This guideline is intended for the dynamic calibration of force transducers which are used, for example, as transfer standards for the dynamic calibration of materials testing machines.

This guideline does not restrict the validity of existing calibration requirements and the need to implement them. Even when being dynamically calibrated, force measuring devices and materials testing machines should also have a static calibration (e.g. DIN EN ISO 376 or DIN EN ISO 7500) as proof of traceability.

2 Symbols

Abbreviations/ formula symbols	Unit	Explanation
A	N	Force amplitude
b_c	kg/s	Damping constant of the connection between force transducer and load mass
b_f	kg/s	Damping constant of the connection base mass - head mass of the force transducer
c_{p_1}, c_{p_2}	(N.N.)	Sensitivity coefficient
F	N	Force
f	Hz	Frequency
f_0	Hz	Resonance frequency
k	1	Coverage factor
k_c	N/m	Stiffness of the connection force transducer – load mass
K_{corr}	1	Correction parameters in the event of acceleration distribution of the load mass
k_f	N/m	Stiffness of the connection base mass - head mass of the force transducer
m_b	kg	Base mass of the force transducer
m_i	kg	Head mass of the force transducer
m_t	kg	Load mass
p_1, p_2	(N.N.)	Parameters of the dynamic sensitivity
$\hat{p}_1$, $\hat{p}_2$	(N.N.)	Estimated parameter values
Q	1	Quality factor, ratio of the resonance frequency to the half-width of the resonance peak
S_f	(N.N.)	Dynamic sensitivity of the force transducer
$\hat{S}_f$	(N.N.)	Estimated value of the dynamic sensitivity
S_{f0}	(N.N.)	Static sensitivity of the force transducer
t	s	Time
u_{S_f}	(N.N.)	Uncertainty of the dynamic sensitivity
u_{p_1}, u_{p_2}, $u_{p_{1,2}}$	(N.N.)	Uncertainty of the parameters of the dynamic sensitivity

Abbreviations/ formula symbols	Unit	Explanation
U_{S_f}	(N.N.)	Expanded uncertainty of the dynamic sensitivity
$\mathbf{U}_p$	(N.N.)	Covariance matrix
U_f	(N.N.)	Output signal of the force transducer
x_b	m	Vertical displacement of the base mass
$\ddot{x}_\mathrm{b}$	m/s^2	Acceleration of the base mass
x_i	m	Vertical displacement of the head mass
x_t	m	Vertical displacement of the load mass
$\ddot{x}_\mathrm{t}$	m/s^2	Acceleration of the load mass
Δf	Hz	Half-width of the resonance peak
μ	kg	Reduced mass from combination of load mass and head mass
ρ	(N.N.)	Correlation coefficient
φ	rad	Angle
ω	rad	Angular frequency

3 Calibration example

3.1 Model and basic conditions

The dynamic behaviour of a force transducer is described by a spring-mass model. Figure 1 shows the simplified schematic model of a force transducer which is additionally loaded by a load mass m_t. The fastening of the load mass is represented as a damped spring by the connection stiffness k_c and the damping constant b_c. The force transducer itself consists of base mass m_b and head mass m_i which are connected via stiffness k_f and damping b_f.

If a purely vertical movement is exerted during dynamic calibration, the vertical displacements of the three masses can be described by the vector (x_t, x_i, x_b).

The sinusoidal force excitation takes place from underneath the base mass m_b. This force is generated, for example, by an electrodynamic vibration exciter. The accelerations of the load mass $\ddot{x}_t$ and the base mass $\ddot{x}_b$ on the mounting plate of the vibration exciter as well as the signal of the force transducer U_f are recorded for calibration.

