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VNA Tools II: Calibrations Involving Eigenvalue Problems

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***Abstract*— Many calibration algorithms for vector network analyzers using partially unknown standards can be stated as an eigenvalue problem. The construction of the eigenvalue problem is described and examples for line reflect match (LRM) and thru reflect line (TRL) calibrations are given. Advantages of the new algorithm are that all uncertainties can be taken into account and that it is fully analytic. A further advantage is that the same approach can be used for different calibration schemes. The algorithm is implemented in METAS VNA Tools II and METAS UncLib is used for the linear propagation of uncertainties.**

***Index Terms*— Vector Network Analyzer, S-parameters, Calibration, Uncertainty, Traceability**

I. Introduction

Calibration algorithms for vector network analyzers (VNAs) which require only partially known standards pose problems for uncertainty calculation. Examples are line reflect match (LRM) and through reflect line (TRL) calibrations where the reflectivity of the line can not be specified because in the algorithm it is assumed that the line has a characteristic impedance of $50\,\Omega$. In reality the line in use will not have exactly the required characteristic impedance and thus this needs to be taken into account for uncertainty computation.

This limitation of the TRL algorithm [1] can be partly lifted. In [2] uncertainties are propagated from the standards to raw measured results and then through the TRL algorithm to the error terms. This approach ignores that the TRL algorithm is not valid for lines with reflection. This problem has inspired the development of the algorithm described here.

II. Constructing the Calibration Matrix

As pointed out in [3], the measurement model for VNAs can be written as a matrix equation

$$\mathbf{M} = \mathbf{E_{00}} + \mathbf{E_{01}} \left(\mathbf{I} - \mathbf{S}\mathbf{E_{11}}\right)^{-1} \mathbf{S}\mathbf{E_{10}}. \tag{1}$$

Here $\mathbf{M}$ denotes the matrix of raw measured S-parameters, $\mathbf{S}$ represents the S-parameter matrix of the device under test (DUT), $\mathbf{I}$ is the identity matrix, and $\mathbf{E_{00}}$, $\mathbf{E_{01}}$, $\mathbf{E_{10}}$, $\mathbf{E_{11}}$ are matrices containing the error parameters of the VNA. By rearranging one finds

$$\mathbf{M}\mathbf{E_{10}}^{-1}\mathbf{S}^{-1} - \mathbf{E_{00}}\mathbf{E_{10}}^{-1}\mathbf{S}^{-1} = \mathbf{E_{01}} \left(\mathbf{I} - \mathbf{S}\mathbf{E_{11}}\right)^{-1}. \tag{2}$$

Expansion with $\left(\mathbf{I} - \mathbf{S}\mathbf{E_{11}}\right)\mathbf{S}$ and rearranging yields

$$\begin{aligned} &\mathbf{M}\underbrace{\mathbf{E_{10}}^{-1}}_{\mathbf{A}} - \underbrace{\mathbf{E_{00}}\mathbf{E_{10}}^{-1}}_{-\mathbf{B}} - \mathbf{M}\underbrace{\mathbf{E_{10}}^{-1}\mathbf{E_{11}}}_{-\mathbf{C}}\mathbf{S} + \cdots \\ &\qquad \underbrace{\left(\mathbf{E_{00}}\mathbf{E_{10}}^{-1}\mathbf{E_{11}} - \mathbf{E_{01}}\right)}_{\mathbf{D}}\mathbf{S} = 0. \end{aligned} \tag{3}$$

This development is chosen instead of the one in [4] because now it is easy to set $e_{10_{11}} = 1$ by setting $a_{11} = 1$. Here $e_{10_{11}}$ and a_{11} denote the upper left elements of the matrices $\mathbf{E_{10}}$ and $\mathbf{A}$ respectively. Note that $\mathbf{E_{10}}$ is most often a diagonal matrix.

Measuring the i-th calibration standard with definition $\mathbf{S}^i$ yields the raw measured S-parameters $\mathbf{M}^i$. Plugging $\mathbf{S}^i$ and $\mathbf{M}^i$ in (3) yields a matrix equation which is linear with respect to the unknown parameters $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$. From this matrix equation one can form as many complex non-matrix equations as there are elements in $\mathbf{S}^i$. Repeating this for all used standards and setting $a_{11} = 1$ yields a system of equations.

