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Reality – governed by six invariant fundamental constants

– A metrological view on the unity of physics –

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The author felt the need to republish his former report

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as an **online edition**. He took this opportunity to revise the text while maintaining the pagination and layout. Only a few typesetting inconsistencies had to be corrected, in one place the wording was changed for clarity, a forgotten publication year was added to a citation and also a number referencing a chapter was inserted. As this is an online edition the header and footer information were added to state the source on each page. Also this verso title page is unique for this online edition of the report.

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Frank Spieweck was born in 1937 in Fichtenau near Berlin. He received a Dipl.-Phys. degree from the Technical University of Braunschweig and a Dr. rer. nat. degree from the Technical University of Hannover. In 1966 he joined the Physikalisch-Technische Bundesanstalt (PTB) in Braunschweig. There he first constructed and built pressure controlled $^{136}Xe^+$ lasers. Then he developed mode selection and wavelength stabilization methods for Kr^+ and Ar^+ lasers. His wavelength determination of an I_2 stabilized Ar^+ laser line was used for calibrating the atlas of I_2 absorption lines by Gerstenkorn and Luc in 1978. His Ar^+ laser, stabilized to an external I_2 absorption cell, with its high wavelength or frequency reproducibility (Helmholtz Prize 1981) was an important step towards the new definition of the metre in 1983. As head of the Density Laboratory of PTB he worked "on the silicon path" leading to a new definition of the kilogram. Frank Spieweck retired from PTB in 1999, but he has remained a client of the PTB-Library to pursue his interests in metrology especially and physics in general.

Descriptors

Metrology, Constants, Measurement Standards, Physics Fundamentals, Units (Measurement), Measurement Theory, Quantum Theory, Quantum Gravity, General Relativity, Cosmology

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Reality – governed by six invariant fundamental constants
– A metrological view on the unity of physics –

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Abstract

It is proposed that in classic, general relativistic and quantum physics the same nomenclature shall be used. This may be achieved by making use of a slightly modified International System of Units SI, which should be adapted to the requirements of general relativistic and quantum mechanics. Then, general relativistic problems will be simply treated like phenomena of classic physics. The system of possible future SI units should be based on fixed values for the general relativistic constant $8\pi/\kappa$ ($= c^4/G$, κ: Einstein constant, c: speed of light in vacuum, G: Newton's gravitational constant), Planck's constant h, the elementary charge e, Boltzmann's constant k, the atomic mass unit u, $1/N_A$ (N_A: Avogadro constant), and for a certain atomic energy W_a. In part I, a metrological basis is presented, where reference frame dependent and reference frame independent – or invariant – physical quantities are defined, whereas in parts II and III resulting consequences are discussed, e.g. concerning the grainy structure of empty space and a metrologically reformulated path to general relativistic physics.
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Zusammenfassung

Es wird vorgeschlagen, zur Behandlung der klassischen und allgemein-relativistischen Physik sowie der Quantenphysik eine einheitliche Nomenklatur zu verwenden. Dies könnte dadurch erreicht werden, dass das Internationale Einheitensystem SI gemäß den Erkenntnissen der allgemeinen Relativitätstheorie und der Quantenmechanik geringfügig modifiziert wird, so dass sich allgemein-relativistische Probleme so einfach behandeln lassen wie Probleme der klassischen Physik. So ließe sich ein System möglicher zukünftiger SI-Einheiten gründen auf festgelegte Werte für die allgemein-relativistische Konstante $8\pi/\kappa$ ($=c^4/G$, κ: Einsteinkonstante, c: Lichtgeschwindigkeit im Vakuum, G: Newtons Gravitationskonstante), die Planckkonstante h, die Elementarladung e, die Boltzmannkonstante k, die atomare Masseneinheit u, $1/N_A$ (N_A: Avogadrokonstante) und eine bestimmte atomare Energie W_a. Im Teil I werden die metrologischen Grundlagen vorgestellt, wobei unterschieden wird zwischen physikalischen Größen, die vom Bezugssystem abhängen und Invarianten, die nicht vom Bezugssystem abhängen. In den Teilen II und III werden hieraus sich ergebende Folgerungen diskutiert, die z.B. die körnige Struktur des Vakuums und einen metrologisch neu formulierten Weg zur allgemein-relativistischen Physik betreffen.

Preface

The Global Positioning System GPS has shown that the results of general relativity must be taken into account in today's practical metrology. Therefore, it is here proposed that on the one hand the nomenclature of general relativistic physics should be adapted to the usual metrological "language", which on the other hand should be adapted to the requirements of general relativity. For this purpose, the primary elements to be used for the treatment of physical problems shall be physical units (instead of coordinates), which will be defined in accordance with the requirements of general relativistic, quantum and classic physics.

The today's treatment of physics in a euclidean or non-euclidean space may be compared to the situation in the 19th century, when it was believed that physics in the whole can be mechanically explained. In this case, the cgs system was established as an adequate system of units, in which several electric units are resulting as derived units with broken exponents. However, meanwhile the majority of physicists are convinced that electrical phenomena are at least as fundamental as mechanical phenomena. Therefore, in the International System of Units SI, an electric base unit is defined besides the mechanical base units. Then, electric phenomena must not be treated by using units with broken exponents.

A similar situation is concerning general relativistic gravitational physics. Up to the creation of general relativity it was believed that physics in the whole can be based on laws of euclidean or pseudo-euclidean geometry. And the present SI seems to be an adequate system of units for treating all physical problems. In this case, however, general relativistic problems must be handled by making use of the mathematically complex metric tensor $g_{\alpha\beta}(x^{\mu})$, see Guinot (1997). Therefore, it is now proposed that the SI should be slightly modified in a way that general relativistic problems can be treated in the same simple way as problems of classic physics – without the necessity for making use of the Riemannian tensor formalism.

The proposed (future) change of the SI is mainly concerning the units of inertial and gravitational mass. In the present SI, the unit kilogram (kg) was originally defined as unit of inertial mass m, see Fleischmann (1980). Then, the units of force and energy are obtained as coherent units, that means, as units which are directly derived from the units of length, time and inertial mass – without an additional factor $\neq 1$. In contrast, the unit of gravitational mass m_g, which shall be written as kg_g (or kg_s, see Fleischmann 1980), is – incoherently – derived from Newton’s law $F = Gm_g^2/r^2$, where G is the Newtonian gravitational constant, which is a dimensional factor $\neq 1$. In euclidean space, respectively in a laboratory in which the Newtonian gravitational potential is assumed to be constant within the whole volume, the unit kg_g can be set equal to the unit kg.

It may be interesting to read the remarks in a book of J.L. Synge (1960) on the equivalence of inertial and gravitational mass, respectively on the principle of equivalence: “I have never been able to understand this principle. Does it mean that the signature of the space-time metric is +2 (or – 2 if you prefer the other convention)? If so, it is important, but hardly a Principle. Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer’s acceleration? If so, it is false. In Einstein’s theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property; it has nothing to do with any observer’s world-line. Space-time is either flat or curved, and in several places in the book I have been at considerable pains to separate truly gravitational effects due to curvature of space-time from those due to curvature of the observer’s world-line (in most ordinary cases the latter predominate). The Principle of Equivalence performed the essential office of a midwife at the birth of general relativity, but, as Einstein remarked, the infant would never have got beyond his long-clothes had it not been for Minkowski’s concept. I suggest that the midwife be now buried with appropriate honours.”

Here, it is proposed that not only general relativity but the whole physics may be built up without the necessity for setting the inertial mass equal to the gravitational mass: Besides the unit of inertial mass not only a separate unit of electric charge, coulomb (C), but also a separate unit of gravitational (charge or) mass should be used. However, contrary to the original definition of the kilogram as a unit of inertial mass m, the kilogram, should be now defined as the unit of the – scalar – gravitational mass m_g, whilst the unit of the – possibly tensorial (Goldstein 1963) – inertial mass should become a derived unit, namely N s^2/m in euclidean space and N sr s^2/m in a non-euclidean space, see below, part I.

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Contents

I. Basic concept for a uniform view on six branches of physics

Abstract. Similarly, as Einstein unified electrodynamics and relativistic mechanics by interpreting the speed of light in vacuum *c* as a common constant of electrodynamics and relativistic mechanics, it is now proposed that six fundamental constants, namely Planck's constant *h*, the elementary charge *e*, Boltzmann's constant *k*, the atomic mass unit u, $1/N_A$ (N_A: Avogadro constant) and $8\pi/\kappa$ (= c^4/G, κ: Einstein constant, *G*: Newton's gravitational constant) may be interpreted and used as common invariants of discontinuous or single-particle quantum physics and continuous or many-particle – non-euclidean – relativistic physics. In addition, one reference frame dependent physical quantity is given, namely a certain atomic proper energy W_a. The six invariants can be used for the physical description of an objective reality, whereas the seven physical quantities $8\pi/\kappa$, *h*, *e*, *k*, u, $1/N_A$ and W_a can be used as microscopic – so-called natural – physical base units $8\pi/\kappa$, h, e, k, u, $1/\mathrm{N_A}$ and $\mathrm{W_a}$ for a consistent description of the general relativistic, quantum mechanical and classic reality.
PACS: 06.20.F-

Zusammenfassung. Ähnlich wie Einstein Elektrodynamik und relativistische Mechanik vereinen konnte, indem er die Vakuumlichtgeschwindigkeit *c* als deren gemeinsame Konstante erkannte, so wird hier vorgeschlagen, sechs Fundamentalkonstanten, nämlich die Planckkonstante *h*, die Elementarladung *e*, die Boltzmannkonstante *k*, die atomare Masseneinheit u, $1/N_A$ (N_A: Avogadrokonstante) und $8\pi/\kappa$ (= c^4/G, κ: Einsteinkonstante, *G*: Newtons Gravitationskonstante) als gemeinsame Invarianten der Diskontinuums-, Ein-Teilchen- oder Quanten-Physik und der Kontinuums-, Mehr-Teilchen- oder Relativitäts-Physik zu interpretieren und zu benutzen. Zusätzlich wird eine bezugssystemabhängige physikalische Größe gewählt, nämlich eine bestimmte atomare Eigenenergie W_a. Die sechs Invarianten können dazu dienen, eine objektive Realität zu beschreiben, während sich mit Hilfe der sieben physikalischen Größen $8\pi/\kappa$, *h*, *e*, *k*, u, $1/N_A$ und W_a sogenannte natürliche Basiseinheiten $8\pi/\kappa$, h, e, k, u, $1/\mathrm{N_A}$ und $\mathrm{W_a}$ eines mikrophysikalischen Einheitensystems definieren lassen, um die allgemein-relativistische Physik, die Quantenphysik und die klassische Physik in konsistenter Weise zu beschreiben.

1. Introduction

During the last decades of his life, Einstein tried to find a complete theory of reality, unifying continuous field and discontinuous quantum physics. In December 1954, he suggested that reality cannot be described by a continuous field, because quantum phenomena can be fully described by a finite number of (quantum) numbers. Therefore, reality should not be described by a continuum theory, but by an algebraic theory. He concluded his remarks by stating that nobody has an idea how to get a basis for such a theory (Einstein 1965 p 110).

However, perhaps he himself gave a hint towards such a theory. In a letter to Mrs. Rosenthal-Schneider, dated 11 May 1945, he wrote that universal constants should be mere numbers, as π and e (Rosenthal-Schneider 1988 p 24). – Now, let us look at the common features of π and quantum numbers as n/2 (n = 1, 2, 3, ...). Obviously, both kinds of numbers are characterizing an objective reality, being independent of an observer's experimental conditions.

2. A mathematical analogy

In euclidean geometry, the ratio of a circle's circumference *u* and its diameter *d* is equal to π, irrespective of the circle's size or the material of the observer's measuring sticks. Moreover, the ratio of two algebraic numbers with an infinite number of digits (e.g. 0.6666... and 0.3333...) may be equal to a finite integer (= 2).

Similarly, in physics, the ratio of an atomic proper energy W_a, which may be also the energy difference of two atomic levels, $W_a = W_2 - W_1$, and the oscillation frequency *f* of a correspondingly emitted light wave is equal to a constant finite value ($W_a/f = h$), namely Planck's constant *h* (Planck 1900), irrespective of the dimensions of the light source or the individual measuring instruments of the observer. Therefore, the crucial question seems to be the following: Do there exist similar constants like *h* in all branches of physics, including general relativistic physics?