The signal of the force transducer is assumed to be proportional to the deformation of the spring element of the force transducer $(x_i - x_b)$.
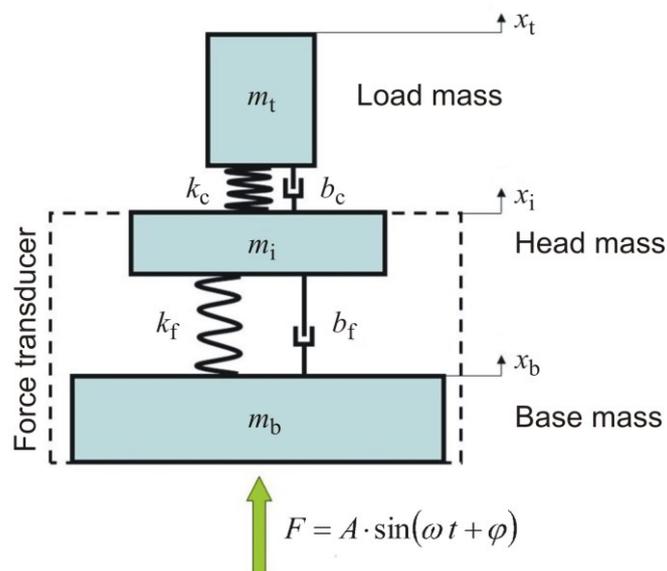
Figure 1: Schematic model of a force transducer with load mass. The transducer itself is shown as a damped spring-mass system with head and base mass. The connection to the load mass is depicted as a damped spring; however, its stiffness is much greater than that of the force transducer. The sizes of the drawn masses are not to scale; as a rule, the head mass is much smaller than the load mass.

The model of the force transducer with load mass shown in Figure 1 is described by the following system of linear differential equations with constant coefficients:

$$m_t \ddot{x}_t = -k_c (x_t - x_i) - b_c (\dot{x}_t - \dot{x}_i)$$
$$m_i \ddot{x}_i = k_c (x_t - x_i) + b_c (\dot{x}_t - \dot{x}_i) - k_f (x_i - x_b) - b_f (\dot{x}_i - \dot{x}_b) \qquad (1)$$
$$m_b \ddot{x}_b = k_f (x_i - x_b) + b_f (\dot{x}_i - \dot{x}_b) + F$$

The desired calibration quantity, namely the dynamic sensitivity S_f, is the frequency-dependent ratio of the force transducer output signal U_f (bridge signal in mV/V, charge in pC, voltage in mV) to the acting dynamic force F:

$$S_f = \frac{U_f}{(m_t + m_i) \cdot \ddot{x}_t} \qquad (2)$$

According to the model in Figure 1, the signal U_f of the force transducer is proportional to the difference between the coordinates $(x_i - x_b)$, and the dynamic force applied to the head mass m_i is proportional to the acceleration $\ddot{x}_t$ of the load mass m_t.

$$S_f \propto \left| \frac{x_i - x_b}{\ddot{x}_t} \right| \qquad (3)$$

By using the trial function $\ddot{x} = i\omega \dot{x} = -\omega^2 x$ for sinusoidal movements, the system of equations (1) can be solved for the required quantity:

$$\left| \frac{x_i - x_b}{\ddot{x}_t} \right| = \frac{m_t + m_i}{k_f} \cdot \sqrt{ \frac{1 - 2\omega^2 \frac{\mu}{k_c}\left(1 - \frac{b_c^2}{2\mu k_c}\right) + \left(\omega^2 \frac{\mu}{k_c}\right)^2}{\left(1 + \left(\omega \frac{b_f}{k_f}\right)^2\right) \cdot \left(1 + \left(\omega \frac{b_c}{k_c}\right)^2\right)} }, \qquad (4)$$

with the reduced mass $\mu = (m_t \, m_i)/(m_t + m_i)$.

