The uncertainty evaluation of homodyne signals

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Abstract

The simulation results of errors introduced by non-ideal quadrature signals in homodyne interferometers are presented. The empirical formula for uncertainty of uncorrected signals was obtained using the Monte Carlo method (MCM). Generally, the phase error of real uncorrected quadrature signals can be obtained, independently of their physical nature.

1   Introduction

The elliptic correction for real homodyne signals of interferometers is known for many years [1], [2] and it can be used to decrease the fringe interpolation error of quadrature signal. A similar procedure can be used for heterodyne interferometers. Previous works shows that correction can reach 10 pm non-linearity error level (i.e. 0.2 mrad in phase) for optical interferometers [3]. However the parameters of corrections can vary with e.g. the interferometer adjustment and then they change with time and position. Thus a real time correction is needed, which is not common case of commercial optical interferometers. And quadrature signal outputs are also present e.g. in line scale encoders and then a general approach was applied and the uncertainty of uncorrected quadrature signals was evaluated by the MCM because of the nonlinearity of the problem.

2   Results

The following uncertainty analysis is based on the results obtained from the software ECunc developed in the Subnano project. The estimations of errors in homodyne interferometers (or encoders etc.) by this software are based on the Monte Carlo method calculations. The software generates signal with a selected noise level and for a selected elliptical parameters and signal digitalisation steps. The uncertainty and noise propagation from elliptic signal parameters is evaluated as a difference from the elliptically corrected quadrature signal. These calculations are repeated by a given number of trials. All ellipse parameters are changed with distributions and uncertainties selected by user. All non-linearity errors of all data from all ellipses are sorted and drawn in the histogram. Then percentile ranks are calculated and expanded uncertainty for approximately 95% coverage probability is used as output.

In the simplest case, when the systematic bias of ellipse parameters is not expected, we can simplify the MCM results by the following empirical relationship. The result is the root of the sum of squares of components as it is expected from the central limit theorem. The total expanded uncertainty for interferometer (in units relative to the fringe, i.e. 2π) is given by

$$U_{tot} = \sqrt{U_p^{\,2} + U_d^{\,2} + U_c^{\,2}} \tag{1}$$

where U_p is the expanded uncertainty of phase evaluation (including the phase noise), U_d is contribution from the digitalisation error and U_c is the expanded uncertainty of elliptic correction (i.e. fringe non-linearity) given by

$$U_c = \sqrt{(c_x u_x)^2 + (c_y u_y)^2 + (c_{cx} u_{cx})^2 + (c_{cy} u_{cy})^2} \tag{2}$$

where u_x and u_y are standard uncertainties (including the signal noise) of X and Y "detector" sensitivities (relative to the signal – the radius of ideal signal) and u_{cx} and u_{cy} are standard uncertainties

of X and Y detector offsets (relative to the correct signal). Sensitivity coefficients c_x and c_y equals approximately to 0.122 (nearly independent whether sources for u_x and u_y have the normal or the rectangular distribution) and for fully correlated errors (u_x and u_y) are equal to zero (as it is expected for the circular signal). Sensitivity coefficients c_{cx} and c_{cy} equals approximately to 0.221 in the uncorrelated case and they are 0.243 (little bit larger) for fully correlated u_{cx} and u_{cy}.

The quadrature signal is often processed by an analog-to-digital converter (see figure 1). Thus the contribution of digitalisation to the measurement uncertainty must be also taken into account. The signal rounding step d (in units relative to the signal amplitude) contributes to the uncertainty by

$$U_d = c_d d \tag{3}$$

where the coefficient c_d is approximately 0.092 for rounding and 0.142 for truncating in a conversion process. These sensitivity coefficient values were evaluated with rectangular distributions of ellipse parameters and thus represent upper limit from the MCM calculations. They are smaller by about 5 % for normal distributions of ellipse parameters.

The application of empirical coefficients for interferometer fringe interpolation uncertainty estimation can be demonstrated on specific examples. As the first example we can evaluate 8 bit analog-to-digital converter. I.e. 7 bit effective per radius in the optimal case. Then the digitalisation uncertainty for 633 nm interferometer is 0.23 nm in the case of rounding and 0.35 nm in the case of truncating. We can also show that at least a 14 bits digitalisation is necessary to reach 10 pm uncertainty (for the fringe equals to half of 633 nm).

For another example with the single-pass 633 nm interferometer let be with following inputs. Standard uncertainties of semi-axes and the centre coordinates equal to 5 % of the signal amplitude and they are uncorrelated. The result obtained by formula is 0.0179 of fringe. It corresponds to the MCM results 0.0175 and 0.0166 for normal and rectangular distributions respectively. I.e. the expanded uncertainty of single-pass 633 nm interferometer (without elliptic correction) is then 5.6 nm. The contribution of signal digitalisation is negligible in this case. Nevertheless the phase error, given by interferometer amplitude, phase and polarisation instabilities and noises, should be evaluated and present in the summation in quadrature of the uncertainty contributions.

The presented formula is relative simple and allows the calculation of total interferometer expanded uncertainties for the most common cases even though various distributions of errors were obtained in the MCM calculations. Examples of these distributions are shown in figure 2.

The linearity of empirical formula was also tested for various combinations of ellipse parameter uncertainties. The non-linearity of this formula is less than about 5% up to the expanded uncertainty of fringe equal to 5 %. I.e. it is valid within 5 % for example for variations by about 50 % (for the coverage factor $k=2$) of two ellipse parameters (or numerically up to 16 nm errors for the single-pass interferometer with 633 nm wavelength).