3. Description of reality by an external observer

Mössbauer experiments – and meanwhile also experiments with transported atomic clocks – have demonstrated the following situation. Between two events, an atomic clock, located on Earth in a lower local non-inertial proper frame *K*, is showing the time $t = \{t\}[t]$, e.g., $t = 10^{13}$ s, where $[t] = \mathrm{s}$ is the unit of *t*, and $\{t\} = 10^{13}$ is the numerical value of *t*, if *t* is expressed in the unit $[t]$, which is realized in the proper frame *K*. However, an external observer in an upper inertial or local non-inertial frame K^*, looking down to the lower clock, will see on the lower clock the same number ($\{t^*\} = \{t\} = 10^{13}$) of "effective" second beats, i.e. $\{t^*\} = 10^{13}$ "effective" second beats $[t^*]$. Therefore, for an external observer the "effective" time unit $[t^*]$ seems to be enlarged to $[t^*] > [t]$, e.g., $[t^*] = (1 + 10^{-13})[t]$, if the observer is positioned about 1000 m above the lower clock. Those "effective" physical units have been – probably first – introduced by Dehnen *et al* (1960).

In contrast, in the theory of relativity, Einstein (1965) only considered so-called "naturally measured" physical quantities: In this case, processes occurring in the proper frame *K* are measured by using measuring sticks and clocks, which are located in *K*, and processes occurring in another proper frame *K*' are measured by using measuring sticks and clocks, which are located in *K*'. Then, in the proper frames *K* and *K*', e.g., the same units of proper time can be used: $[t] = [t'] = \mathrm{s}$ (see Spieweck 1978 or Pierseaux 2003).

According to the considerations made here, physics taking place in an inertial or in a local non-inertial proper frame of reference *K* may have been, first, described or measured by an observer in the proper frame *K* by using naturally measured physical quantities $Q_i = \{Q_i\}[Q_i]$. Then, this physics, occurring in the proper frame *K*, can be also described by an external observer, staying in another inertial or local non-inertial frame of reference K^*, by using so-called "effective" physical quantities $Q_i^* = \{Q_i^*\}[Q_i^*]$, where $[Q_i^*]$ is an "effective" unit, which is meaning the unit $[Q_i]$ in the proper frame *K*, however being judged or measured by the external observer, who is using units, being realized in his own frame of reference. In this case, the "effective" unit $[Q_i^*]$ and the "naturally measured" unit $[Q_i]$ will differ by a numerical factor f_i, which will be derived in part III (e.g.: $f_i = W_0^*/W_0$, where W_0 is the naturally measured proper energy in the frame *K*): $[Q_i^*] = f_i[Q_i]$. It is $f_i \neq 1$, if the Newtonian potential in *K* is differing from the Newtonian potential in K^*, or if *K* is moving against K^* with the velocity *v*. Because the numerical values $\{Q_i\}$ and $\{Q_i^*\}$ are identical, $\{Q_i\} = \{Q_i^*\}$, it results $Q_i^* = \{Q_i\}(f_i[Q_i]) = \{Q_i\}f_i[Q_i]$. E.g., in case of time it is $t^* = \{t\}f_i[t]$. Note, that the same situation is equivalently – however inversely – described in the theory of relativity. There, the units $[t]$ and $[t']$ are the same, but the numerical coordinate values $\{t\}$ and $\{t'\}$ are differing: $\{t'\} = \{t\}f_i$. Thus, it results $t' = (\{t\}f_i)[t] = \{t\}f_i[t]$.

Not only time *t*, but the majority of physical quantities Q_i – as length *s*, (spherical) plane angles (in a horizontal plane) λ and (in a vertical plane) ϕ as well as solid angle Ω – will be judged by an external observer as altered physical quantities s^*, λ^*, ϕ^* and Ω^*. If a value Q_i seems to be changed to Q_i^*, this physical quantity Q_i shall be called a reference frame dependent physical quantity. If the value Q_i^* coincides with the value Q_i, as e.g. in case of action (Wirkung), $S^* = S$, this physical quantity Q_i shall be defined as a reference frame independent physical quantity or as an invariant, see Dehnen *et al* (1960). It must be emphasized that, there, – probably first – a new kind of invariance had been defined: A physical quantity Q_i shall be called a metrological invariant, $Q_i^* = Q_i$, if its "effective" unit $[Q_i^*]$ is equal to the naturally realized unit $[Q_i]$, $[Q_i^*] = [Q_i]$. Because this contribution is dealing with those metrological invariants instead of other invariants like Lorentz invariants (resulting from *mathematical* operations), in the following metrological invariants will be merely called invariants.

4. Unconventional ideas for combining different branches of physics

Perhaps, a view on Einstein's ideas of special relativity may give us a hint for a solution of the present physical problems, concerning the discrepancies between relativistic and quantum physics.

4.1. Unification of electrodynamics and relativistic mechanics

The physical quantities length *s*, time *t* and speed of light in vacuum *c* were interpreted by Lorentz in a conventional way, which satisfactorily explained the phenomena of electrodynamics. In this case, Lorentz maintained *t* as an absolute physical quantity, whereas *c* was believed to be a reference frame dependent physical quantity, defined as the velocity of electro-magnetic waves against the ether. The interpretation of *t* as an absolute physical quantity could be made by Lorentz, because within a single branch of physics (here, electrodynamics) at least one – arbitrarily selected – physical quantity can be defined as an absolute physical quantity or as a physical quantity having the same value in different frames of reference.

For this reason, however, it was possible, too, to define – instead of *t* – another physical quantity, which has the same value in different frames of reference, namely *c* (Einstein 1905b). In this case, Einstein unconventionally treated *t* as a reference frame dependent physical quantity. The interchanged interpretation of *t* and *c* – as a reference frame dependent physical quantity and as a physical quantity having the same value in different frames of reference – had the following advantage. Not only electro-magnetic phenomena but also mechanical experiments with high-speed particles, then, could be described in a consistent way. That means, by this new interpretation of *t* and *c*, Einstein unified electrodynamics and relativistic mechanics.

4.2. Proposed new view on relativistic, classic and quantum physics

The physical quantities *c* and Newton's gravitational constant *G*, contained in Einstein's general relativistic field constant κ (= $8\pi G/c^4$, see, e.g. Dehnen *et al* 1960 or Goenner 1996), were interpreted by Einstein in a way, which satisfactorily explained the phenomena of general relativity. In this case, κ was believed to be only a constant of a single branch of physics, namely general relativistic continuum field physics.

Now, again, an unconventional proposal is made, namely to treat general relativistic, classic and quantum physics on the basis of common invariants. In this case, $8\pi/\kappa$ and *h*, as well as Boltzmann's constant *k*, the elementary electric charge *e*, a corresponding elementary gravitational charge or atomic mass unit u and the amount of a single particle $1/N_A$, where N_A is the Avogadro constant, are considered as common invariants of the whole physics: $(8\pi/\kappa)^* = 8\pi/\kappa$, $h^* = h$ (see also Leonard 2006), $e^* = e$, $k^* = k$, $u^* = u$ and $(1/N_A)^* = 1/N_A$ (see also Leonard 2006). Then, *c*, the inertial proper mass *m* and the proper energy $W_0 = mc^2$, respectively, c^*, m^* and $W_0^* = m^*c^{*2}$ are obtained as reference frame dependent physical quantities, which depend, e.g., on the gravitational potential Φ, see Dehnen *et al* (1960) and part III.

5. Six aspects of proper energy

5.1. Natural physical units à la Planck

Planck (1899) introduced the invariants *h* and *k* as so-called "natural units" of action and entropy, which, therefore, should be written – like the units eV and u – as h and k. Here, besides *h* and *k*, four further physical quantities are proposed as common invariants of discontinuous or single-particle physics and continuous or many-particle physics: *e*, u, $1/N_A$ and $8\pi/\kappa$. Then, the six invariants *h*, *k*, *e*, u, $1/N_A$ and $8\pi/\kappa$ shall be used as natural physical units: h, k, e, u, $1/\mathrm{N_A}$ and $8\pi/\kappa$.

However, contrary to Planck's choice to select besides *h* and *k* the – reference frame dependent – speed of light in vacuum *c* and the Newtonian gravitational constant *G* as further natural units (c and G), here, besides the six natural units – of reference frame independent physical quantities – h, k, e, u, $1/\mathrm{N_A}$ and $8\pi/\kappa$, additionally a certain atomic proper energy W_a shall be chosen as – reference frame dependent – natural unit (of single-particle physics). Thus, the seven natural units should be used: $8\pi/\kappa$, h, e, k, u, $1/\mathrm{N_A}$ and $\mathrm{W_a}$. From these natural primary (or base) units secondary natural units of length *s*, time *t*, frequency *f*, voltage *U*, temperature *T*, gravitational potential Φ and chemical potential μ_i, as well as of velocity v and inertial mass *m* can be derived, namely $s_u = W_a/(8\pi/\kappa)$, $t_u = h/W_a$, $f_u = W_a/h$, $U_u = W_a/e$, $T_u = W_a/k$, $\Phi_u = W_a/u$, $\mu_{iu} = W_a/(1/N_A)$, $v_u = W_a^2/(8\pi h/\kappa)$ and $m_u = (8\pi h/\kappa)^2/W_a^3$, see figure 1. That means, especially a natural unit of length should no longer be obtained from the reference frame dependent constant *c*, but from the reference frame independent general relativistic constant κ.

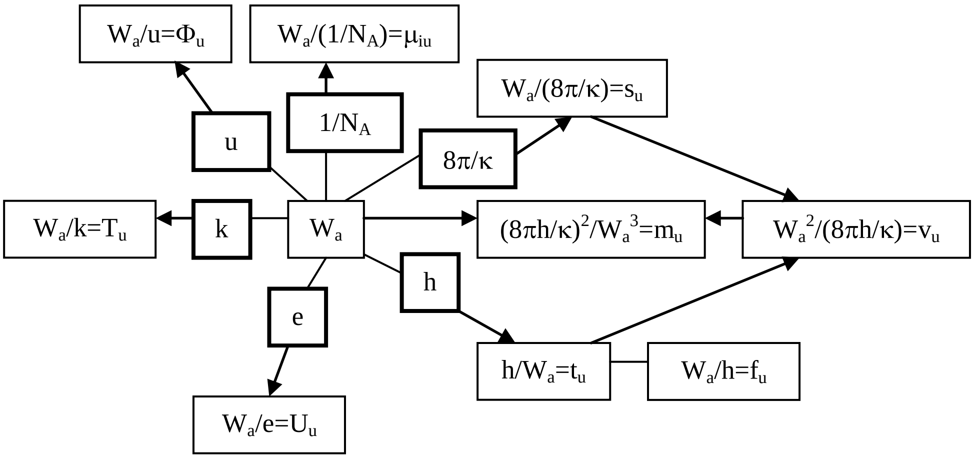

Figure 1. Flow diagram for deriving natural physical units of length (s_u), time (t_u), frequency (f_u), voltage (U_u), temperature (T_u), gravitational potential (Φ_u), chemical potential (μ_{iu}), velocity (v_u) and inertial (proper) mass (m_u) from the central natural unit of a certain proper energy (W_a) and from the surrounding natural units $8\pi/\kappa$, h, e, k, u and $1/N_A$.

Principally, the proper energy $W_0 = mc^2$ (Einstein 1905c) – of a single atomic particle ($W_0 = W_a$) – may be composed of tensional energy $(8\pi/\kappa)s$, oscillational energy hf (Einstein 1905a), electro-magnetic energy eU, thermal energy kT, gravi-inertial energy $\mathrm{u}\Phi$ and chemical energy $(1/N_A)\mu_i$:

$$W_0 = mc^2 = (8\pi/\kappa)s + hf + eU + kT + \mathrm{u}\Phi + (1/N_A)\mu_i. \tag{1}$$

Perhaps, this equation may be a generalization of the de Broglie (1925 p 33) relation $mc^2 = hf$. That means, matter (with the energy mc^2) does not only have an oscillational component (de Broglie: hf), but also a spatial ($(8\pi/\kappa)s$), an electro-magnetic (eU), a thermal (kT), a gravi-inertial ($\mathrm{u}\Phi$) and a (chemical or) particle ($(1/N_A)\mu_i$) aspect. Each of the six energy contributions is – symmetrically – composed of a discontinuous component ($8\pi/\kappa$, h, e, k, u and $1/N_A$) and a continuous one (s, f, U, T, Φ and μ_i). Note, that each of the continuous physical quantities (s, f, U, T, Φ and μ_i) may have arbitrary values – including zero. By the way, the chemical energy $(1/N_A)\mu_i$ should be better called – nuclear, atomic or molecular – particle energy.

In macroscopic – or many-particle – physics ($W_0 = n_0 W_a$), in a closed system, equation (1) may be written as

$$W_0 = mc^2 = n_1(8\pi/\kappa)s + n_2 hf + n_3 eU + n_4 kT + n_5 \mathrm{u}\Phi + n_6(1/N_A)\mu_i = \text{constant}. \tag{2}$$

That means, the sum of all different kinds of energy should remain constant. E.g., in a nuclear explosion, the primary nuclear particle energy of uranium is partly changed into five other kinds of energy and the residual particle energy of barium: Besides a pressure burst, phonons and photons are emitted, the atmosphere's electro-magnetic properties are changed, and a thermal storm, a swirling up (gravitational lifting) of dust (mushroom cloud) and a new chemical element (barium) are created.