For simplicity's sake, an example with fully known standards and a non-leaky VNA model is given. Thus $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are all diagonal matrices. All unknowns can by concatenated to a column vector

$$\begin{aligned} \mathbf{x} = (&a_{11}, a_{22}, \cdots, a_{nn}, b_{11}, b_{22}, \cdots, b_{nn}, \\ &c_{11}, c_{22}, \cdots, c_{nn}, d_{11}, d_{22}, \cdots, d_{nn})^T \end{aligned} \tag{4}$$

where n is the number of ports.

The resulting system of equations is

$$\mathbf{F}\mathbf{x} = 0 \tag{5}$$

where $\mathbf{F}$ is constructed from $\mathbf{S}^i$ and $\mathbf{M}^i$. The following example shows the matrix $\mathbf{F}$ for a two-port QSOLT [5] calibration

$$\left(\begin{array}{cccccccc} m_{11}^o & 0 & 1 & 0 & m_{11}^o s_{11}^o & 0 & s_{11}^o & 0 \\ m_{11}^s & 0 & 1 & 0 & m_{11}^s s_{11}^s & 0 & s_{11}^s & 0 \\ m_{11}^l & 0 & 1 & 0 & m_{11}^l s_{11}^l & 0 & s_{11}^l & 0 \\ \hline m_{11}^t & 0 & 1 & 0 & m_{11}^t s_{11}^t & m_{12}^t s_{21}^t & s_{11}^t & 0 \\ m_{21}^t & 0 & 0 & 0 & m_{21}^t s_{11}^t & m_{22}^t s_{21}^t & 0 & s_{21}^t \\ 0 & m_{12}^t & 0 & 0 & m_{11}^t s_{12}^t & m_{12}^t s_{22}^t & s_{12}^t & 0 \\ 0 & m_{22}^t & 0 & 1 & m_{21}^t s_{12}^t & m_{22}^t s_{22}^t & 0 & s_{22}^t \end{array}\right) \tag{6}$$

where m^i are the elements of the switch corrected measurements $\mathbf{M}^i$ and s^i are the elements of the definitions $\mathbf{S}^i$. The first three rows are three one-port standards (open, short, load) and the other four rows are the transmission standard (thru).

For the final solution $\mathbf{F}$ is separated in into two parts because $a_{11} = 1$

$$\mathbf{F} = \left(\begin{array}{cc} \mathbf{F}_{:,1} & \mathbf{F}_{:,2:n} \end{array}\right). \tag{7}$$

Here the vector $\mathbf{F}_{:,1}$ is the first column of matrix $\mathbf{F}$. Finally the following system can be solved

$$\mathbf{F}_{:,2:n}\mathbf{x}_{2:n} = -\mathbf{F}_{:,1}. \tag{8}$$

III. Constructing the Eigenvalue Problem

A calibration with unknown standards means that $\mathbf{M}$ is fully known for each calibration standard and $\mathbf{S}$ is known for some standards and partly unknown for others.

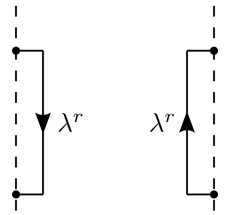

Fig. 1. Two one-port calibration standards with the same unknown reflection λ^r.

Figure 1 shows a flowchart of the same reflection standard attached to two ports. The following sub-matrix describes this measurement

$$\begin{pmatrix} m_{11}^r & 0 & 1 & 0 & m_{11}^r\lambda^r & 0 & \lambda^r & 0 \\ 0 & m_{22}^r & 0 & 1 & 0 & m_{22}^r\lambda^r & 0 & \lambda^r \end{pmatrix}. \tag{9}$$

λ^r is the unknown reflection coefficient.

Figure 2 shows a line standard with known reflection and unknown propagation constant (transmission λ^l), which is described by the following sub matrix

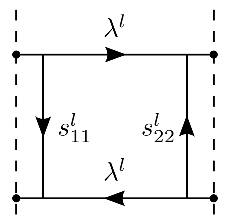

Fig. 2. Line standard with unknown transmission λ^l.

$$\begin{pmatrix} m_{11}^l & 0 & 1 & 0 & m_{11}^l s_{11}^l & m_{12}^l\lambda^l & s_{11}^l & 0 \\ m_{21}^l & 0 & 0 & 0 & m_{21}^l s_{11}^l & m_{22}^l\lambda^l & 0 & \lambda^l \\ 0 & m_{12}^l & 0 & 0 & m_{11}^l\lambda^l & m_{12}^l s_{22}^l & \lambda^l & 0 \\ 0 & m_{22}^l & 0 & 1 & m_{21}^l\lambda^l & m_{22}^l s_{22}^l & 0 & s_{22}^l \end{pmatrix}. \tag{10}$$