Assuming the connection to the load mass to be very stiff and thus only having a small influence (b_b / k_c towards zero), and assuming b_f / k_f to be small, (4) can approximately be written as follows:

$$\left| \frac{x_i - x_b}{\ddot{x}_t} \right| \approx \frac{m_t + m_i}{k_f} \cdot \left(1 - \frac{\mu}{k_c}\omega^2\right) \qquad (5)$$

Using (2), then the following expression is obtained for the dynamic sensitivity:

$$S_f = p_1 \cdot \left(1 - p_2 \cdot \omega^2\right) \tag{6}$$

By means of regression, the two parameters p_1 and p_2 can be determined from the measurement of the dynamic sensitivity S_f. It is obvious that S_f decreases with the square of the frequency ω. The transition to $\omega = 0$ yields the static sensitivity S_{f0}.

Note: Strictly speaking, the model (Figure 1) and its equations of motion (1) apply to rigid bodies. In an elastic structure, however, the individual mass points experience – to a small extent – different accelerations in vertical direction. The acceleration which in practice is measured on the upper side of the load mass by means of a laser vibrometer therefore slightly differs from the value used in (1). To take account of this elastic influence, a correction factor K_{corr} can be defined, e.g. by using finite element methods. This factor can be used to correct the measured acceleration values. Further information regarding primary calibration with sinusoidal forces is described in [1] - [3].

3.2 Calibration procedure

The schematic set-up of a sinusoidal calibration is shown as an example in Figure 2. The sinusoidal force is generated by an electrodynamic vibration exciter which is controlled by a power amplifier with an upstream waveform generator. The acceleration measurement of the head mass is carried out by means of a laser vibrometer, for example; the acceleration measurement of the vibration table (shaker) is carried out by means of an acceleration sensor.
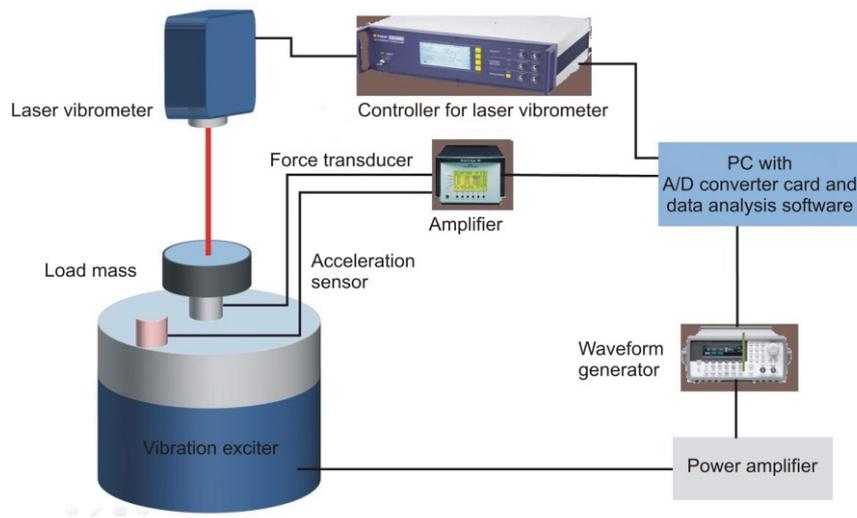
Figure 2: Schematic set-up for the dynamic calibration of a force transducer according to the sinusoidal method

Laser vibrometers record the acceleration of the vibrating surface by means of optical measurement (optical probing), i.e. without force shunt and additional mass loading. The surface can be scanned at various points (especially with specially developed scanning vibrometers) to determine the average acceleration of the upper side of the load mass. If, instead, an accelerometer mounted on the load mass is used, the mass of the accelerometer must be considered accordingly.

The measurement of the acceleration of the load mass is required to determine the dynamic sensitivity. The additional acceleration measurement of the shaker that is illustrated above is required to determine the stiffness and damping parameters of the force transducer.

Since there is no interdependence between the parameters of the sinusoidal calibration and since these parameters largely depend on the technical equipment of the calibration laboratory, the following conditions must be agreed upon with the customer and specified prior to calibration:

- Which dynamic force amplitudes (i.e. different load masses and/or accelerations) shall be used for calibration? Note: To determine the head mass m_i of the force transducer, at least two different load masses m_t must be used.
- Which frequency range should be used for calibration?
- Which accelerations ($\ddot{x}_b$ or $\ddot{x}_t$) should be used for calibration?