Because W_0 may have continuous as well as discrete values, the dualism of W_0 seems to be the common origin of all kinds of dualisms – including the dualism of the empty space, see also part II. Moreover, inertia seems to be one of the main properties of all kinds of energy, and not of the other different properties of matter, which are characterized by the fundamental constants $8\pi/\kappa$, h, e, k, u and $1/N_A$.

5.2. Proposed possible future macroscopic physical base units

Now, it is proposed that – in the (far) future – the units metre (m), second (s), volt (V), kelvin (K) etc. of the International System of Units SI may be derived from the natural base units $8\pi/\kappa$, h, e, k, u, $1/N_A$ and W_a. For this purpose, these natural units of single-particle physics should be multiplied by appropriate factors ($\{\kappa/8\pi\}$, $\{h\}^{-1}$, $\{e\}^{-1}$, $\{k\}^{-1}$, $\{u\}^{-1}$, $\{N_A\}$ and $\{W_a\}^{-1}$), in order to get corresponding macroscopic or many-particle (SI) base units. Note, that in 1983, when the speed of light in vacuum was defined to be c = 299 792 458 m/s, similarly, de facto the SI unit of velocity was defined by making use of the "natural unit of velocity" c = 299 792 458 m/s, to be m/s = $\{c\}^{-1}$c = (1/299 792 458)c.

Obviously, from h, a base unit of action can be derived: (J s) = $\{h\}^{-1}$h. Similarly, from e, k, u and N_A, the following base units can be obtained, the base unit of electric charge coulomb: C = $\{e\}^{-1}$e (= J/V), a base unit of thermal charge or entropy, J/K = $\{k\}^{-1}$k, the base unit of gravitational charge or gravitational mass m_g: kg = $\{u\}^{-1}$u (= J/[Φ]) and the base unit for the amount of substance mol = $\{N_A\}$(1/N_A) (= J/[μ_i]). Note, that in all cases, the base units (J s), C, J/K, kg and mol are units of invariant physical quantities. That means, the units (J s), C = J/V, J/K, kg = J/[Φ] and mol = J/[μ_i] are products or quotients of two units of reference frame dependent physical quantities. Therefore, a corresponding fraction of 8π/κ should yield a product or fraction of the units of two reference frame dependent physical quantities, too. As it will be shown in section *6.1*, the invariant 8π/κ seems to be composed of several reference frame dependent physical quantities (and one invariant). The reference frame dependent physical quantities can be combined to the reference frame dependent physical quantity force on the one hand, and solid angle Ω on the other. Then, the unit of the physical quantity force, newton (N), and the unit of solid angle, steradian (sr), shall be combined to a macroscopic base unit, being a fraction of 8π/κ, namely (N sr) = J/m = $\{\kappa\}\kappa^{-1}$ = $\{\kappa/8\pi\}$(8π/κ) = $\{G/c^4\}$(c^4/G). – Of course, if in a restricted spatial region the non-euclidean structure of space (see section *6.2*) can be approximated by using the euclidean geometry, the unit steradian (sr) becomes one: In this case, the base unit (N sr) is equal to the unit of force newton (N). The base units (N sr), (J s) and J/K, being fractions or multiples of natural units, which are based on Einstein's constant κ, Planck's constant h and Boltzmann's constant k, in honour of Einstein, Planck and Boltzmann, could be called "einstein" (E = N sr = J/m), "planck" (P = J s = J/Hz) and "boltzmann" (B = J/K).

In addition to the – macroscopic – base units (J s), C, J/K, kg, mol and (N sr), only one additional base unit for a reference frame dependent physical quantity is needed. Because energy is the common physical quantity in all different branches of physics, a certain value for the proper energy W_0, namely 1 Joule, is chosen. Therefore, the unit joule (J) may be chosen as the central macroscopic physical base unit. The unit joule (J) should be defined as a multiple of a certain atomic proper energy $W_0 = W_a$. E.g., W_a may be the energy difference $W_2 - W_1 = W_a$ of two hyperfine structure levels of the caesium-133 atom, or – in the future – of an atom, promising an even lower uncertainty. That means, the unit joule may be defined as J = $\{W_a\}^{-1}W_a$ (e.g. $W_a = W_{Cs}$, i.e. J = $\{h\}^{-1}\{f_{Cs}\}^{-1}W_{Cs}$ = $\{h\}^{-1}h\{f_{Cs}\}^{-1}f_{Cs}$).

For the sake of continuity, the proposed definitions of possible future SI base units, to be defined by an international metrological committee, for (proper) energy J, action (J s), electric charge C, entropy (J/K), gravitational mass kg, amount of substance mol and force times solid angle (N sr) – see section 6 – should be based on the actual CODATA values for h, $W_a = W_{Cs} = hf_{Cs}$ (f_{Cs} = 9 192 631 770 Hz), e, k, u, 1/N_A and 8π/κ = c^4/G. If the definitions of new SI base units would be made now and not in the (far) future, the following exact values for the so-called natural physical units h, W_a, e, k, u, 1/N_A and 8π/κ should be defined, see Mohr and Taylor (2005): h = 6.626 0693×10^{-34} J s, W_a = W_{Cs} = hf_{Cs} = (6.626 0693×9 192 631 770)×10^{-34} J, e = 1.602 176 53×10^{-19} C, k = 1.380 6505×10^{-23} J/K, u = 10^{-3} kg mol^{-1} N_A^{-1} = (1/6.022 1415)×10^{-26} kg, 1/N_A = (1/6.022 1415)×10^{-23} mol and 8π/κ = c^4/G = (299 792 458^4/6.6742)×10^{11} N sr. Therefore, the new SI units would be (J s) = (1/6.626 0693)×10^{34} h, J = (1/6.626 0693)×(1/9 192 631 770)×10^{34} W_a, C = (1/1.602 176 53)×10^{19} e, J/K = (1/1.380 6505)×10^{23} k, kg = 6.022 1415×10^{26} u, mol = (6.022 1415×10^{23})×(1/N_A) and (N sr) = (6.6742/299 792 458^4)×10^{-11} (8π/κ).

Then, the (SI) units of all other physical quantities can be derived from the central base unit J and the surrounding base units (N sr), (J s), C, J/K, kg and mol, see figure 2: the unit of (spatial extension, expansion, elongation or) length m = J/(N sr), the unit of (temporal extension or) time s = (J s)/J, the unit of electric tension or voltage V = J/C, the unit of (thermal tension or) temperature K = J/(J/K), and – so far missing – units of (gravitational tension or) gravitational potential [Φ] = J/kg and (chemical tension or) chemical potential [μ_i] = J/mol.

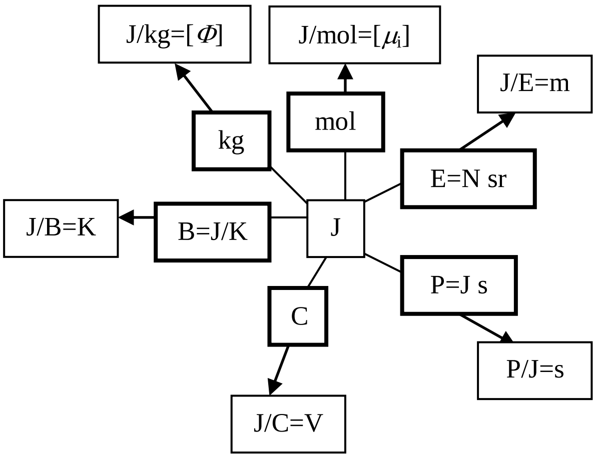

Figure 2. Flow diagram for deriving possible (future SI) units of length (m), time (s), voltage (V), temperature (K), gravitational potential ([Φ]) and chemical potential ([μ_i]) from the central (future) base unit for a certain proper energy, namely joule J = $\{W_a\}^{-1}W_a$, and from the surrounding additional (future) base units "einstein" E = N sr = $\{\kappa/8\pi\}(8\pi/\kappa)$, "planck" P = J s = $\{h\}^{-1}$h, coulomb C = $\{e\}^{-1}$e, "boltzmann" B = J/K = $\{k\}^{-1}$k, kilogram kg = $\{u\}^{-1}$u and mol = $\{N_A\}(1/N_A)$.

The system of the seven base units J, (N sr), (J s), C, J/K, kg and mol would have the advantage that any improvement of this system of units could be achieved by a redefinition of only one unit, namely joule J = $\{W_a\}^{-1}W_a$, because the six other base units would have been internationally defined to the fixed values (N sr) = $\{\kappa/8\pi\}(8\pi/\kappa)$, (J s) = $\{h\}^{-1}$h, C = $\{e\}^{-1}$e, J/K = $\{k\}^{-1}$k, kg = $\{u\}^{-1}$u and mol = $\{N_A\}(1/N_A)$.

6. Requirements of general relativistic physics

6.1. Detailed reformulation of Einstein's constant κ

First, it shall be mentioned that (the SI unit of) length (metre) is defined as the length of the path travelled by light in vacuum during a certain time. Usually, the realization of the unit of length is achieved by using a slightly diverging laser beam of non-zero cross-section. That means, in practical physics, we are confronted with diverging light bundles instead of light lines. Similarly, we should look on force lines, which are diverging into the solid angle Ω. Therefore, in Newton's gravitational law, the unit of solid angle sr should not be suppressed: $F = 4\pi G m_g^2/(4\pi \text{ sr } r^2) = G m_g^2/(\text{sr } r^2)$. Thus, G should be measured in N sr m^2/kg^2, or – according to the considerations made in section *5.2* [m = J/(N sr)] – in J^2 (N sr)$^{-1}$ kg^{-2} [or J^2 E^{-1} kg^{-2}].

Now, the principle of equivalence shall be formulated as equivalence of gravitation and inertia. Then, gravitation is represented by $G m_g^2$ or by $G^{1/2} m_g$, whereas inertia is given by mc^2. Therefore, the quotient $G^{1/2} m_g/mc^2$ or $G m_g^2/(mc^2)^2$ should be an invariant. Of course, $G m_g^2/m^2c^4$ is an invariant, as it should be $G m_g^2/m^2c^4 = \kappa/8\pi$.

If the relation $\kappa/8\pi = G m_g^2/m^2c^4$ is rewritten as $m_g = (\kappa c^4/8\pi G)^{1/2} m$, the proportionality factor $(\kappa c^4/8\pi G)^{1/2}$ between gravitational mass m_g and inertial mass m will depend on Φ if the two terms c^4 and G are depending on Φ to a different degree. That means, whereas Einstein's constant κ does not depend on Φ, the quotient m_g/m, however, may depend on Φ.

Similarly as in electrodynamics, which can be treated with one more degree of freedom by making a difference between the units weber (Wb) and voltsecond (V s), resulting into a proportionality factor (Döring 1962) called electro-magnetic linkage γ_{em} (Rang 1985), here, the proportionality factor between gravitational and inertial mass may be called gravi-inertial linkage $\gamma_{gi} = m_g/m = c^2(\kappa/8\pi G)^{1/2}$. Then, κ can be also written as $\kappa = 8\pi G \gamma_{gi}^2/c^4$. – As the division of electro-magnetic energy in electric and magnetic energy and of gravi-inertial energy in gravitational and inertial energy depends on the (observer's) frame of reference, the electro-magnetic linkage γ_{em} and the gravi-inertial linkage γ_{gi} are reference frame dependent physical quantities.

6.2. Physical quantities to be used in the real non-euclidean world

According to the considerations made in section *6.1*, in the expression for $\kappa/8\pi$ (= Gm_g^2/m^2c^4), the – scalar – gravitational mass m_g had not been cancelled against the – possibly tensorial – inertial mass m. By this means, physics is getting one more degree of freedom, as the rigid link between gravitational and inertial mass is removed. Thus, not only Newton's (1687) ideas concerning space and time, but also the second Newtonian relic, namely the rigid proportionality between gravitational and inertial mass, can be revised. Then, the overdetermination of reality due to the laws of euclidean geometry, will be avoided: If the inertial mass will be allowed to have a dimension, being different from the dimension of gravitational mass, plane and solid angles will be allowed to have their own physical dimensions, too. Thus, the so far rigid coupling of curved and straight lengths (fixed quotient of a circle's circumference u and its diameter d, $u/d = \pi$) or of rotational and linear physical quantities (fixed quotient of angular velocity ω and frequency f, $\omega/f = 2\pi$), being true only in the special case of an empty, euclidean space, will be abolished. – Note, that Gauss considered plane angle (λ) as a physical quantity, the euclidean or non-euclidean nature of which should result from measuring the three angles of a large terrestrial triangle (Brocken, Hoher Hagen, Inselsberg, see Born 1969 p 283).