Depending on the measured standards, $\mathbf{F}$ can be assembled from the mentioned sub-matrices. $\mathbf{F}$ contains λ and thus it is separated into the matrices $\mathbf{G}$ and $\mathbf{H}$

$$\mathbf{F}\mathbf{x} = (\mathbf{G} + \lambda\mathbf{H})\,\mathbf{x} = 0. \tag{11}$$

If suitable standards are measured and $\mathbf{G}$ is invertible, then the generalized eigenvalue problem can be written in the form

$$-\mathbf{G}^{-1}\mathbf{H}\mathbf{x} = \frac{1}{\lambda}\mathbf{x}. \tag{12}$$

This eigenvalue problem can be solved.

IV. TRM and LRM Calibration

The following example shows the matrix $\mathbf{G} + \lambda^r\mathbf{H}^r$ for TRM and LRM [6] calibration

$$\left(\begin{array}{cccccccc} m_{11}^t & 0 & 1 & 0 & m_{11}^t s_{11}^t & m_{12}^t s_{21}^t & s_{11}^t & 0 \\ m_{21}^t & 0 & 0 & 0 & m_{21}^t s_{11}^t & m_{22}^t s_{21}^t & 0 & s_{21}^t \\ 0 & m_{12}^t & 0 & 0 & m_{11}^t s_{12}^t & m_{12}^t s_{22}^t & s_{12}^t & 0 \\ 0 & m_{22}^t & 0 & 1 & m_{21}^t s_{12}^t & m_{22}^t s_{22}^t & 0 & s_{22}^t \\ \hline m_{11}^r & 0 & 1 & 0 & m_{11}^r\lambda^r & 0 & \lambda^r & 0 \\ 0 & m_{22}^r & 0 & 1 & 0 & m_{22}^r\lambda^r & 0 & \lambda^r \\ \hline m_{11}^m & 0 & 1 & 0 & m_{11}^m s_{11}^m & 0 & s_{11}^m & 0 \\ 0 & m_{22}^m & 0 & 1 & 0 & m_{22}^m s_{22}^m & 0 & s_{22}^m \end{array}\right). \tag{13}$$

Here the first four rows describe the transmission standard (known line or thru), the next two rows describe the one-port standards with unknown reflection λ^r and the last two rows describe the matched one-port standards. This matrix can be split up into the matrix $\mathbf{G}$

$$\left(\begin{array}{cccccccc} m_{11}^t & 0 & 1 & 0 & m_{11}^t s_{11}^t & m_{12}^t s_{21}^t & s_{11}^t & 0 \\ m_{21}^t & 0 & 0 & 0 & m_{21}^t s_{11}^t & m_{22}^t s_{21}^t & 0 & s_{21}^t \\ 0 & m_{12}^t & 0 & 0 & m_{11}^t s_{12}^t & m_{12}^t s_{22}^t & s_{12}^t & 0 \\ 0 & m_{22}^t & 0 & 1 & m_{21}^t s_{12}^t & m_{22}^t s_{22}^t & 0 & s_{22}^t \\ \hline m_{11}^r & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & m_{22}^r & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline m_{11}^m & 0 & 1 & 0 & m_{11}^m s_{11}^m & 0 & s_{11}^m & 0 \\ 0 & m_{22}^m & 0 & 1 & 0 & m_{22}^m s_{22}^m & 0 & s_{22}^m \end{array}\right) \tag{14}$$

and the matrix $\lambda^r\mathbf{H}^r$

$$\lambda^r \left(\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & m_{11}^r & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{22}^r & 0 & 1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right). \tag{15}$$

In this example $\mathbf{G}$ is invertible and thus the eigenvalue problem can be solved. Usually the result contains more than one eigenvalue. One uses an estimate of the eigenvalue in order to select the right result.

The concept has been tested with a real measurement example by computing the error terms with VNA Tools II [7] for unknown 2.4 mm coaxial standards (TRM calibration). The resulting value and uncertainty region (k = 2) of the eigenvalue is shown in fig 3.

