Dynamic Torque Calibration

Leonard Klaus*

1 Introduction

The demand from industry for dynamic torque calibration has significantly increased over the past years. However, dynamic torque calibrations are not available at present and the respective research activities have started only a few years ago. As part of the European Metrology Research Programme (EMRP), research in dynamic torque calibration took part in work package 3 Dynamic Torque of the joint research project IND09 Traceable Dynamic Measurement of Mechanical Quantities.

2 Definition of the Unit Torque

The International System of Units (SI) [1] defines the torque $M$ as a derived unit given by the cross product of the position vector $\mathbf{r}$ and the force vector $\mathbf{F}$:

$$
M = \mathbf{r} \cdot \mathbf{F}.
$$

(1)

Maybe easier to understand is the simplified scalar representation as a force $F$ acting on a lever arm of length $l$, which leads to

$$
M = l \cdot F.
$$

(2)

The unit of torque is newton metres (N·m).

3 Design of Torque Transducers

The vast majority of torque transducers used in dynamic applications and for high-precision measurements are based on strain gauges. Strain gauge torque transducers have a specific mechanical design consisting of rigid components and of compliant measuring elements at which the strain gauges are located. In case of a change of the applied torque load, the measuring element's deflection will change and the strain gauges will detect a change in the surface strain. The mechanical elongation of a strain gauge proportionally changes its electrical resistance. Usually, four strain gauges are electrically connected to a full Wheatstone bridge circuit in order to compensate for temperature drift and to obtain an output signal with a good signal-to-noise ratio, which is proportional to the applied load. The output voltage of such a bridge circuit is conditioned by a bridge amplifier. Based on the transducer, this amplifier can be included in integrated electronics. The traceable dynamic calibration of bridge amplifiers is described in a dedicated contribution in this issue [pp. 52–61].

In general, two easily distinguishable structural designs of strain gauge torque transducers can be found: shaft-type transducers and flange-type transducers. The two types are depicted in figure 1.

The classic shaft-type transducer consists of a cylindrical shaft with specific structural parts of reduced stiffness used for the strain measurement. The measuring elements can be carried out as a tapered solid shaft, a hollow shaft or as a cage shaft as displayed in the figure.

Flange type transducers have a much shorter design with larger diameters. They possess a significantly higher torsional stiffness and mass moment of inertia than shaft-type transducers. The shear elements at which the strain is picked up are positioned between the two flanges, and are exposed to radial or axial strain.

* Leonard Klaus, Working Group “Impact Dynamics”, PTB, e-mail: leonard.klaus@ptb.de

Figure 1: Schematic view of a shaft type (top) and flange type (bottom) torque transducer.
4 Applications with Dynamic Torque Excitation

The demand for research on the traceable dynamic calibration of torque transducers comes from two main applications with dynamic torque signals.

4.1 Impulse wrenches

Screw connections in industrial assembly lines are often fastened by impulse wrenches. For safety-relevant screw connections, the traceable measurement of the fastening torque is required.

Impact wrenches fasten the screw connection by applying a sequence of short impulses generated by the release of a pressurized hydraulic fluid. The operator's exposition to the high fastening torque is limited due to the steepness of the pulses and the tools’ inertia. The measurement of a typical torque pulse is shown in figure 2. The pulse duration of impulse wrenches is in the range of milliseconds which results in an associated frequency content of several hundred hertz. The fastening torque can amount up to one kilo newton metre.

4.2 Motor test rigs

The output of electric motors and internal combustion engines (ICE) is analyzed in research and development, but for type-approval as well. The mechanical power $P$ transferred by a rotating shaft can be measured by means of the rotational speed $n$ and the torque $M$ as

$$P = n \cdot M.$$

This relationship is often used for the mechanical output power measurement of the aforementioned electric motors as well as of ICEs. The output torque level of both types does not only contain static components. ICEs rather generate a highly fluctuating torque output due to their working principle with several strokes and combustion processes, where the dynamic torque components can range in the same magnitude as the mean torque level.