Therefore, let us have a look on rotational physical quantities. As the magnetic flux density B is measured in units of $\mathrm{V\,s\,m^{-2} = J^{-2}\,(N\,sr)^2\,(J\,s)\,C^{-1}}$ [or $\mathrm{J^{-2}\,E^2\,P\,C^{-1}}$], the corresponding rotational quantity of so-called angular velocity ω should be measured in the corresponding units of $\mathrm{J^{-2}\,(N\,sr)^2\,(J\,s)\,kg^{-1}}$ [or $\mathrm{J^{-2}\,E^2\,P\,kg^{-1}}$]. Then, the plane angle λ may be obtained from the physical quantities ω and time t: $\lambda = \int\omega \mathrm{d}t$. That means, in a real, not assumed to be a euclidean space, the plane angle λ may result from a rotational movement with the so-called angular velocity ω during the time t. Consequently, the plane angle λ should be generally considered as a reference frame dependent physical quantity to be measured in units of $\mathrm{J^{-3}\,(N\,sr)^2\,(J\,s)^2\,kg^{-1}}$ [or $\mathrm{J^{-3}\,E^2\,P^2\,kg^{-1}}$]. Because the unit of velocity can be written as $\mathrm{m/s = J^2\,(N\,sr)^{-1}\,(J\,s)^{-1}}$ [or $\mathrm{J^2\,E^{-1}\,P^{-1}}$] and, thus, inertial mass can be written as $\mathrm{J\,m^{-2}\,s^2 = J^{-3}\,(N\,sr)^2\,(J\,s)^2}$ [or $\mathrm{J^{-3}\,E^2\,P^2}$], the unit of the plane angle λ can be written as $[\lambda] = [m]/[m_g] = \gamma_{gi}^{-1}$, and its unit rad should be considered as an abbreviation of $\mathrm{J^{-3}\,(N\,sr)^2\,(J\,s)^2\,kg^{-1}}$ [or $\mathrm{J^{-3}\,E^2\,P^2\,kg^{-1}}$] rather than of m/m, which is true only in the euclidean approximation of reality. Note, that in the real space of our solar system one revolution, e.g. of Mercury around the Sun, takes more than 360 °, known as the so-called perihelion advance, see part III.

Whereas static physical quantities, like potential energy ($m_g\Phi$), and linear physical quantities, like momentum (mv) and inertial energy (mc^2) can be written in the usual way as before, rotational physical quantities, however, should be multiplied with the factor $\gamma_{gi} = c^2(\kappa/8\pi G)^{1/2}$. Thus, e.g., the moment of inertia becomes $J = \gamma_{gi}m_g r^2$, to be measured in units of $\mathrm{J^5\,(N\,sr)^{-4}\,(J\,s)^{-2}\,kg^2}$ [or $\mathrm{J^5\,E^{-4}\,P^{-2}\,kg^2}$]. Then, the angular momentum $L = J\omega$, as well as the spin (n $\hbar/2$ = (n/2)$h/(2\pi$ rad)), should be measured in units of $\mathrm{J^3\,(N\,sr)^{-2}\,(J\,s)^{-1}\,kg}$ [or $\mathrm{J^3\,E^{-2}\,P^{-1}\,kg}$]. That means, angular momentum and spin turn out to be reference frame dependent physical quantities, whereas the product $L\lambda$ is an invariant (Spieweck 1992), to be measured in (J s) [or P].

The situation concerning gravi-inertial physics in the real, non-euclidean space seems to be quite similar to the situation in electro-magnetic physics, in which the first two Maxwellian equations should be written in a manner, that the rotational terms rot$\boldsymbol{H}$ and rot$\boldsymbol{E}$ should be multiplied by $\gamma_{em} = \gamma$ (Döring 1962, Rang 1985). That means, in case of circularly bent field lines, the situation in the real space, obviously, should not be described by using the laws of euclidean geometry, in which $\gamma_{em} = \gamma$ can be set equal to one. That means, in the real, non-euclidean space the terms rot$\boldsymbol{H}$ and rot$\boldsymbol{E}$ should be replaced by γrot$\boldsymbol{H}$ and γrot$\boldsymbol{E}$.

7. Discussion

It was shown that reality may be centrally governed by one reference frame dependent physical quantity, namely the proper energy W_0 (of a certain atom) W_a (= W_0), and by the six invariants $8\pi/\kappa$, h, e, k, u and $1/N_A$. This metrological consistent view on all branches of physics may replace the geometric view on (relativistic) physics, which is characterized by the one infinitesimal line element $\mathrm{d}s = [g_{\alpha\beta}(x^\mu)\mathrm{d}x^\alpha \mathrm{d}x^\beta]^{1/2}$, where d$s$ is implicitly containing several reference frame dependent physical quantities as v^2/c^2, $\omega^2 r^2/c^2$ or Φ/c^2.

The six constants $8\pi/\kappa$, h, e, k, u and $1/N_A$ were interpreted as common invariants of discontinuous – single-particle – quantum physics and continuum – many-particle – relativistic field physics. As the six constants can be considered as invariants of single-particle physics or quantum constants, all derived physical quantities, resulting from them, should be considered as quantum quantities, too. In addition, there are also other reasons, why the quantum aspect of physics should be considered in all parts of physics: Physical units – needed for building up physical quantities – are per se portions or quanta, and, moreover, all kinds of measurements can only be performed with a finite uncertainty. That means, in continuum or field physics, too, physical quantities can be only known down to a finite fraction. Therefore, the role of the measuring observer should not only be basically considered in quantum mechanics but in all parts of physics, including special and general relativistic physics.

Obviously, Einstein was right, when he said: "God does not play dice." That means, the invariants $8\pi/\kappa$, h, e, k, u and $1/N_A$ are characterizing physical properties of an objective reality (of single-particle physics). Note, that in this picture reality is not described by a continuous field but by discrete, algebraic elements, as suggested by Einstein in December 1954 (Einstein 1965 p 110). – The invariants $8\pi/\kappa$, h, e, k, u and $1/N_A$, respectively the numerical values $\{\kappa/8\pi\}$, $\{h\}^{-1}$, $\{e\}^{-1}$, $\{k\}^{-1}$, $\{u\}^{-1}$ and $\{N_A\}$), and, thus, the proposed (future) base units (N sr), (J s), C, J/K, kg and mol, may be fixed by an international metrological committee to exact values – with zero uncertainty, provided that G – and thus κ – is known to the required accuracy.

By the way, in quantum mechanics, even hundred years after the creation of special relativity, curiously enough, time is still treated as an absolute physical quantity, see Kiefer (2005). Thus, it can be stated: In quantum mechanics the relativity of time is not taken into account, whereas in the theories of relativity the quantum aspect of nature is not taken into account. Moreover, in the whole physics except in general relativity, the relativity of angles is so far not taken into account, because our present SI units are based on the laws of euclidean geometry. – The figures 1 and 2 have shown a symmetric – metrological – view on the general relativistic units $8\pi/\kappa$ and (N sr), on the quantum units h and (J s) and on the older (classic) units e and C, k and J/K, u and kg as well as on $1/N_A$ and mol, whereas so far usually asymmetric – mathematical – relations are given, concerning the transitions from relativistic physics and from quantum physics to classic physics, namely $c \to \infty$ and $h \to 0$.

8. Conclusion

Proposed was a new concept for the description of our partly non-euclidean reality by the six invariants $8\pi/\kappa$, h, e, k, u and $1/N_A$ and one reference frame dependent physical quantity, namely a certain proper energy W_0, which may be chosen as an atomic energy W_a (= W_0). The six invariants may be characterizing objective parts of reality, whereas the seven natural units $8\pi/\kappa$, h, e, k, u, $1/N_A$ and W_a can be used as microscopic physical base units for a consistent physical description of the whole reality, an observer is confronted with. Macroscopic (SI) base units can be obtained by multiplying the microscopic natural base units with appropriate numerical factors: Thus, the possible future base units could be "einstein" E = (N sr) = J/m = $\{\kappa/8\pi\}(8\pi/\kappa)$, "planck" P = (J s) = J/Hz = $\{h\}^{-1}$h, coulomb C = J/V = $\{e\}^{-1}$e, "boltzmann" B = J/K = $\{k\}^{-1}$k, kg = $\{u\}^{-1}$u, mol = $\{N_A\}(1/N_A)$ and J = $\{W_a\}^{-1}W_a$. Then, all further (SI) units like m, s, m/s, V, A, Ω, K etc. can be derived from these seven base units.

An uncoupling of the – mathematical – laws of euclidean geometry from the laws of physics, respectively a giving up of setting the units of plane and solid angle, radian (rad) and steradian (sr), equal to one, made it possible to propose a system of units, which can be commonly used in classic, general relativistic and quantum physics. In this part I, first of all, the basic metrological concept for a symmetrical view on all branches of physics (including general relativistic, quantum and classic physics) was given, whereas resulting consequences will be discussed in parts II and III.

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II. The second nature of empty space: Suggestions on the grainy structure inside and outside of matter

Abstract. In part I, it was shown that physics in the whole can be metrologically mainly based on common invariants of continuous and discontinuous physics. Now, a revised view on Einstein's field constant κ, which is partly interpreted as an invariant of single-particle physics, is leading to several new ideas. First, a so-called natural unit is derived from κ. Second, many-particle or macroscopic physics is considered: Ideas are given concerning the grainy structure inside and outside of matter, as well as an uncertainty relation of general relativistic physics. The latter two suggestions are confirming the removal of the differences between the quantum and relativistic view on physics.
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Zusammenfassung. Im Teil I wurde gezeigt, dass sich die Physik insgesamt metrologisch gesehen weitestgehend auf gemeinsame Invarianten der Kontinuums- und der Diskontinuums-Physik gründen lässt. Hier eröffnet nun eine neue Sicht auf die Einsteinsche Feldkonstante κ, die auch als Invariante der Ein-Teilchen-Physik interpretiert wird, verschiedene neue Aspekte. Zunächst wird eine sogenannte natürliche Einheit aus der Konstante κ abgeleitet. Zweitens wird sich der Mehr-Teilchen- oder makroskopischen Physik zugewendet: Diskutiert wird insbesondere die körnige Struktur innerhalb und außerhalb von Materie sowie eine mögliche Unschärfebeziehung der allgemein-relativistischen Physik. Dies lässt hoffen, dass sich die Diskrepanzen zwischen den beiden unterschiedlichen Betrachtungsweisen der Quanten- und Relativitäts-Physik beseitigen lassen.

1. Introduction

So far open questions in modern physics are concerning the unification of the relativistic and the quantum view on physics (see part I) and, e.g., a grainy structure of so-called space-time (Rauner 2004). Based on common invariants of discontinuous (single-particle) and continuous (many-particle) physics, presented in part I, now, suggestions will be made concerning the physical description of the empty space by making use of those invariants.

That means, obviously physics in the whole can be treated according to two different aspects. First, Einstein revealed in 1905 that the physical properties of light cannot only be described on the basis of the wave picture but also by using the quantum or particle picture. Then, de Broglie (1925 p 33) has shown that matter cannot be only considered in the particle picture, but that matter may be simultaneously interpreted in the wave picture. Now, it is suggested that the empty space, too, cannot only be considered as a continuum but may be also described by using discontinuous physical quantities.

2. A natural unit derived from Einstein's constant κ

First, it should be mentioned that the value of Einstein's constant κ, namely $\kappa = 8\pi G/c^4$ (c: speed of light in vacuum, G: Newton's gravitational constant), see e.g. Dehnen *et al* (1960), Lenk and Gellert (1974), Goenner (1996), Schmutzer (1996) and Greulich (1999), is only seldom correctly presented. Because Einstein and several other authors were setting c, and thus also c^2, equal to one, concerning κ, obviously a factor c^2 is usually omitted or forgotten. That means, in almost all relevant books and papers, κ is incorrectly given as $\kappa = 8\pi G/c^2$. This may be the reason, why so far nobody detected a relationship between κ^{-1} and force, respectively the product of force times solid angle.

In the preceding part I, the so-called natural units W_a, h, e, k, u, $1/N_A$ and $8\pi/\kappa$ had been proposed, where $8\pi/\kappa$ may be interpreted as the natural unit for the product of force times solid angle $8\pi/\kappa = c^4/G$. By the way, the natural unit, which could have been already given by Planck (1899), would (also) be c^4/G – with the enormously high value of 1.21×10^{44} N sr (see part I). As $8\pi/\kappa$ was partly considered as an invariant of single-particle physics, perhaps, the corresponding bundle of force lines is holding the quarks of one nuclear particle together. Because κ is containing c and G, κ may be also considered as a combination, linkage – or unification – of electro-magnetic phenomena, being characterized by the propagation velocity of electro-magnetic waves in vacuum c, and gravitational phenomena, being characterized by G. Note, that the product of force times solid angle $8\pi/\kappa = c^4/G$ is proportional to c^4, whereas nuclear force mc^2/s is only proportional to c^2, and atomic force $hf/\lambda_W = hc/\lambda_W^2$ is merely proportional to c (λ_W: wavelength).