- Are there further parameters to be determined in addition to the dynamic sensitivity S_f and head mass m_i?
- Should a laboratory-owned amplifier be used for the force transducer?
- In case of calibrating the measuring chain: Is the customer's own amplifier suited for integration into the calibration set-up?

3.3 Determinable parameters

3.3.1 Head mass

The head mass m_i of the force transducer (see Figure 1) is a parameter required to determine the dynamic response characteristic (1). A procedure for the dynamic determination of the head mass m_i from the sine calibration is described below.

By transposing (2) according to the ratio of the force transducer signal U_f and the acceleration of the load mass $\ddot{x}_t$, we obtain an equation for determining the head mass m_i. The following applies to the transfer function $(U_f / \ddot{x}_t)_0$ extrapolated to zero frequency:

$$\left(\frac{U_f}{\ddot{x}_t}\right)_0 = S_{f0} \cdot m_t + S_{f0} \cdot m_i \qquad (7)$$

This mathematical relationship is graphically illustrated in Figure 3. If the measured transfer function $(U_f / \ddot{x}_t)_0$ extrapolated to frequency zero is plotted over the load mass m_t, the head mass m_i can be calculated from the intersection of the regression line with the ordinate $m_t = 0$.

The ratio between force transducer signal and acceleration is to be determined by measurement at several frequencies in the lower frequency range, i.e. frequencies at which the dynamic sensitivity does not yet change with the frequency, for example at 50 Hz, 75 Hz and 100 Hz.
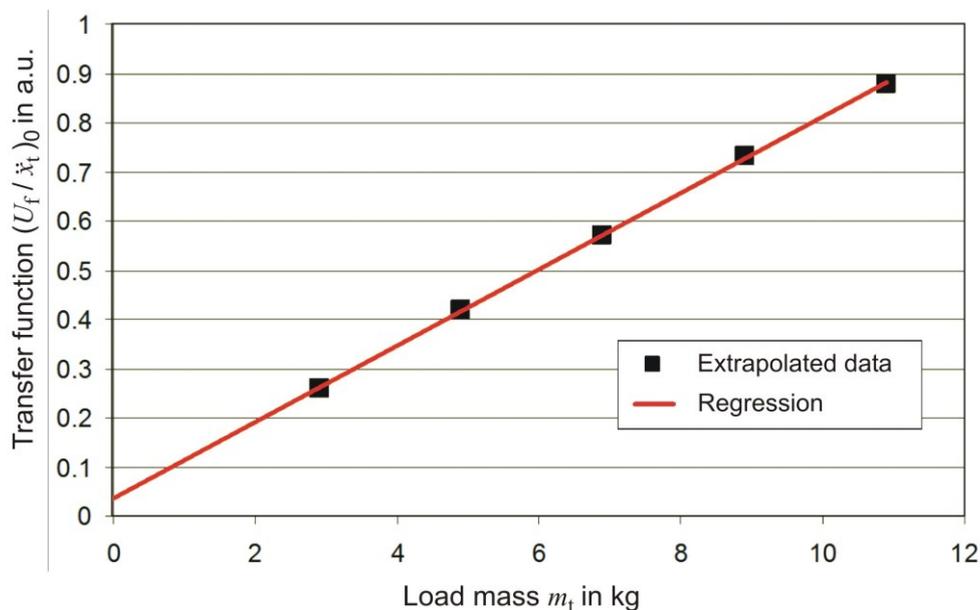
Figure 3: The transfer function $(U_f / \ddot{x}_t)_0$ for different load masses m_t to determine the transducer head mass m_i.

The head mass including its associated uncertainty is to be stated in the calibration certificate together with the determination procedure.

With very small head masses – in relation to the load masses used – the described procedure might fail if the errors of the measured transfer function are too large; this would result, for example, in a negative head mass. In this case, the internal mass should be neglected ($m_i = 0$) and a corresponding uncertainty contribution for the internal mass should be estimated.

