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DKD-L 13-3

Rounding of Results and Measurement Uncertainties in Calibration Certificates

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Deutscher Kalibrierdienst (DKD) – German Calibration Service

Since its foundation in 1977, the German Calibration Service has brought together calibration laboratories of industrial enterprises, research institutes, technical authorities, inspection and testing institutes. On 3rd May 2011, the German Calibration Service was reestablished as a *technical body* of PTB and accredited laboratories.
This body is known as *Deutscher Kalibrierdienst* (DKD for short) and is under the direction of PTB. The guidelines and guides developed by DKD represent the state of the art in the respective areas of technical expertise and can be used by the *Deutsche Akkreditierungsstelle GmbH* (the German accreditation body – DAkkS) for the accreditation of calibration laboratories.
The accredited calibration laboratories are now accredited and supervised by DAkkS as legal successor to the DKD. They carry out calibrations of measuring instruments and measuring standards for the measurands and measuring ranges defined during accreditation. The calibration certificates issued by these laboratories prove the traceability to national standards as required by the family of standards DIN EN ISO 9000 and DIN EN ISO/IEC 17025.

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Foreword

DKD guides provide recommendations on technical issues that arise in connection with the practical work of accredited calibration laboratories. The guides describe procedures which may serve accredited calibration laboratories as a model for defining internal procedures and regulations. DKD guides may become an essential component of the quality management manuals of calibration laboratories. By using these guides, the state of the art in the respective field may be implemented into laboratory practice. This contributes to the harmonisation of procedures and may thus help to make work in calibration laboratories more efficient.

DKD guides should not impede the further development of calibration procedures and processes. Deviations from these guides as well as new procedures are permitted if there are technical reasons to support this action.

This guide has been drawn up by DKD's Technical Committee *Measurement Uncertainty* and approved by the Board of the DKD.

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1 Terms and definitions

Term	Meaning	Definition according to
Indication	Quantity value provided by a measuring instrument or a measuring system	VIM [1], 4.1
Result value	Measured value of the measurement result	own definition
Quantity value	Number and reference together expressing magnitude of a quantity Example: Length of a given rod: 5.34 m or 534 cm	VIM [1], 1.19
Measurement result	A measurement result is generally expressed as a single measured quantity value and a measurement uncertainty. …	VIM [1], 2.9, Note 2
Measured quantity value	Quantity value representing a measurement result	VIM [1], 2.10
Nominal value	Rounded or approximate value of a characterizing quantity of a measuring instrument or measuring system that provides guidance for its appropriate use	VIM [1], 4.6
Estimated value of an input quantity	Measured value attributed to an input variable as best value and used to determine the measurement result.	DAkkS-DKD-3 [9], B11
Digit accuracy	Number of digits to approximately represent a real number as fixed-point number.	own definition

2 Purpose and scope of application

Measurement results are expressed as measured quantity value and measurement uncertainty (cf. International Vocabulary of Metrology [1], section 2.9) and form part of every calibration. Since representation with infinite accuracy (infinite number of places) is not possible, measurement results must inevitably be rounded. At the same time, it is necessary to adjust the number of digits of the measured value of the measurement result (result value) to the measurement uncertainty to avoid suggesting an uncertainty below its actual magnitude. This guide summarises normative rules and guideline specifications offering viable strategies and examples as a recommendation for proper rounding. In this guide, the term "measurement results" refers to all measured values and their associated measurement uncertainty clearly identified as such in the report stating the results. The guide does not describe the representation and rounding of indicated values (readings of a display), nominal values or (calculated) intermediate or auxiliary values for determining the measurement result.

3 Steps for rounding measurement results

To represent measurement results (best estimates and their associated uncertainties) that have previously been calculated or represented with maximum available floating-point precision (so-called unrounded measurements or unrounded uncertainties), the following steps are applied:

1. rounding the measurement uncertainty to two significant places (digits)[1] using the rounding rules proposed here

2. rounding the measured value of the measurement result (result value) to the last digit that can be determined by the rounded measurement uncertainty (rounding digit)

3. stating the rounded result value in the calibration certificate

4. stating the rounded measurement uncertainty in the calibration certificate

4 Recommended rules for rounding

As a matter of fact, rounding always changes the value of the exact number. There are guides [2], guidelines [6] and standards [3] [4] providing specifications on how to apply the rounding rules. These have been summarised here to offer a consistent approach for the preparation of calibration certificates (result reports):

4.1 Selecting the rounding digits of measurement uncertainty and result value

As a first step, the expanded measurement uncertainty of the result value is rounded. It is recommended to always round to two significant digits. Significant digits are *"all digits [...] from the first non-zero digit to the rounding digit"* [3], i.e. the first non-zero digit and the following digit of the measurement uncertainty.