The reason, why Einstein obviously did not notice the unifying role of κ, linking gravitational physics and electrodynamics, may be his opinion that c is a physical constant, which primarily expresses the properties of space and time – and not primarily of electrodynamics. In contrast, in part I, it was shown that length and time may be considered as derived – or secondary – physical quantities, whereas the characteristic constant of electrodynamics, the elementary charge e, and thus electrodynamics itself, should be considered as primary physics, similarly as gravi-inertial physics, too. – By the way, force is a somehow neglected physical quantity in modern physics. However, it should be emphasized that due to the principle of causality force is always the cause for any conversion from one kind of – tensional, oscillational, electro-magnetic, thermal, gravi-inertial or chemical – energy into another.

3 The grainy or quantum structure of macroscopic physics inside and outside of matter

The invariants $8\pi/\kappa$, h, e, k, u and $1/N_A$ are concerning single-particle physics. Planck's constant h is, e.g., characterizing the invariant physical properties of one photon (Spieweck 1992). In contrast, special quotients of these invariants are concerning many-particle or macroscopic physics. Note, that the Josephson effect and the von Klitzing effect, in which quotients of h and e, respectively h and e^2, are concerned, are called macroscopic quantum effects.

Many-particle physics, macrophysics or macroscopic physics is based on multiples of $8\pi/\kappa$, h, e, k, u and $1/N_A$, namely $n_1(8\pi/\kappa)$, n_2h, n_3e, n_4k, n_5u and n_6/N_A, where n_i may be an arbitrarily large number, e.g., $n_i = 10^{23}$. Consequently, the quotient of any two of the invariants of many-particle physics, $n_1(8\pi/\kappa)$, n_2h, n_3e, n_4k, n_5u and n_6/N_A, must be equal to the quotient of the corresponding invariants of single-particle physics.

3.1. Quantum structure inside of matter

Quotients of h and e (respectively h and e^2) are concerning solid matter. In addition, it should be mentioned that it may be proposed to define the unit kilogram with the aid of a single crystal ($1\ \text{kg} = 10^3\ \text{u}/\{1/N_A\} = 10^3\ \{N_A\}\ \text{u}$). Moreover, the quotient of h and $1/N_A$ is the molar Planck constant N_Ah. In case of liquid matter, the quotient of e and $1/N_A$ is resulting into the Faraday constant $F = eN_A$, and in case of gaseous matter, the quotient of k and $1/N_A$ is concerned, resulting into the molar gas constant $R = kN_A$. Therefore, not only the Josephson constant $K_J = 2\,e/h$ and the von Klitzing constant $R_K = h/e^2$, but also N_Ah, F and R may be considered as macroscopic quantum constants, see figure 1.

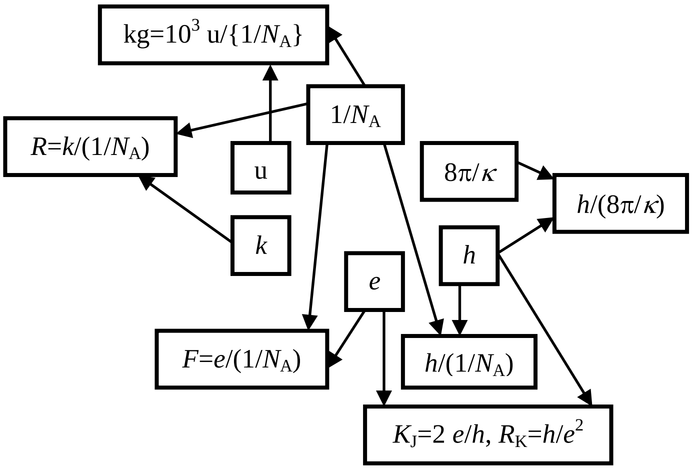

Figure 1. Grainy – or quantum – structure of single-particle physics, concerning the invariants or quantum constants $8\pi/\kappa$, h, e, k, u and $1/N_A$, and of many-particle (or macroscopic) physics inside and outside of matter, concerning quotients of single-particle physics, namely gaseous physics (molar gas constant $R = k/(1/N_A)$), liquid physics (Faraday constant $F = e/(1/N_A)$), solid-state physics (molar Planck constant $N_A h$, Josephson constant $K_J = 2\ e/h$, von Klitzing constant $R_K = h/e^2$, single crystal of 1 kg: kg = 10^3 u/{$1/N_A$}) and vacuum physics: $h/(8\pi/\kappa)$. Note, that all elements are concerning physics of an objective reality.

3.2. Grainy or quantum structure outside of matter

Similarly, outside of matter, a further quotient of single-particle invariants may play a role, namely the quotient of h and $8\pi/\kappa$, i.e. $h/(8\pi/\kappa)$, see figure 1. It may be an interesting fact that the values of all macroscopic quantum constants ($R = kN_A$, $F = eN_A$, $K_J = 2\ e/h$, $R_K = h/e^2$, $N_A h$, kg = 10^3 {N_A} u and $(\kappa/8\pi)h$ are independent of the actual value chosen for the man-made units W_a or joule (J). That means, the grainy or quantum structure inside and outside of mater seems to be an objective feature of reality, being independent of the position of an observer in a certain frame of reference.

Perhaps, the quotient $h/(8\pi/\kappa)$ may be interpreted according to the following consideration. Due to an idea of Einstein (1959), the concept of space is derived from the preceding concept of a solid body: The interspace between two bodies can be filled with a third body, or this third body may be taken away. Thus, space should be considered as being real in the same sense as solid bodies are real (Einstein 1959). Therefore, the non-euclidean structure of space – and the relativity of time – should be known from the behaviour of a test particle (moving) in this space, which may be a massive body (as, e.g., Mercury) or a photon near the sun's surface.

Solid bodies are composed of atoms, which are known to be structured. Perhaps, in an atom, length and time are only existing as unrolled physical quantities – or physical dimensions. Thus, an atom is a totally symmetrical particle. After an interaction with the physical surroundings, however, a symmetry breaking will take place: Molecules have at least one less degree of symmetry. And atoms, e.g., within a laser, may be either used as length standards or as time standards. Note, that symmetry breaking is also occurring in case of the weak interaction, when a neutrino is emitted.

Because space should be considered as being real in the same sense as solid bodies, length and time should not only exist as unrolled physical quantities within an atom or within a solid body, but also outside of a solid body, i.e. in the space surrounding this body. As the product of the microscopic units of length and time is equal to the product of $(8\pi/\kappa)^{-1}$ and h,

$$\mathrm{s_u \times t_u} = (8\pi/\kappa)^{-1} \times \mathrm{h} = 5.47\times10^{-78}\ \mathrm{m\ s}, \tag{1}$$

perhaps, the surroundings of the body has a grainy – or quantized – structure, the microscopic element of which is as small as 5.47×10^{-78} m s. That means, besides the Einsteinian linkage of length and time, which was first formulated by Minkowski in 1908 as a four-dimensional generalization of the theorem of Pythagoras – $\mathrm{d}s = [x_1^2 + x_2^2 + x_3^2 + (\mathrm{i}ct)^2]^{1/2}$ –, equation (1) may be considered as a quantum law, describing the grainy structure of so-called space-time.

3.3. An uncertainty relation of general relativistic physics

In the area of quantum gravity, an uncertainty relation between space and time is mentioned (Kiefer 2000). Such an uncertainty relation may result from the product of the man-made or natural units of length (s_u or Δs) and of time (t_u or Δt):

$$\Delta s \times \Delta t \geq (8\pi/\kappa)^{-1} \times h = 5.47 \times 10^{-78} \text{ m s.} \qquad (2)$$

This result, may be interpreted in the following way. As any measurement means a disturbance – and thus an alteration – of the measured physical object, the observer does not know exactly what was the object's state before doing the measurement and what it will be afterwards. In contrast, in case of a measurement in which energy and time – or momentum and length – are involved, the minimum uncertainty is in the order of the Planck constant h. Perhaps, if only values for length and time are measured, the disturbance may be smaller. Correspondingly, also the degree of information should be smaller, and the minimum uncertainty will be derived from the value of the product $(8\pi/\kappa)^{-1} \times h$.

4. Discussion

4.1. General remarks

One of the reasons, why the considerations made in parts I and here had not been made previously, may be the following. Einstein commented his field equations (Einstein 1916) in the following manner: The left side, governed by Riemannian mathematics, is "built on granite", whereas the right side (containing κ), describing the physical situation concerning the distribution of masses and energies in the universe, is "built on sand" (Rauner 2004). In contrast, the author is believing that the Riemannian mathematics, which is used for the description of a continuous field structure, may be more easily compared with a continuous matter like sand, whereas κ (for the first time) is identified as the keystone in the edifice of physics, being one of the granite stones of physics, respectively one of the common invariants of discontinuous (single-particle) and continuous (many-particle) physics ($8\pi/\kappa$, h, e, k, u and $1/N_A$). Moreover, the physical quantity proper energy, which can be only known with respect to an observer's frame of reference, is now believed to play a central role in the physics of the observer, who is doing experiments with a non-zero uncertainty.

The Einstein constant κ may also play another important role in modern physics: Perhaps, the nowadays discussed dark energy in the universe (Wetterich 2004, Rauner 2004) may be identified with a so far widely neglected kind of energy, namely tension or torsion energy, being accompanied by the so-called curvature and tapering of space. That means, the (strain or) tension and torsion energy in the whole universe may be calculated by using Einstein's constant κ, respectively the invariant $8\pi/\kappa$. Obviously, this energy has the same properties as the so-called dark energy: It is transparent as well as unstructured (Wetterich 2004).

Newton as well as Einstein were looking for "fixed points" outside of physics in a pre-physical or mathematical world. Before Newton (1687) formulated his three laws, he claimed time as an absolute, true or mathematical quantity ("Tempus absolutum verum & Mathematicum"). – Similarly, Einstein believed in the reality of the four-dimensional tensor $g_{\alpha\beta}(x^\mu)$ (Schmutzer 1996). Note, that the use of time as a fourth spatial dimension Einstein called a mathematical trick (Einstein 1965, footnote on p 67). – In contrast, here, respectively in part I, "fixed points" – of reality – had been detected within the picture of physics, as the physical quantities $8\pi/\kappa$, h, e, k, u and $1/N_A$ had been used as invariants. By the way, as all these invariants – in contrast to the invariant infinitesimal line element $ds = [g_{\alpha\beta}(x^\mu)dx^\alpha dx^\beta]^{1/2}$ – have finite values, singularities should be avoidable, especially as, in addition, bundles of field lines with non-zero cross-section should be considered.

Perhaps, the picture of (the whole) physics may be identified with a jigsaw puzzle. In classic physics, its elements had been mixed with elements of a mathematical jigsaw puzzle, called euclidean geometry. In special relativity, its elements had been mixed with (Minkowski's) elements of another mathematical jigsaw puzzle, called pseudo-euclidean geometry. In general relativity, elements of a further mathematical jigsaw puzzle, called non-euclidean geometry, were added to the physical elements, whilst simultaneously two physical elements, namely gravitational and inertial mass, were eliminated. In contrast, here, it was tried to form a complete picture of the whole physics without using elements of the mathematical jigsaw puzzles mentioned. Thus, in figure 1, the physical element $h/(8\pi/\kappa)$ could be directly connected to the general relativistic element $8\pi/\kappa$ and the quantum mechanical element h, because there do no more exist disturbing and separating intermediate mathematical elements.

In addition, the following should be stated: Whereas in the 19th century it was believed that the whole physics can be mechanically explained, in the 20th century the opinion was continually growing that physics must be mathematically founded: First, Einstein explained the properties of space and time by creating a new, geometrical interpretation of the Lorentz transformation. Then, in general relativity, he presented his Riemannian field equations. Ten years later, the interest focussed on Schrödinger's equation. In the second half of the 20th century, Heisenberg presented a so-called world formula, and the string theories are operating with eleven (or up to 46) mathematical dimensions.

In contrast, here (in parts I to III), mathematics is only considered as an assistant or auxiliary science, and physics should be explainable on a common metrological basis. – Finally, it may be stated that Einstein was obviously right, when he was convinced that quantum mechanics is an incomplete theory: If quantum mechanics is primarily founded on Planck's quantum constant h, then, according to figure 1, quantum physics is only concerning one sixth of the whole physics. That means, quantum mechanics seems to be a rather one-sided view on physics.