There are different methods to determine the head mass (e.g. by rotating the transducer in the Earth's gravitational field). Depending on the procedure, other values may result. For evaluation purposes, it is not allowed to use the head mass indicated in data sheets or older calibration certificates. Information obtained from other sources may not be valid for the test carried out during calibration.

3.3.2   Dynamic sensitivity

The dynamic sensitivity is the most important calibration result when calibrating force transducers according to the sinusoidal method. The determination is carried out according to (2) by using differently sized load masses and passing through the required frequency range. The head mass m_i of the force transducer must be taken into account. The dynamic sensitivities can be determined based on (6). The dynamic sensitivities determined with the different load masses may differ and should be stated separately. For a dynamic calibration that is independent of the load mass, the stiffness and damping parameters in equation (1) have to be determined from the calibration, see e.g. [4].

In the calibration certificate, the parameters p_1 and p_2 – which have been determined by way of regression from the measurement data – are to be indicated for each load mass. According to the underlying measurement uncertainty, the parameter p_1 should correspond to the static sensitivity.

As shown in Figure 4, the decrease in dynamic sensitivity with frequency should be graphically represented by the ratio S_f / S_{f0}.

The dynamic behaviour of the force transducer amplifier used for calibration must be known since it is directly included in the calibration result.

- If a laboratory-owned amplifier is used during calibration, the dynamic behaviour is determined using a dynamic calibrator and taken into account when calculating the calibration quantity.
- In case that the customer's own amplifier is to be used together with the force transducer as measuring chain during calibration, it must be ensured that the signal propagation times and sampling rates of the measuring chain to be calibrated comply with the conditions of the calibration set-up.
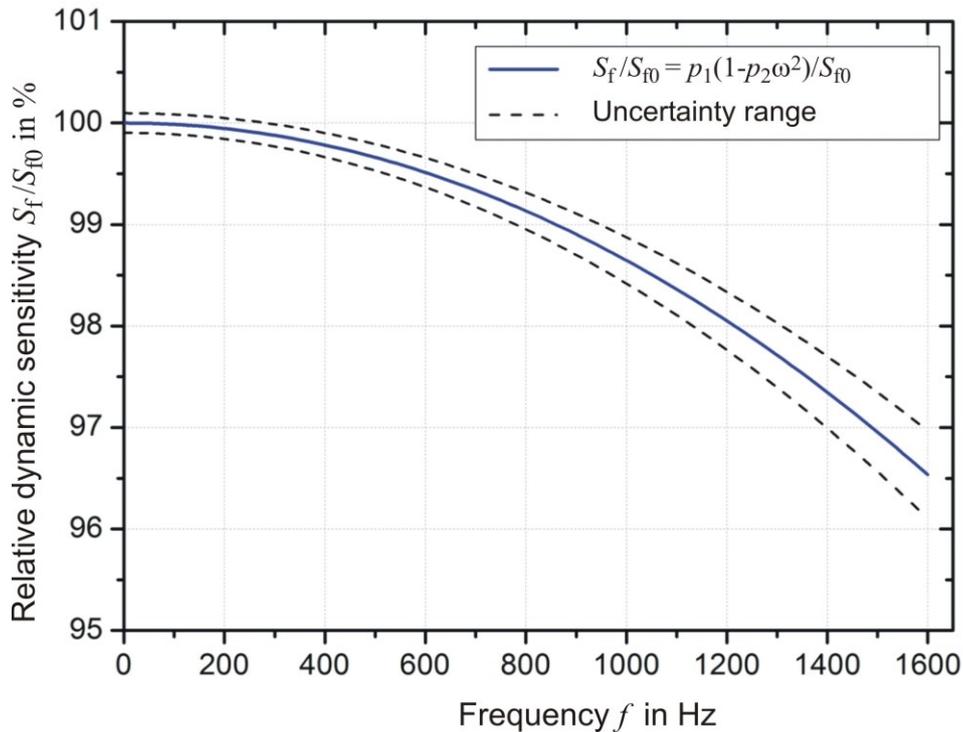
Figure 4: Decrease of dynamic sensitivity with frequency.