The frequency content of the dynamic torque components can be up to the kilohertz range, and the respective torque amplitude can be up to some kilonewton metres. In the case of electric drives, strong dynamic components in the torque output can be caused by the electronic frequency converters, which are used to control the rotational speed and the torque output of synchronous and asynchronous electric motors. The dynamic components are usually smaller than those of ICEs, however, they can be in the range of several per cent of the mean torque. Both frequency content and mean torque level are comparable to ICEs. A typical test rig for electric motors is shown in figure 3. The measured torque output of a synchronous electric motor is depicted in figure 4.

5 State of the Art in Torque Calibration

At present, commonly accepted methods for a dynamic calibration of torque transducers, or even documentary standards, do not exist. All torque transducers that are used in the above-mentioned
applications with dynamic torque signals are calibrated only statically.

Static torque calibrations can be carried out in a wide torque range and with small measurement uncertainties. For a precise primary static torque calibration, deadweight facilities are used. A deadweight with the mass $m$ is connected to a lever arm of a known length $l$. With the known local gravitational acceleration $g_{loc}$, equation 2 leads to

$$M = l \cdot m \cdot g_{loc}.$$ (4)

However, in reality, the air density has to be compensated and many more influences have to be taken into account. A detailed overview of the realization of the static torque can be found in [2].

6 Measurement Principle for the Dynamic Torque Calibration

The primary dynamic calibration is based on Newton’s second law. The generated dynamic torque $M(t)$ equals the mass moment of inertia $J$ times the time-dependent angular acceleration $\dot{\varphi}(t)$, and it holds

$$M(t) = J \cdot \ddot{\varphi}(t).$$ (5)

A dedicated dynamic torque measuring device based on this principle was developed and commissioned at PTB [3]. At the beginning of the joint research project, the dynamic torque device was improved by installing a new exciter and a larger air bearing with higher capacity.

The design and the components of the measuring device are depicted in figure 5. The device consists of a freely rotatable vertical drive shaft at which all components are arranged in series. At the bottom, a rotational exciter is mounted on a platform which can be moved vertically. Attached to the exciter is the transducer under test (device under test, DUT), which is mounted between two single cardanic couplings on both sides. These couplings are torsionally stiff, but compliant for bending to reduce parasitic lateral forces and bending moments. At the top end of the drive shaft, an arrangement is mounted of a radial grating disk (which is used for the incremental angle measurement) and of an air bearing (which enables low-friction support of the components). The angle position at the top is measured by interferometric methods and afterwards converted to the angular acceleration. The two laser beams of a rotational vibrometer pass through the radial grating disk at diametrically opposed positions and get diffracted. The first order of diffraction of the beam is coupled back into the vibrometer. When a grating line passes the beam, a phase shift of $2\pi$ will be detected.

The angular acceleration at the bottom of the drive shaft is measured by an angular accelerometer which is embedded in the rotor of the rotational exciter. For the dynamic calibration measurement, the rotational exciter generates monofrequent sinusoidal excitations. The angular acceleration at the bottom, the output signal of the transducer, and the angle position at the top are acquired simultaneously by a data acquisition system.

![Figure 5: Dynamic torque measuring device and its components.](image-url)
7 Physical Model

Torque transducers are always coupled to their mechanical environment on both sides when used for measurements. The mechanical environment may have influence on the transducer’s dynamic behaviour, and vice versa. To understand this influence, the mechanical system of torque transducer and measuring device can be described by model calculations.

The model uses a system consisting of two elements of mass moment of inertia $J_H$, $J_B$ connected by a torsional spring $c_T$ and damper $d_T$ in parallel, which represent the transducer under test, and an additional rigidly coupled mass moment of inertia $J_0$ at the top. The dynamic behaviour is described by the magnitude response which is the ratio of the magnitudes of the angular accelerations $\ddot{\varphi}_0$ and $\ddot{\varphi}_B$ at the top and the bottom, respectively. The model and the magnitude response are depicted in figure 6 and figure 7.

Different mass moments of inertia $J_0$ (which represent different mechanical environment properties connected to the torque transducer) lead to a change in the magnitude response of the system, as illustrated in figure 7. Marked in red is the sensitivity value which would have been obtained by a static calibration. The dynamic behaviour of such a system is obviously dependent on both the properties of the coupled components and of those of the transducer. Deviations from the static calibration value can occur even if the excitation frequency is significantly below the resonance frequency (illustrated by the grey box).