In part III, the extended region of the physical jigsaw puzzle, called relativistic physics, shall be built up without using pure mathematical elements. Then, in the real, non-euclidean world the relativistic behaviour of area and volume will be related to the relativistic behaviour of length, plane angle and solid angle, and not of length alone. That means, in a real, non-euclidean world, the units of area and volume should be m^2 rad and m^2 rad m sr = m^3 rad sr, respectively.

4.2. Physical properties of empty space

In sections *3.1* and *3.2*, the product $(8\pi/\kappa)^{-1}\times h$ was given as 5.47×10^{-78} m s, that means, as a result concerning so-called space-time. Moreover, it should be noted that m and s are units of reference frame dependent physical quantities. However, in part I, m and s were obtained as secondary units, being derived from the primary or base units of invariant physical quantities, namely J and (J sr) = E [m = J/E], respectively (J s) = P and J [s = P/J]. Therefore, the product $(8\pi/\kappa)^{-1}\times h$ can be also written as

$$(8\pi/\kappa)^{-1}\times h = 5.47\times10^{-78}\ \mathrm{E}^{-1}\ \mathrm{P}. \qquad (3)$$

In this case, the physical properties of empty space seem to be characterized by $8\pi/\kappa$ [or E] on the one hand, and h [or P] on the other. Obviously, the invariant $8\pi/\kappa$ is representing the possibility for the "conduction" of (bundles of) force lines, which may be originating either from gravitational or electric charges. As already mentioned in section 3, the invariant h may be characterizing the main physical property of a photon (Spieweck 1992). Therefore, the second physical property of empty space seems to be possibility for the "conduction" of photons (and other particles).

5. Conclusion

A revised view on the Einstein constant κ, which is interpreted as a common invariant of discontinuous (single-particle) and continuous (many-particle) physics, was leading to new ideas. First, a so-called natural unit for the product of force times solid angle was considered, and second, suggestions were made concerning the grainy structure of the world inside and outside of matter, as well as an uncertainty relation of general relativistic physics. The latter two suggestions, again, are confirming the removal of the discrepancies between the quantum and relativistic view on physics.

In addition, the following may be stated: Whereas in the 19th century it was believed that the whole physics can be mechanically explained, in the 20th century the opinion was continually growing (finally in the string theories) that physics should be (geometrically, i.e.) mathematically founded or explained. In contrast, here, as well as in parts I and III, mathematics is considered as an assistant or auxiliary science. That means, physics should be explainable by making use of a consistent set of (six invariant and one reference frame dependent) physical quantities.

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III. A "second leg" of relativistic physics: Simple reformulation of relativistic physics by making use of invariant constants

Abstract. In part I, it was shown that physics in the whole can be metrologically mainly based on invariant fundamental constants. In part II, it was suggested that not only light and matter but also the empty space can be treated by using those invariants. Here, instead of the infinitesimal invariant mathematical line element $\mathrm{d}s = [g_{\alpha\beta}(x^{\mu})\mathrm{d}x^{\alpha}\mathrm{d}x^{\beta}]^{1/2}$, Einstein's constant κ, Planck's constant h, the elementary charge e and Boltzmann's constant k shall be used as invariants. By this means, formulae for all so far experimentally verified special and general relativistic effects are simply derived from the postulate $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$, where W_0^* is the proper energy as judged by an external observer and ΔW is the difference of a particle's energy, assigned in the frames K and K^*. Suggestion are made concerning new general relativistic conservation laws.
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Zusammenfassung. Im Teil I wurde gezeigt, dass sich die Physik insgesamt metrologisch gesehen weitestgehend auf invariante Fundamentalkonstanten gründen lässt. Im Teil II wurde vermutet, dass nicht nur Licht und Materie, sondern auch der leere Raum mit Hilfe dieser Invarianten behandelt werden kann. Hier werden nun anstelle des infinitesimalen invarianten mathematischen Linienelements $\mathrm{d}s = [g_{\alpha\beta}(x^{\mu})\mathrm{d}x^{\alpha}\mathrm{d}x^{\beta}]^{1/2}$ die Einsteinkonstante κ, die Planckkonstante h, die Elementarladung e und die Boltzmannkonstante k als Invarianten benutzt. So werden Formeln für alle bisher experimentell bestätigten speziell- und allgemein-relativistischen Effekte einfach aus dem Postulat $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ hergeleitet, wobei W_0^* die Eigenenergie bedeutet, die ein externer Beobachter einem im Bezugsystem K befindlichen Teilchen zuordnet, und ΔW die Differenz der Energien ist, die dem Teilchen in den Bezugssystemen K und K^* zugeordnet werden. Vermutungen werden angestellt über neue allgemein-relativistische Erhaltungssätze.

1. Introduction

In part I, it was shown that physics in the whole can be metrologically mainly based on common invariant constants of continuous and discontinuous physics. In part II, it was suggested that not only light and matter but also the empty space can be treated by using those invariants. That means, obviously physics in the whole can be treated according to the continuous (or wave) aspect and the (discontinuous or) quantum aspect. Because quantum mechanics can be treated due to the quantum aspect of Heisenberg (1925), as well as to the wave aspect of Schrödinger (1926), now also relativistic physics shall be described on a second basis of discontinuous invariant constants instead of the infinitesimal line element $\mathrm{d}s = [g_{\alpha\beta}(x^{\mu})\mathrm{d}x^{\alpha}\mathrm{d}x^{\beta}]^{1/2}$ (Schmutzer 1996) of continuous field physics. In this case, physics occurring in a distant proper frame K shall be described in an external observer's frame by using so-called "effective" physical quantities, see section 3 of part I.

2. Elementary building up special and general relativistic formulae

2.1. Laboratory relativistic physics

The principle of general relativity may be formulated as follows: Physics in all inertial and local non-inertial proper frames of reference can be described in the same way. Due to this principle, which was verified, e.g., by Mössbauer experiments, in any physical laboratory local reference frames can be used as laboratory systems, irrespective of whether they are located in an inertial or in a non-inertial frame of reference. Then, a relativistic problem to be solved, consists in obtaining a relationship between the physical quantities Q_i, describing the considered physics in an inertial or local non-inertial proper frame of reference K, and the corresponding "effective" physical quantities Q_i^* which are used for the description of this physics in an external frame of reference K^*, that means in an external observer's laboratory. – According to the principle of general relativity, in any inertial and local non-inertial frame of reference the laws of physics can be formulated in the same way. Therefore, the interrelations of "effective" physical quantities Q_i^* will have the same shape as the interrelations of the corresponding physical quantities Q_i, see Schmutzer (1996).

2.2. Relativity of a particle's energy

Energy, already in classic mechanics, was a reference frame dependent physical quantity. Therefore it should be suited also for preliminary relativistic considerations. – A certain value of a physical quantity as length, time, inertial mass or energy can be given only with respect – or in relation – to an inertial or to a local non-inertial frame of reference. That means, especially any kind of energy must be considered as a reference frame dependent physical quantity.

2.2.1. Relativity of energy: A preliminary approach

The proper energy (Einstein 1905c) $W_0 = mc^2$ (m: inertial proper mass) shall be considered as a pre-relativistic term, because Einstein has shown that the equation $W_0 = mc^2$ can be obtained already without using the formalism of the special theory of relativity (Einstein 1905b), simply by evaluating the radiation pressure within a moving resonator (Born 1969).

On Earth, if we are going upstairs from the first to the second floor, with respect to the first floor – or to the frame of reference K, where our proper energy has been $W_0 = mc^2$, – our total energy amounts to $mc^2 + m_g g\Delta H$, where m_g is the gravitational mass, g is the gravitational acceleration and ΔH is the difference in height between the first and second floor. The total energy can be also written as $W_0 + m_g g\Delta H$.

With respect to the second floor – or to the frame of reference K^* – , however, due to the principle of general relativity, our total energy again only consists of our proper energy $W_0 = mc^2$. Therefore, with respect to the second floor or to the frame of reference K^*, our energy in the first floor – being measured in physical units realized in the frame of reference K^* and therefore called "effective" proper energy W_0^* – should be $W_0^* = W_0 - m_g g\Delta H$. For similar reasons, the energy of a particle positioned in an inertial frame K, from another inertial frame of reference K^* – moving with the velocity $\boldsymbol{v}$ against the frame of reference K – will be judged as the – lower – "effective" energy $W_0^* = W_0 - mv^2/2$, because the energy $\Delta W = mv^2/2$ must be afforded if the particle is transferred from a resting position in an observer's frame of reference K into an – again – resting position in another observer's frame of reference K^*. That means, ΔW is denoting a particle's energy difference with respect to two different observers in the frames of reference K and K^*.

It should be emphasized that in special relativistic problems it is always $\Delta W > 0$, because ΔW is used for achieving either acceleration or braking, e.g. of a space ship, whereas in general relativistic physics it is $\Delta W > 0$, if it is, e.g., $\Delta H > 0$, and $\Delta W < 0$, if it is $\Delta H < 0$, that means, if we are looking upward (from an observer's frame of reference K^* to a proper frame K, where the considered physics takes place).

2.2.2. Relativistic energy formulae

In Newtonian physics energies can be additively summed up to arbitrarily high values. In relativistic physics, however, it must be taken into account that the velocity v between two inertial frames of reference cannot exceed c. Therefore, the – so far given – classic formula $W_0^* = W_0 - mv^2/2$ must be replaced by a relativistic formula, yielding real values only if the velocity v is lower than c. Similarly as in classic mechanics, here, too, an energy formula – for the "effective" proper energy – may be guessed (Schmutzer 1996):

$$W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}. \tag{1}$$

This basic energy formula, given by the author already in 1988 (Spieweck 1988, 2000a, 2000b) is the only relativistic *postulate* needed for building up further relativistic formulae. In contrast to a transformation of coordinates of two different coordinate systems, the relation $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ may be understood as a transition or change from a proper frame of reference K to an external observer's frame of reference K^*. It shall be mentioned, that ΔW may be composed of different kinds of energy, e.g., gravitational energy and kinetic energy of translation or rotation, as in case of the twin paradox (Spieweck 1988, 1992, 2000b).

As already mentioned in part I, the proper energy $W_0 = mc^2$ may be composed of tensional (and torsional), oscillational, electro-magnetic, thermal, gravi-inertial and chemical energy: $W_0 = mc^2 = (8\pi/\kappa)s + hf + eU + kT + \mathrm{u}\Phi + (1/N_\mathrm{A})\mu_\mathrm{i}$. – In contrast to the proper energy W_0, which in most cases can be treated in the proper frame as a scalar, the "effective" proper energy $W_0{}^*$ may be a tensor. Therefore, in special relativistic physics, two kinds of tensor components for the "effective" proper energies should be considered, namely $W_{0\|\boldsymbol{e}}{}^*$ and $W_{0\perp\boldsymbol{e}}{}^*$, where $\boldsymbol{e}$ is the unit vector in the direction of the velocity $\boldsymbol{v}$ between the frames of reference K and K^*: $W_{0\|\boldsymbol{e}}{}^* = W_0(1 - v^2/c^2)^{1/2}$, and $W_{0\perp\boldsymbol{e}}{}^* = W_0$, see section *2.3.1*. In general relativistic physics, however, $W_0{}^*$ mostly can be treated as a scalar.

Whereas the usual special relativistic formulae, which are describing a transition from one proper frame of reference to another proper frame, are symmetric, general relativistic formulae, e.g., for frequency shifts due to a gravitational field, are asymmetric: If we are looking upward to a spectral lamp in a higher laboratory, we are observing blue shifted spectral lines, whereas, if we are looking downward to a spectral lamp in a lower laboratory, we are observing red shifted lines. Therefore, the corresponding transitions are non-reciprocal. However, gravitational transitions, based on the formula $W_0{}^* = W_0(1 - 2\Delta W/W_0)^{1/2}$, are transitive (Spieweck 2000b).

According to the principle of correspondence, for values of $\Delta W/W_0$ small compared to 1 the relativistic formula $W_0{}^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ merges into the classic formula $W_0{}^* = W_0 - \Delta W$. The relativistic formula for W_0 is not explicitly given in the theory of relativity. In the special theory of relativity, the physical situation is described in the frame of reference K, where a particle resting in the frame of reference K^* is considered to be a moving particle, which consequently possesses a total energy $W > W_0$. Therefore, if the situation is described in the frame of reference K, in the formula $W_0{}^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ which describes the situation judged from the frame of reference K^*, the term W_0 must be replaced by W, and the term $W_0{}^*$ must be replaced by W_0, resulting into the formula $W_0 = W(1 - 2\Delta W/W)^{1/2}$.