In the second step the last digit of the rounded measurement uncertainty determines the rounding digit of the measured value (result value).

4.2 Rounding according to DIN EN ISO 80000-1:2013 (Annex B, Rule B)

The steps outlined in section 3 require the rounding of the result value and the measurement uncertainty. The use of consistent rounding rules is recommended. For metrological quantity values and measurement uncertainties, the rounding rule according to DIN EN ISO 80000-1:2013 (ISO 80000-1:2009 + Cor 1:2011), Annex B using Rule B has turned out to be viable:

> "B.2 If there is only one integral multiple[2] nearest the given number, then that is accepted as the rounded number."
>
> "B.3 If there are two successive integral multiples equally near the given number, two different rules are in use.
> Rule B: The greater in magnitude multiple is selected as the rounded number."

These two rules are implemented, for example, in commercially available spreadsheet programmes and correspond to the so-called "commercial rounding".

To avoid multiple rounding, only the measurement result is to be rounded. Rounding must always be carried out in only one step.

[1] Taking into account EA-4/02 M S9.14 para. 3 [7] and DAkkS-DKD-3:2010 para. 6.3 [9], a lower number of digits of the rounded uncertainty is not recommended because the rounding error might then be more than 5 %. A higher number of digits is not permissible according to [2] para. 7.2.6.

[2] The multiple refers to the rounding range of the given number. "B.1 Rounding means replacing the magnitude of a given number by another number called the *rounded number*, selected from the sequence of integral multiples of a chosen rounding range" (DIN EN ISO 80000-1:2013), i.e. products of whole number and rounding range. See examples in Annex B of the standard.

5 Examples

5.1 *Rounding of absolute values (number and unit)*

Determining the output voltage of a 1 V source using a voltmeter or digital multimeter (DMM). The deviation of the source from the nominal value is determined.

Nominal value of the source: $N = 1\ \mathrm{V}$
Measured value[3]: $M = 1.0054321\ \mathrm{V}$
Result value[4]: $E = 0.0054321\ \mathrm{V}$
Unrounded uncertainty: $U = 0.00012499\ \mathrm{V}$

1. Rounding the measurement uncertainty to two significant digits:
   $U_{\mathrm{rounded}} = 0.00012\ \mathrm{V}$ (rounding range $0.00001\ \mathrm{V}$)

2. Rounding the result value to the rounding digit:
   $E_{\mathrm{rounded}} = 0.00543\ \mathrm{V}$

3. Indication of measured value, result value and measurement uncertainty in the calibration certificate

		Measurement result	
Nominal value	**Measured valued**	**Error**	**Expanded uncertainty**
1 V	1.0054321 V	0.00543 V	0.00012 V

5.2 *Relative indications (without units)*

Determining the output voltage of a 5 V source using a voltmeter or digital multimeter (DMM). The relative deviation of the measured value of the source from the nominal value is determined.

Nominal value: $N = 5\ \mathrm{V}$
Measured value[3]: $M = 5.00054521\ \mathrm{V}$
Result value[5]: $E = 109.0301111 \cdot 10^{-6}$
Unrounded relative uncertainty[6]: $U = 24.99527447 \cdot 10^{-6}$

1. Rounding the relative (no units) measurement uncertainty to two significant digits:
   $U_{\mathrm{rounded}} = 25 \cdot 10^{-6}$ (rounding range $1 \cdot 10^{-6}$)

2. Rounding the relative result value (no units) to the rounding digit:
   $E_{\mathrm{rounded}} = 109 \cdot 10^{-6}$

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[3] Mean value of the indication of the DMM
[4] In this example, the result value is defined as the deviation from the nominal value $E = M - N$
[5] In this example, the result value is defined as the relative deviation from the nominal value in relation to the nominal value $E = \frac{M-N}{N}$
[6] in relation to the measured value

3. Indication of measured value, result value and measurement uncertainty in the calibration certificate

		Measurement result	
Nominal value	**Measured value**	**Relative error**	**Relative expanded uncertainty**
5 V	5.00054521 V	$109 \cdot 10^{-6}$	$25 \cdot 10^{-6}$

6 Bibliography

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[4] DIN EN ISO 80000-1: Größen und Einheiten - Teil 1: Allgemeines (ISO 80000-1:2009 + Cor 1:2011, Quantities and units, Part 1: General); Deutsche Fassung EN ISO 80000-1, 2013, Beuth Verlag GmbH, Berlin-Wien-Zürich

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