3.3.3   Base mass

The base mass m_{b} of the force transducer (see Figure 1) is a parameter which is insignificant for the determination of the dynamic sensitivity when calibrating according to the sinusoidal method as described here.

However, the knowledge of the base mass could be interesting or even required for future utilization of the dynamically calibrated force transducer. There is, for example, a procedure for the dynamic determination of the base mass in which the force transducer is mounted upside down with the evaluation being carried out analogously to the determination of the head mass.

3.3.4   Stiffness

The stiffness k_{f} of the force transducer (see Figure 1) is a parameter that is only determined on request when calibrating according to the sinusoidal method. In addition to using the method for dynamic determination described below, the stiffness can also be determined by static loading. The method used has to be specified in the calibration certificate.

To determine the dynamic stiffness, the ratio $\ddot{x}_{\mathrm{t}} / \ddot{x}_{\mathrm{b}}$ of the accelerations of load mass and vibration table (shaker) is to be plotted over frequency (see Figure 5). This amplitude response shows a pronounced resonance with the resonance frequency f_0.

The dynamic stiffness k_f of the force transducer is given by the following relationship

$$k_f = (2\pi \cdot f_0)^2 \cdot (m_i + m_t) \tag{8}$$

Note: The connection between load mass and force transducer is assumed to be rigid (coupling stiffness $k_c \to \infty$), i.e. the oscillatory system behaves like a damped single degree of freedom system (SDOF).

3.3.5 Damping

The damping b_f of the force transducer (see Figure 1) is a parameter that is only determined on request when calibrating according to the sinusoidal method. Below, a procedure for the dynamic determination of the damping from the sinusoidal calibration is described. The procedure used must be specified in the calibration certificate.

To determine the damping from the resonance curve obtained in Chapter 3.3.4, the ratio of the resonance frequency to the half-width of the resonance peak needs to be evaluated. This ratio forms the quality factor Q. The value of the quality factor must be at least 20 to obtain an uncertainty contribution of less than 0.1 %.
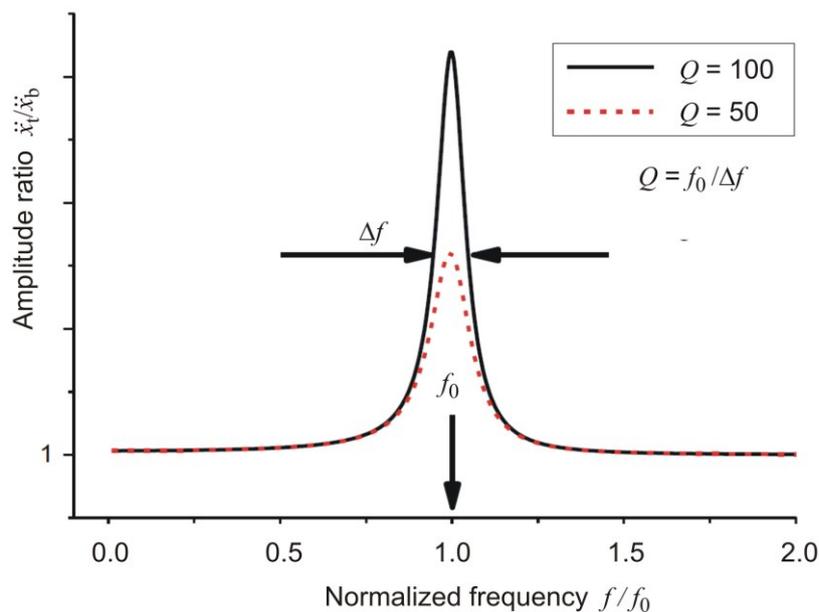
Figure 5: Amplitude response of the ratio of accelerations $\ddot{x}_t / \ddot{x}_b$ of load mass and vibration table with resonance frequency f_0 and half-width Δf of the resonance peak. Two resonance curves with different quality factor Q are shown as examples.