Thus, the total relativistic energy becomes

$$W = W_0(1 - v^2/c^2)^{-1/2} = mc^2(1 - v^2/c^2)^{-1/2}. \qquad (2)$$

If this equation is written as $W = [m(1 - v^2/c^2)^{-1/2}]c^2$, the term $[m(1 - v^2/c^2)^{-1/2}]$ is the so-called relativistic mass m_r (Schmutzer 1996, Spieweck 1971). Then, the relativistic momentum can be written as $\boldsymbol{p} = (W/c^2)\boldsymbol{v} = m_\mathrm{r}\boldsymbol{v}$ (Landau and Lifschitz 1967).

2.3. Building up special and general relativistic formulae by making use of invariant constants

In the preceding part I, natural physical units of proper energy W_0, length s, time t, voltage U, temperature T, as well as of velocity v and inertial mass m were given as $\mathrm{W_{0u} = W_a}$, $\mathrm{s_u} = (\kappa/8\pi)\mathrm{W_a}$, $\mathrm{t_u = h/W_a}$, $\mathrm{U_u = e^{-1}W_a}$, $\mathrm{T_u = k^{-1}W_a}$, $\mathrm{v_u} = (\kappa/8\pi\mathrm{h})\mathrm{W_a}^2$ and $\mathrm{m_u} = (8\pi\mathrm{h}/\kappa)^2/\mathrm{W_a}^3$. Then, the physical quantities s, t, U, T, v and m, being multiples of $\mathrm{s_u}$, $\mathrm{t_u}$, $\mathrm{U_u}$, $\mathrm{T_u}$, $\mathrm{v_u}$ and $\mathrm{m_u}$, can be used for the description of physics occurring in a proper or eigen frame of reference K. Now, if the physical quantities used in the proper frame of reference K are judged from another frame of reference K^* – using units realized in an observer's frame of reference K^*, the physical units $\mathrm{s_u}$, $\mathrm{t_u}$, $\mathrm{U_u}$, $\mathrm{T_u}$, $\mathrm{v_u}$ and $\mathrm{m_u}$ – due to the invariance of the constants κ, h, e and k – will be judged as "effective" length $\mathrm{s_u}^* = (\kappa/8\pi)\mathrm{W_a}^*$, "effective" duration $\mathrm{t_u}^* = \mathrm{h/W_a}^*$, "effective" voltage $\mathrm{U_u}^* = \mathrm{e^{-1}W_a}^*$, "effective" temperature $\mathrm{T_u}^* = \mathrm{k^{-1}W_a}^*$, "effective" velocity $\mathrm{v_u}^* = (\kappa/8\pi\mathrm{h})\mathrm{W_a}^{*2}$ and "effective" inertial mass $\mathrm{m_u}^* = (8\pi\mathrm{h}/\kappa)^2/\mathrm{W_a}^{*3}$.

Then, it can be written $W_0 = \mathrm{n\,W_a}$, as well as $W_0{}^* = \mathrm{n\,W_a}^*$, where n is equal to $\{W_0\} = \{W_0{}^*\}$, see I, section 3. Therefore, if the "effective" physical units $\mathrm{s_u}^*$, $\mathrm{t_u}^*$, $\mathrm{U_u}^*$, $\mathrm{T_u}^*$, $\mathrm{v_u}^*$ and $\mathrm{m_u}^*$ are divided by the corresponding units $\mathrm{s_u}$, $\mathrm{t_u}$, $\mathrm{U_u}$, $\mathrm{T_u}$, $\mathrm{v_u}$ and $\mathrm{m_u}$, to be used in the proper frame of reference, we get $\mathrm{s_u^*/s_u = W_a^*/W_a} = W_0{}^*/W_0 = s^*/s$, $\mathrm{t_u^*/t_u = W_a/W_a^*} = W_0/W_0{}^* = t^*/t$, $\mathrm{U_u^*/U_u = W_a^*/W_a} = W_0{}^*/W_0 = U^*/U$, $\mathrm{T_u^*/T_u = W_a^*/W_a} = W_0{}^*/W_0 = T^*/T$, $\mathrm{v_u^*/v_u = W_a^{*2}/W_a^2} = W_0{}^{*2}/W_0{}^2 = v^*/v$ and $\mathrm{m_u^*/m_u = W_a^3/W_a^{*3}} = W_0{}^3/W_0{}^{*3} = m^*/m$. That means, if the judgement of proper energy by an external observer, i.e. if the quotient $W_0{}^*/W_0$, is known from the formula $W_0{}^* = W_0(1 - 2\Delta W/W_0)^{1/2}$, then the relativistic expressions for the quotients s^*/s, t^*/t, U^*/U, T^*/T, v^*/v and m^*/m, will be known, too.

2.3.1. Alterations of length and time

According to the formulae $W_{0\|\boldsymbol{e}}^* = W_0(1 - v^2/c^2)^{1/2}$ and $W_{0\perp\boldsymbol{e}}^* = W_0$, as well as to the relations $s^*/s = W_0^*/W_0$ and $t^*/t = W_0/W_0^*$, the special relativistic formulae for length contraction and time dilatation immediately turn out to be

$$s_{\|\boldsymbol{e}}^* = s(1 - v^2/c^2)^{1/2},\ s_{\perp\boldsymbol{e}}^* = s, \text{ and}$$

$$t^* = t(1 - v^2/c^2)^{-1/2}. \tag{3}$$

In case of $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ and $\Delta W = m_\mathrm{g} g \Delta H$, the general relativistic formulae for alterations in length and time result to be

$$s^* = s(1 - 2m_\mathrm{g} g\Delta H/mc^2)^{1/2} \approx s(1 - g\Delta H/c^2) \text{ and}$$

$$t^* = t(1 - 2m_\mathrm{g} g\Delta H/mc^2)^{-1/2} \approx t(1 + g\Delta H/c^2). \tag{4}$$

Therefore, a proper length s, measured in a frame of reference K, in an external observer's frame of reference K^* will be judged as s^*, and a proper duration t, measured in a frame of reference K, in an external observer's frame of reference K^* will be measured as $t^* > t$. E.g., if in a proper frame of a space ship, t is measured as $t = 10^7$ seconds or heartbeats, in an observer's frame K^* on earth, the larger time $t^* > t$, i.e. more than 10^7 seconds or heartbeats will be measured.

In general relativistic physics besides the term $\Delta W = m_\mathrm{g} g\Delta H$ also the term $\Delta W = m_\mathrm{g}\Delta\Phi = m_\mathrm{g}(\Phi_\infty - \Phi)$ is used, where Φ is the Newtonian gravitational potential $\Phi = -Gm_\mathrm{g}/r$ (r: distance to the gravitational centre) in a local non-inertial frame of reference K, and $\Phi_\infty = -Gm_\mathrm{g}/\infty = 0$ is the Newtonian gravitational potential in an observer's – inertial – frame of reference K^*. Then, the general relativistic length contraction and time dilatation formulae become

$$s^* = s(1 + 2m_\mathrm{g}\Phi/mc^2)^{1/2} \approx s(1 + \Phi/c^2) \text{ and}$$

$$t^* = t(1 + 2m_\mathrm{g}\Phi/mc^2)^{-1/2} \approx t(1 - \Phi/c^2). \tag{5}$$

2.3.2. Relativistic frequency shifts

The special and general relativistic redshift formulae will be obtained in the following way. If an atom – positioned in a proper frame of reference K – is emitting an electromagnetic wave with the proper frequency $f = \Delta W_0/h$ (Einstein 1905a), in an observer's frame of reference K^* the frequency will be judged as "effective" frequency $f^* = \Delta W_0^*/h = f\Delta W_0^*/\Delta W_0$ or – because the quotient $\Delta W_0^*/\Delta W_0$ will be equal to the quotient W_0^*/W_0 – as $f^* = fW_0^*/W_0$.

In the classic Doppler formula (Otting 1939) $f_\mathrm{D} = f[1 - (v/c)\cos\theta]^{-1}$ the frequency f, therefore, must be replaced by the "effective" frequency f^* (θ: angle of the direction of motion of the light source with the velocity v against the direction of observation). Then, according to formulae $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ and $\Delta W = mv^2/2$, the relativistic Doppler formula results to be

$$f_\mathrm{D} = f(1 - v^2/c^2)^{1/2}[1 - (v/c)\cos\theta]^{-1}. \tag{6}$$

In a gravitational field, according to formulae $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ and $\Delta W = m_\mathrm{g} g\Delta H$ the observed radiation frequency, e.g., measurable via the Mössbauer effect, will be

$$f^* = f(1 - 2m_\mathrm{g} g\Delta H/mc^2)^{1/2} \approx f(1 - g\Delta H/c^2). \tag{7}$$

2.3.3. Light deflection

In any proper frame of reference K the speed of light in vacuum has the same value $c = 299792458$ m/s. This is true not only in an inertial frame of reference K but also in any local non-inertial frame of reference K (Einstein 1965). In an observer's frame of reference K^*, however, where the gravitational potential Φ^* differs from the gravitational potential Φ in the frame of reference K, the "effective" speed of light in a local non-inertial frame of reference K (Dehnen *et al* 1960), will be judged in the frame of reference K^*, where it is $\Phi^* = 0$, as c^*. According to the equation $v^*/v = W_0{}^{*2}/W_0{}^2$ or $c^*/c = W_0{}^{*2}/W_0{}^2$, and due to the formulae $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ and $\Delta W = m_g\Delta\Phi = m_g(\Phi^* - \Phi) = - m_g\Phi$, the "effective" speed of light becomes

$$c^* = c(1 + 2m_g\Phi/mc^2) \approx c(1 + 2\Phi/c^2). \tag{8}$$

This speed of light c^* (respectively c^*/c) – as a result of the theory of relativity – Einstein (1965) was calling "L". That means, one of the basic assumption of special relativity, the constancy of the speed of light in vacuum, is not true in general relativity, as a curvature of light rays is only resulting from a propagation velocity of light that is varying with the local position (Einstein 1969). Thus, a gravitational field – with the gravitational potential Φ – endows space with a variable refractive index (Sciama 1969): Light is travelling like in a medium with a refractive index (Dehnen *et al* 1960, Spieweck 1988) $n(\Phi) = c/c^* = (1 + 2m_g\Phi/mc^2)^{-1} \approx (1 - 2\Phi/c^2)$ or $n(r) = (1 - 2Gm_g{}^2/rmc^2)^{-1} \approx (1 + 2Gm_g/rc^2)$. At the sun's surface, where it is $r = 6.96\times10^8$ m and $m_g = 1.989\times10^{30}$ kg (Zimmermann and Weigert 1995) light is travelling like in a medium with the refractive index $n = 1.0000042$. Therefore, due to the principle of Huygens, light is deflected by an angle of $4Gm/rc^2 \approx 1.75''$ (Dehnen *et al* 1960, Spieweck 1988, 2000b).

As curvature of space is implicitly already contained in Newton's theory, a deflection of light is also resulting from the Newtonian gravitational theory. In this case, however, only half the value of $\approx 1.75''$ is obtained, as Newton was treating time as an absolute instead of a reference frame dependent – and in this case dilated – physical quantity. That means, obviously, the new result of the theory of relativity, is not relativity or curvature of space, but relativity of time.

2.3.4. Further results

According to the equation $m^*/m = W_0{}^3/W_0{}^{*3}$, to the formulae $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$ and $\Delta W = m_g\Delta\Phi = m_g(\Phi^* - \Phi) = - m_g\Phi$, a planet's "effective" inertial mass m^* near the sun in a region with the gravitational potential Φ (in the – proper – frame of reference K) will be judged in an observer's frame of reference K^*, where it is $\Phi^* = 0$, as

$$m^* = m(1 + 2m_g\Phi/mc^2)^{-3/2} \approx m(1 - 3\Phi/c^2). \tag{9}$$

This means that the "effective" inertial mass m^* near gravitating masses will be increased (Einstein 1965). The above formula can be used for the calculation of the advance of perihelion (Dehnen *et al* 1960). But the perihelion advance can be also derived from Kepler's second law $r_P{}^2\lambda/2t = C$ (Spieweck 1988, 2000a, b). As the constant C does not dependent on the momentary distance r_P of the planet from the sun, and thus on the gravitational potential Φ, it results $r_P{}^2\lambda/2t = C = r_P{}^{*2}\lambda^*/2t^*$ or $\lambda^*/\lambda = r_P{}^2t^*/r_P{}^{*2}t$. As it should be $r_P^*/r_P = s^*/s$, the observed angle will be

$$\lambda^* = \lambda(1 + 2m_g\Phi/mc^2)^{-3/2} \approx \lambda(1 - 3\Phi/c^2). \tag{10}$$

This formula may be even more easily obtained according to the following consideration. Because the unit of plane angle λ can be written as the quotient of the units of inertial and gravitational mass (see part I), $[\lambda] = [m]/[m_g]$, and the gravitational mass m_g is thought to be an invariant, the plane angle λ^* will show the same behaviour concerning the dependence on the gravitational potential Φ as the inertial mass m^*.