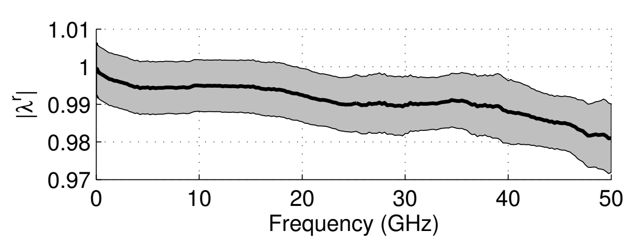

Fig. 3. Reflection coefficient λ^r of the used calibration standard.

As a verification, the same problem was computed with the optimization solver in VNA Tools II. The maximum observed difference between the two solutions is less than 10^{-9}, see fig 4.

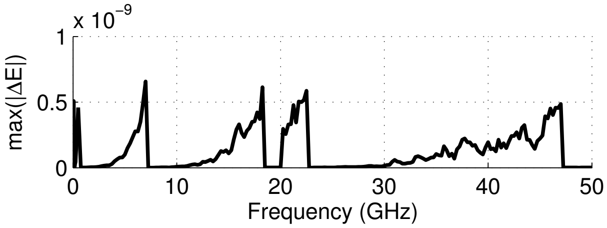

Fig. 4. Maximum difference of all error terms between the TRM calibration and the optimization calibration.

V. TRL and LRL Calibration

The following example shows the matrix $\mathbf{G}+\lambda^r\mathbf{H}^r+\lambda^l\mathbf{H}^l$ for TRL and LRL [1], [6] calibration

$$\left(\begin{array}{cccccccc}
m^t_{11} & 0 & 1 & 0 & m^t_{11}s^t_{11} & m^t_{12}s^t_{21} & s^t_{11} & 0 \\
m^t_{21} & 0 & 0 & 0 & m^t_{21}s^t_{11} & m^t_{22}s^t_{21} & 0 & s^t_{21} \\
0 & m^t_{12} & 0 & 0 & m^t_{11}s^t_{12} & m^t_{12}s^t_{22} & s^t_{12} & 0 \\
0 & m^t_{22} & 0 & 1 & m^t_{21}s^t_{12} & m^t_{22}s^t_{22} & 0 & s^t_{22} \\
\hline
m^r_{11} & 0 & 1 & 0 & m^r_{11}\lambda^r & 0 & \lambda^r & 0 \\
0 & m^r_{22} & 0 & 1 & 0 & m^r_{22}\lambda^r & 0 & \lambda^r \\
\hline
m^l_{11} & 0 & 1 & 0 & m^l_{11}s^l_{11} & m^l_{12}\lambda^l & s^l_{11} & 0 \\
m^l_{21} & 0 & 0 & 0 & m^l_{21}s^l_{11} & m^l_{22}\lambda^l & 0 & \lambda^l \\
0 & m^l_{12} & 0 & 0 & m^l_{11}\lambda^l & m^l_{12}s^l_{22} & \lambda^l & 0 \\
0 & m^l_{22} & 0 & 1 & m^l_{21}\lambda^l & m^l_{22}s^l_{22} & 0 & s^l_{22}
\end{array}\right). \tag{16}$$

Here the first four rows describe the transmission standard (thru or known line), the next two rows describe the one-port standards with unknown reflection λ^r and the last four rows describe the line standard with unknown propagation constant (unknown transmission λ^l).

This problem is solved in two steps because there are two unknown calibration standards, the reflect λ^r and the line λ^l. The first step consists of using only the rows of the transmission standard (thru) and the rows of the line standard. This yields a generalized eigenvalue problem for the unknown propagation constant (unknown transmission λ^l)

$$\mathbf{G}_{[1:4\ 7:10],:}\mathbf{x}+\lambda^l\mathbf{H}^l_{[1:4\ 7:10],:}\mathbf{x}=0. \tag{17}$$

Under the precondition of small measurement error, two eigenvalues have an absolute value less or equal to one. Those two eigenvalues λ^l_j and λ^l_k are used to compute the unknown transmission of the line

$$\lambda^l=\begin{cases} +\sqrt{\lambda^l_j\lambda^l_k}\ , \left|\lambda^l_j-\sqrt{\lambda^l_j\lambda^l_k}\right|\leq\left|\lambda^l_j+\sqrt{\lambda^l_j\lambda^l_k}\right| \\ -\sqrt{\lambda^l_j\lambda^l_k}\ , \left|\lambda^l_j-\sqrt{\lambda^l_j\lambda^l_k}\right|>\left|\lambda^l_j+\sqrt{\lambda^l_j\lambda^l_k}\right| \end{cases}. \tag{18}$$