The damping b_f of the force transducer results from the relationship

$$b_\mathrm{f} = \frac{\Delta f}{f_0}\sqrt{k_\mathrm{f}\cdot(m_\mathrm{i}+m_\mathrm{t})} = \frac{1}{Q}\sqrt{k_\mathrm{f}\cdot(m_\mathrm{i}+m_\mathrm{t})} \qquad (9)$$

Note: The connection between load mass and force transducer is assumed to be rigid (coupling stiffness $k_\mathrm{c} \to \infty$), i.e. the oscillating system behaves like a damped single degree of freedom system.

3.4 Measurement uncertainty

Equation (6) forms the basis for determining the measurement uncertainty of the dynamic sensitivity $S_f(\omega)$. For this purpose, the model parameters p_1, p_2 and their covariance matrix $\mathbf{U}_p$ are determined from a regression:

$$\mathbf{U}_p = \begin{pmatrix} u_{p_1}^2 & u_{p_{1,2}} \\ u_{p_{1,2}} & u_{p_2}^2 \end{pmatrix} \tag{10}$$

with the uncertainties u_{p_1}, u_{p_2} and the covariance $u_{p_{1,2}} = \rho(p_1, p_2) \cdot u_{p_1} \cdot u_{p_2}$, and with $\rho(p_1, p_2)$ denoting the correlation coefficient.

The measurement uncertainty of the model parameters results from the regression to the curve of the measured dynamic sensitivities $S_f(\omega)$. In accordance with (2), the measured values of the dynamic sensitivity include the uncertainty contribution of the electrical measuring chains of the force transducer (signal U_f) and the acceleration measurement (signal $\ddot{x}_t$) as well as the uncertainty of the determination of the masses m_t and m_i. The uncertainty of the measured dynamic sensitivity is then propagated through regression to the uncertainty of the model parameters $\hat{p}_1$ and $\hat{p}_2$ [5].

Using (6) as model equation for the measurand S_f, the following sensitivity coefficients are obtained:

$$c_{p_1} = 1 - \hat{p}_2 \omega^2 , \; c_{p_2} = -\hat{p}_1 \omega^2 \tag{11}$$

with the estimated values $\hat{p}_1$ and $\hat{p}_2$.

The measurement uncertainty of S_f is then calculated according to GUM as

$$u_{S_f} = \sqrt{c_{p_1}^2 u_{p_1}^2 + c_{p_2}^2 u_{p_2}^2 + 2 c_{p_1} c_{p_2} u_{p_{1,2}}}$$
$$= \sqrt{(1 - \hat{p}_2 \omega^2)^2 u_{p_1}^2 + (\hat{p}_1 \omega^2)^2 u_{p_2}^2 + 2(1 - \hat{p}_2 \omega^2)(\hat{p}_1 \omega^2) u_{p_{1,2}}} \tag{12}$$

It should be noted that the uncertainty is a function of the angular frequency ω.

The relative expanded uncertainty is calculated as

$$\frac{U_{S_f}}{\hat{S}_f} = \frac{U_{S_f}}{\hat{p}_1(1 - \hat{p}_2 \omega^2)} = k \cdot \sqrt{\frac{u_{p_1}^2}{\hat{p}_1^2} + \frac{\omega^4 u_{p_2}^2}{(1 - \hat{p}_2 \omega^2)^2} + \frac{2\omega^2 u_{p_{1,2}}}{\hat{p}_1(1 - \hat{p}_2 \omega^2)}} \tag{13}$$

with an appropriately selected coverage factor k.

The measurement uncertainty contribution of the measurement amplifier and accelerometer used indirectly affect the measurement uncertainty consideration by being included in the regression determination of p_1 and p_2 as measurement uncertainty of the data to be fitted [5].

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