3   Conclusions

The evaluation of uncertainty of quadrature signals is useful not only for interferometers. The simple empirical formulae presented in this article can help for practical uncertainty evaluations. However more complex and specific cases directly need the MCM uncertainty calculations and they cannot be evaluated by a simple formula.
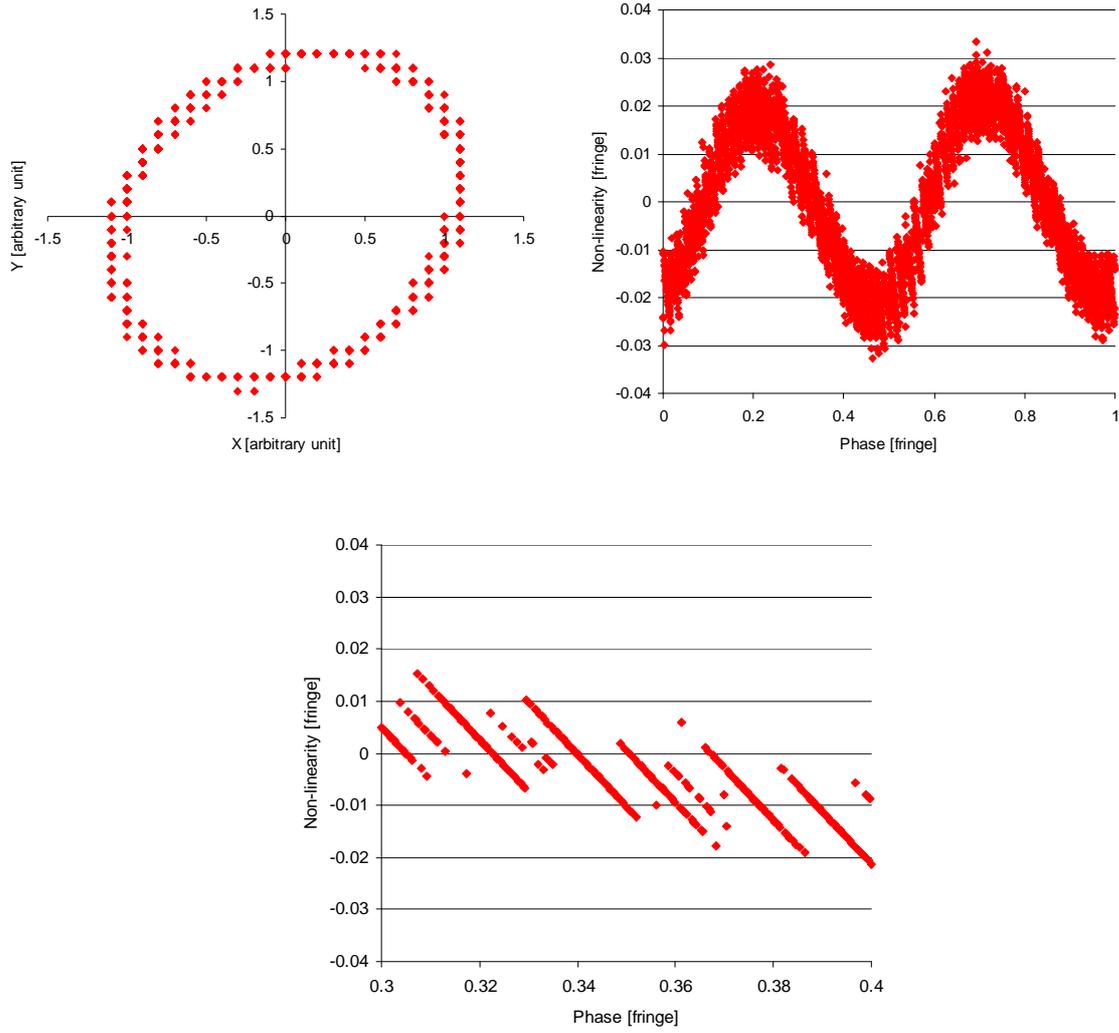
Acknowledgement

Figure 1: The digitalised elliptic signal for a given parameters and noise and the detail of corresponding errors with respect to undigitalized real values are shown.
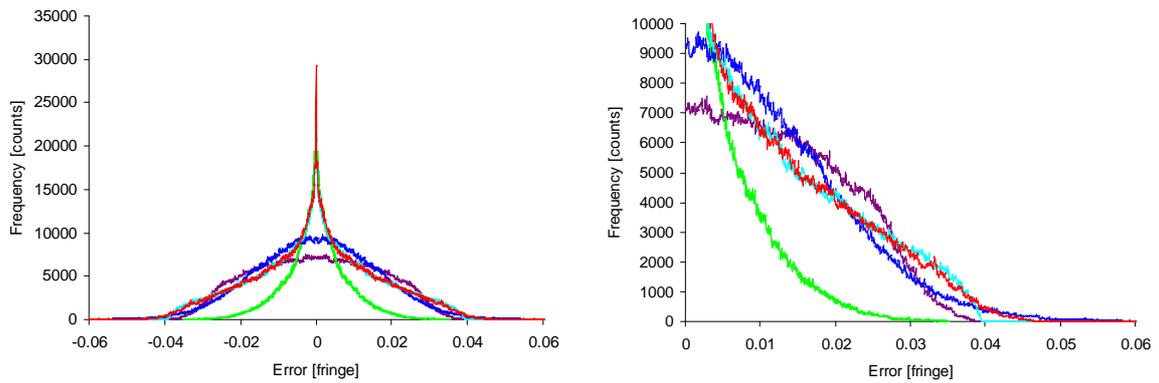

Figure 2: Probability distributions of errors obtained in the MCM and detail of their tails.

3

References

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