The second step consists of using all rows of all standards and the now known transmission of the line λ^l. This yields an over-determined linear eigenvalue problem for the unknown reflection λ^r

$$\left(\mathbf{G}+\lambda^l\mathbf{H}^l\right)\mathbf{x}+\lambda^r\mathbf{H}^r\mathbf{x}=0. \tag{19}$$

VI. Uncertainties

The described concept can capture uncertainties (noise and linearity) which influence the raw measured S-parameters $\mathbf{M}^i$. These uncertainties are just de-embedded from the measured $\mathbf{M}^i$ and the result of this is used instead of $\mathbf{M}^i$. Uncertainties (cable stability, connector repeatability) which act on the S-parameters of the standards $\mathbf{S}^i$ are more difficult to treat. The approach as for $\mathbf{M}^i$ is not possible because the eigenvalues and thus the $\mathbf{S}^i$ are not known beforehand. More details on the used measurement model can be found in [7]. The problem is illustrated with an unknown reflection, which has a certain connector repeatability. Figure 5 shows two different two-ports, c, representing the connector repeatability, which are cascaded to a common unknown reflection λ^r at each port.

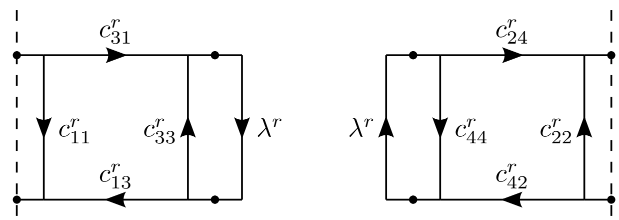

Fig. 5. Two two-ports, c, representing connector repeatability uncertainties, are cascaded to the two one-port standards with the same unknown reflection λ^r.

The sub matrix below describes this constellation

$$\begin{pmatrix} m^r_{11} & 0 & 1 & 0 & m^r_{11}s^r_{11} & 0 & s^r_{11} & 0 \\ 0 & m^r_{22} & 0 & 1 & 0 & m^r_{22}s^r_{22} & 0 & s^r_{22} \end{pmatrix} \tag{20}$$

with

$$s^r_{11} = c^r_{11}+\frac{c^r_{31}c^r_{13}\lambda^r}{1-c^r_{33}\lambda^r} \tag{21}$$

$$s^r_{22} = c^r_{22}+\frac{c^r_{42}c^r_{24}\lambda^r}{1-c^r_{44}\lambda^r}. \tag{22}$$

The series expansion of the equations for cascading a two-port to an one-port are

$$s^r_{11} \approx c^r_{11}+\underbrace{c^r_{31}c^r_{13}}_{d^r_{11}}\lambda^r+\underbrace{c^r_{31}c^r_{13}c^r_{33}}_{e^r_{11}}\left(\lambda^r\right)^2 \tag{23}$$

$$s^r_{22} \approx c^r_{22}+\underbrace{c^r_{42}c^r_{24}}_{d^r_{22}}\lambda^r+\underbrace{c^r_{42}c^r_{24}c^r_{44}}_{e^r_{22}}\left(\lambda^r\right)^2. \tag{24}$$

If c^r_{33} and c^r_{44} are of value zero with non-zero uncertainty, the series expansion is actually exact within a linear uncertainty environment.

Putting (23) and (24) into (20) yields a quadratic eigenvalue problem

$$\mathbf{G}\mathbf{x}+\lambda^r\mathbf{H}^r_{\mathbf{1}}\mathbf{x}+\left(\lambda^r\right)^2\mathbf{H}^r_{\mathbf{2}}\mathbf{x}=0. \tag{25}$$

with sub matrix $\mathbf{G}$

$$\begin{pmatrix} m_{11}^r & 0 & 1 & 0 & m_{11}^r c_{11}^r & 0 & c_{11}^r & 0 \\ 0 & m_{22}^r & 0 & 1 & 0 & m_{22}^r c_{22}^r & 0 & c_{22}^r \end{pmatrix}, \tag{26}$$

sub matrix $\mathbf{H_1^r}$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & m_{11}^r d_{11}^r & 0 & d_{11}^r & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{22}^r d_{22}^r & 0 & d_{22}^r \end{pmatrix}, \tag{27}$$

and sub matrix $\mathbf{H_2^r}$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & m_{11}^r e_{11}^r & 0 & e_{11}^r & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{22}^r e_{22}^r & 0 & e_{22}^r \end{pmatrix}. \tag{28}$$

In a similar way, one can derive a quadratic eigenvalue problem for line standards with unknown transmission. Both cases, unknown reflection and unknown transmission with uncertainty influences, are implemented in VNA Tools II.