Further relativistic effects have been already physically explained in previous papers, the Sagnac effect (Spieweck 2000b), the twin paradox (Spieweck 1978, 1988, 1992, 2000b) and the gravi-magnetic effect (Spieweck 1971, 1985, 2000b), which should be better called gravi-inertial effect. – If the total mass density ρ_g is assumed to be composed of the positive particle density ρ_+ and the negative field mass density of the gravi-inertial field ρ_{-}, gravi-inertial field equations can be written in a form, being identical with the Maxwellian equations (Spieweck 1985, 2000b).

2.3.4.1. Advance of perihelion

The formula for the observed angle λ^* can be used for a simple calculation of the perihelion advance, see Einstein (1915). An increase in the plane angle λ in the planet's region with the gravitational potential Φ (in the proper frame of reference K) will be judged in an external observer's frame of reference K^*, where it is $\Phi^* = 0$, as an "effective" increase in angle by λ^*. The relative increase in angle is measured as $(\lambda^* - \lambda)/\lambda = \Delta\lambda/\lambda \approx (1 - 3\Phi/c^2) - 1 = -3\Phi/c^2$. For one revolution – first assumed to be circular, i.e. $\lambda = 2\pi\text{rad} = 360°$, – the increase in angle is $\Delta\lambda \approx -2\pi\text{rad}\times 3\Phi/c^2 = 6\pi\text{rad}GM/r_P c^2$, where $M = 1.989\times 10^{30}$ kg is the sun's mass. In case of an elliptic orbit with the half-axis a and the numeric eccentricity ε the distance r_P must be replaced by $a(1 - \varepsilon^2)$: $\Delta\lambda \approx 6\pi\text{rad}GM[a(1 - \varepsilon^2)c^2]^{-1}$. For a planet with a period T_P, within 100 Earth years (period $T_E = 365.2425$ d) it is $\Delta\lambda \approx 3\times 100\times 360\times 60\times 60'' \times GMT_E[T_P a(1 - \varepsilon^2)c^2]^{-1}$. In case of Mercury it is $T_P = 87.97$ d, $a = 57.91\times 10^9$ m and $\varepsilon = 0.2056$ (Zimmermann and Weigert 1995). Thus, the advance of perihelion within 100 Earth years results in 43 arc seconds.

2.3.4.2. Einstein's law of gravitation

Newton's third law (Newton 1687) is dealing with pressure (and traction) p. That means, force $\boldsymbol{F}$ is only a secondary physical quantity, which is given as the product of pressure p and area $\boldsymbol{A}$, that should be an axial vector $\boldsymbol{A}$: $\boldsymbol{F} = p\boldsymbol{A}$ or $\boldsymbol{F} = \int p\,\mathrm{d}\boldsymbol{A}$.

Therefore, here, it will be assumed that the pressure (or traction) p is an invariant physical quantity: $p^* = F^*/A^* = F/A = p$. Whereas in special relativity a rigid space is assumed, and consequently a cross-section A, positioned perpendicular to the direction of motion, remains unchanged, $A^* = A$, in general relativistic physics, according to the results of section *2.3.4*, an area $A = r^2\lambda/2$ will be increased to $A^* = A(1 + 2m_g\Phi/mc^2)^{-1/2}$. Thus, in general relativistic physics, a Newtonian gravitational force $F_N = F$, calculated for a proper frame of reference K, in an observer's frame of reference K^*, will be increased to the Einsteinian force $F_E = F^* = F(1 + 2m_g\Phi/mc^2)^{-1/2}$. The Einsteinian gravitational law, therefore, can be written as

$$F_E = F_N(1 - r_s/r)^{-1/2}, \tag{11}$$

where $r_s = 2Gm_g^2/mc^2$ is the Schwarzschild radius (Zimmermann and Weigert 1995).

Perhaps, it will be interesting to look at the relativistic behaviour of solid angle $\Omega = \{\Omega\}\,[\Omega]$. Because $8\pi/\kappa$ is an invariant with the dimension of force times solid angle, it should be $F^*\Omega^* = F\Omega$. For an observer in a frame of reference K^*, therefore, the solid angle Ω will be decreased – or tapered – to

$$\Omega^* = \Omega(1 + 2m_g\Phi/mc^2)^{1/2} \approx \Omega(1 + \Phi/c^2). \tag{12}$$

It must be mentioned, however, that the unit of the solid angle steradian (sr), in the non-euclidean space should not be interpreted as $\text{m}^2/\text{m}^2 = 1$, as the solid angle is a reference frame dependent physical quantity which depends on the gravitational potential $\boldsymbol{\Phi}$. Consequently, the unit sr should be considered as a unit sui generis, see also subsequent section 3.

Because the volume V of a certain part of a sphere may be proportional to $r^2\lambda r\Omega$, it will be $V^*/V = W_0^*/W_0$, and thus $\rho_g^*/\rho_g = W_0/W_0^*$ (ρ_g: density). Therefore, it may be suggested that the plane angle ϕ has the same relativistic behaviour as the solid angle Ω: $\phi^* \approx \phi(1 + \Phi/c^2)$.

2.3.4.3. New general relativistic conservation laws

In the preceding section *2.3.4.2* it was assumed that pressure is an invariant physical quantity. Therefore, pressure is believed to be a conserved physical quantity:

$$p^* = F^*/A^* = F/A = p. \quad (13)$$

Because for the volume V the relation $V^*/V = W_0^*/W_0$ was given, it results:

$$w_0^* = W_0^*/V^* = W_0/V = w_0. \quad (14)$$

That means, the density of proper energy should be conserved. Perhaps, this equation may be a generalization of the special relativistic equation $W_0^2 = W^2 - \boldsymbol{p}^2c^2$, which is only true in every proper frame.

Based on the three equations $c^* = c(1 + 2m_g\Phi/mc^2)$, $\lambda^* = \lambda(1 + 2m_g\Phi/mc^2)^{-3/2}$ and $\Omega^* = \Omega(1 + 2m_g\Phi/mc^2)^{1/2}$, the following general relativistic invariant relation for the bending (or increasing of the plane angle λ to λ^*) and tapering of a light ray (or decreasing of the solid angle Ω to Ω^*) may be suggested:

$$c^*\lambda^*\Omega^* = c\lambda\Omega. \quad (15)$$

This relation may describe the invariant propagation of a light ray in the real, non-euclidean world. Note, that in a euclidean or pseudo-euclidean world it is $\lambda^* = \lambda$, as well as $\Omega^* = \Omega$, and therefore $c^* = c$.

Finally, general relativistic continuity equations may be given:

$$\partial\rho_g^*/\partial t^* + \mathrm{div}^*(\rho_g^*\boldsymbol{v}^*) = 0 = \partial\rho_g/\partial t + \mathrm{div}(\rho_g\boldsymbol{v}) \text{ and}$$

$$\partial\rho^*/\partial t^* + \mathrm{div}^*(\rho^*\boldsymbol{v}^*) = 0 = \partial\rho/\partial t + \mathrm{div}(\rho\boldsymbol{v}), \quad (16)$$

where ρ_g is the density of gravitational (charges or) masses and ρ is the density of electric charges.

2.3.4.4. General relativistic behaviour of voltage, current and resistance

As the energy W_0 may be understood as the product of voltage U and – invariant – charge q, on Earth the "effective" voltage U^* shows the same dependence on height as the energy W_0^*: $U^* \approx U(1 - g\Delta H/c^2)$. Electric current I is the quotient of invariant charge q and time t. Therefore, the "effective" current becomes $I^* \approx I(1 - g\Delta H/c^2)$. Thus, for the "effective" resistance it results $R^* = U^*/I^* = U/I = R$. That means, R^* does not depend on height, respectively on the gravitational potential Φ.

2.3.4.5. General relativistic behaviour of temperature

The energy W_0 may be also understood as the product of the invariant Boltzmann constant k and temperature T. Thus, from the relations $W_0 = \mathrm{n}kT$ and $W_0^* = \mathrm{n}kT^*$, it results $T^* = TW_0^*/W_0 = T(1 + 2m_g\Phi/mc^2)^{1/2} \approx T(1 + \Phi/c^2)$. Therefore, perhaps, the early, hot universe (with the temperature T), nowadays appears as – red shifted or – cooled down to $T^* \approx 3$ K.

3. Discussion

Because the "effective" values of the invariants κ, h, e, k, and u do not depend on Φ ($\kappa^* = \kappa$, $h^* = h$, $e^* = e$, $k^* = k$, and u* = u), these invariants should be proposed as basic elements for a possible future system of units. In contrast, the "effective" values for the so far fixed electro-magnetic field constants μ_0 (permeability of the vacuum) and c do depend on the gravitational potential Φ (see Spieweck 1987b and section *2.3.3*). Therefore, the author is of the opinion, that μ_0 and c should no longer be used for the definition of future SI units.

As already suggested previously (Spieweck 1987a), the so far dimensionless Sommerfeld constant $\alpha = e^2/(4\pi\varepsilon_0\hbar c)$ will be in the 21st century physically understood in a new way. If the "effective" constant α^* is written as $\alpha^* = e^{*2}/[(4\pi\varepsilon_0)^*\hbar^*c^*]$, due to the relations $e^* = e$,

$(4\pi\varepsilon_0)^* = (W_0/W_0^*)(4\pi\varepsilon_0)$, $\hbar^* = (W_0^*/W_0)^3\hbar$ and $c^* = (W_0^*/W_0)^2c$, it becomes $\alpha^* = (W_0/W_0^*)^4\alpha$. (This result may be even easier obtained, if it is taken into account that it can be written $[\alpha] = [\lambda]/[\Omega]$, and therefore $[\alpha^*] = [\lambda^*]/[\Omega^*] = (W_0/W_0^*)^4[\lambda]/[\Omega] = (W_0/W_0^*)^4[\alpha]$.) Therefore, the value $\alpha \approx (1/137.0)$ rad/sr, concerning an inner atomic proper frame of reference K, where the "world" may be extremely non-euclidean, in an observer's frame of reference K^*, may seem to be increased to the "effective" value $\alpha^* \approx (1/128.5)$ rad/sr, see Levine *et al* (1997) and Tobar (2005). A similar situation may also concern an early stage of the universe, which should have been extremely non-euclidean, too.

Finally, on the occasion of the suggestions for new general relativistic conservation formulae, it may be realized that, here, relativistic physics could be treated in a mathematically trivial manner, whereas studying the Riemannian geometry takes years until it can be used as an active tool for a research in general relativistic physics (Schmutzer 1996 p 113). Note, that the perihelion advance is a relatively simple phenomenon, because it is only concerning physics in an area, whereas light rays are propagating in the tapered space.

4. Conclusion

Relativistic problems had been treated by using so-called "effective" physical quantities Q_i^* instead of making use of coordinates (used in a pseudo-euclidean or Riemannian geometry). First, one central postulate concerning the judgement of proper energy W_0 by an external observer was given: $W_0^* = W_0(1 - 2\Delta W/W_0)^{1/2}$, where ΔW is the difference of a particle's energy, measured in the proper frame of reference K and in an observer's frame K^*. Then, without referring to the complex metric tensor $g_{\alpha\beta}(x^\mu)$, the formulae for all so far experimentally verified relativistic effects could be deductively obtained from the formula for the scalar or tensor W_0^* and four invariants, namely Einstein's constant κ, Planck's constant h, the elementary charge e and Boltzmann's constant k: for alterations (or shifts) of length s to $s^* = s(W_0^*/W_0)$, of time t to $t^* = t(W_0/W_0^*)$, of frequency f to $f^* = f(W_0^*/W_0)$, as well as for the following general relativistic effects, namely the deflection of light, an increase of inertial mass m to $m^* = m(W_0/W_0^*)^3$, an increase in gravitational force from F_g to $F_g^* = F_g(W_0/W_0^*)$, the so-called curvature and tapering of space, respectively the relativistic increase of plane angle λ to $\lambda^* = \lambda(W_0/W_0^*)^3$ and a decrease – or tapering – of solid angle Ω to $\Omega^* = \Omega(W_0^*/W_0)$, a decrease of volume V to $V^* = VW_0^*/W_0$, an increase in density from ρ_g to $\rho_g^* = \rho_g W_0/W_0^*$, an increase of α to $\alpha^* = \alpha(W_0/W_0^*)^4$, as well as for a decrease of voltage U to $U^* = UW_0^*/W_0$, of current I to $I^* = IW_0^*/W_0$ and of temperature T to $T^* = TW_0^*/W_0$. In addition, suggestions were made concerning new general relativistic conservation laws.

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