VII. Over-Determined Eigenvalue Problem

The calibration algorithms shown here and the associated uncertainty propagation can lead to over-determined linear and quadratic eigenvalue problems. The most general problem is an over-determined quadratic eigenvalue problem

$$\mathbf{O_0 v} + \lambda \mathbf{O_1 v} + \lambda^2 \mathbf{O_2 v} = 0. \tag{29}$$

The over-determined quadratic eigenvalue problem can be rewritten as an over-determined linear eigenvalue problem

$$\underbrace{\begin{bmatrix} \mathbf{O_0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}}_{\mathbf{P_0}} \underbrace{\begin{bmatrix} \mathbf{v} \\ \lambda \mathbf{v} \end{bmatrix}}_{\mathbf{w}} + \lambda \underbrace{\begin{bmatrix} \mathbf{O_1} & \mathbf{O_2} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}}_{\mathbf{P_1}} \underbrace{\begin{bmatrix} \mathbf{v} \\ \lambda \mathbf{v} \end{bmatrix}}_{\mathbf{w}} = 0 \tag{30}$$

where $\mathbf{I}$ is the identity matrix. This step can be omitted when all elements of $\mathbf{O_2}$ are zero.

The over-determined linear eigenvalue problem can be rewritten as a quadratic eigenvalue problem

$$\underbrace{\mathbf{P_0^* P_0}}_{\mathbf{Q_0}} \mathbf{w} + \lambda \underbrace{(\mathbf{P_0^* P_1} + \mathbf{P_1^* P_0})}_{\mathbf{Q_1}} \mathbf{w} + \lambda^2 \underbrace{\mathbf{P_1^* P_1}}_{\mathbf{Q_2}} \mathbf{w} = 0 \tag{31}$$

by squaring the linear over-determined problem. The operator * denotes the conjugate transpose.

The quadratic eigenvalue problem can be rewritten as a generalized eigenvalue problem

$$\underbrace{\begin{bmatrix} \mathbf{Q_0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{bmatrix}}_{\mathbf{R_0}} \underbrace{\begin{bmatrix} \mathbf{w} \\ \lambda \mathbf{w} \end{bmatrix}}_{\mathbf{z}} + \lambda \underbrace{\begin{bmatrix} \mathbf{Q_1} & \mathbf{Q_2} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}}_{\mathbf{R_1}} \underbrace{\begin{bmatrix} \mathbf{w} \\ \lambda \mathbf{w} \end{bmatrix}}_{\mathbf{z}} = 0 \tag{32}$$

which yields a standard eigenvalue problem

$$\underbrace{-\mathbf{R_0}^{-1} \mathbf{R_1}}_{\mathbf{T}} \mathbf{z} = \underbrace{\frac{1}{\lambda}}_{\lambda'} \mathbf{z}. \tag{33}$$

The eigenvalue computation with linear uncertainty propagation is described in [8] and it is fully implemented in METAS UncLib [9], [10].

VIII. Conclusion

In this paper a generalization of calibration schemes with partly unknown standards has been presented. The generalization consists of constructing an eigenvalue problem for each calibration scheme. One obvious advantage is that the same algorithm can be used for different schemes as TRM, LRM, TRL and LRL. Another advantage is that partly unknown lines can now be described with non-zero reflection and uncertainty, which is a clear improvement over the traditional TRL algorithm. Over-determined calibration with several partly unknown lines is as well possible with this algorithm. This is an advantage if broad frequency ranges have to be covered. The propagation of uncertainties coming from instrument noise, instrument linearity, drift, cable stability, connection repeatability is fully supported by solving quadratic eigenvalue problems. This algorithm is analytic and thus can be used to generate starting values for an optimization calibration involving offset shorts and multiple lines. The here described concept could be extended to multi-port and full-leaky calibration.

Software

The calibration algorithms involving eigenvalue problems with partially unknown standards and linear propagation of uncertainties are implemented in VNA Tools II. The software is available online: http://www.metas.ch/vnatools.

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