To properly describe such an interaction of transducer and environment and to be able to determine these influences, it is necessary to describe the transducer by a proper physical model. For this purpose, a model similar to that of the simulation calculation was developed.

The model approach is linear and time-invariant (LTI). These assumptions are valid, because transducers are designed to be linear and to not change their properties over time significantly (time invariance).

The model is based on the mechanical design of torque transducers. As described in section 3, these transducers have a characteristic mechanical design consisting of torsionally compliant measuring elements, which are modelled as a torsional spring and a damper in parallel, and of rigid components, which are modelled as coupled mass moments of inertia. The model components of the transducer are marked in red in figure 6.

To be able to identify the torque transducer’s model properties, the model needs to be extended to include the measuring device, which is the mechanical environment in the case of the calibration.

The chosen model represents the mechanical design of the components in the drive train. The coupling elements are assumed to be the components with the lowest torsional stiffness and, therefore, are represented by a mass-spring-damper element, whereas all other components are assumed to be rigidly coupled mass moment of inertia elements. The model and the corresponding components of the drive train are shown in figure 8.
The modelled measuring device can be described mathematically as an inhomogeneous ordinary differential equation (ODE) system

\[ J \ddot{\phi} + D \dot{\phi} + C \phi = M. \] (6)

\( J \) denotes the mass moment of inertia matrix, \( D \) the damping matrix, and \( C \) the stiffness matrix. The angle vector \( \phi \) describes the angle values at the different positions as depicted in figure 8, and \( \dot{\phi} \) and \( \ddot{\phi} \) denote the vectors of the corresponding derivatives. The forced excitation of the rotational exciter is given in the load vector \( M \).

8 Model Parameter Identification

The model parameters of the transducer under test are going to be identified by means of measurement data. For this purpose, all angle positions, angular velocities and angular accelerations at the four positions marked in figure 8 would have to be measured. Due to the fact that the excitation is sinusoidal, it is possible to calculate the angular acceleration or the angular velocity from the angle position, and vice versa. Technically, it is not possible to measure the torsion angles directly above and below the device under test (\( \phi_H, \phi_B \)) with sufficient precision.

The output voltage signal of the transducer \( U_{DUT}(t) \) is assumed to be proportional to the torsion of the transducer \( \Delta \phi_{HB} = \phi_H - \phi_B \) giving

\[ U_{DUT}(t) = \rho \cdot \Delta \phi_{HB}(t). \] (7)

The proportionality factor \( \rho \) has to be identified by the model parameter identification as well, in order to use the transducer’s voltage output instead of two angular positions.

Prior to the identification, the model properties of the measuring device need to be identified. For this purpose, three dedicated auxiliary measurement set-ups were developed to determine mass moment of inertia, torsional stiffness [4], and damping [5]. Based on the measurement results of the three set-ups, all necessary model parameters could be determined.

With this knowledge of the model parameters of the dynamic torque measuring device, it is now possible to identify the unknown parameters of the device under test from measurement data. A summary of all known and unknown model parameters is given in table 1.

<table>
<thead>
<tr>
<th>known parameters of the measuring device</th>
<th>unknown parameters of the DUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>MMOI ( J_{M2}, J_{M1}, J_E ) ( J_H ) ( J_B )</td>
<td>( J_{H2}, J_{H1} )</td>
</tr>
<tr>
<td>torsional stiffness ( c_M, c_E, c_T )</td>
<td>( c_T )</td>
</tr>
<tr>
<td>damping ( d_M, d_E, d_T )</td>
<td>( d_T )</td>
</tr>
<tr>
<td>proportionality factor ( \rho )</td>
<td>( \rho )</td>
</tr>
</tbody>
</table>

The simultaneously acquired data samples of the three measurement channels (transducer signal, angular acceleration at the bottom, angular acceleration at the top) are processed applying a sine fit to identify magnitude and phase of each signal. All three signals are fitted in a combined process, applying a common frequency [7]. As the corresponding
equations are non-linear in their parameters, an iterative fitting algorithm is required.

Based on the results from the three measurement channels, two complex frequency response functions can be calculated (see figure 9). The response function $H_{\text{top}}(i\omega)$ represents the top part of the measuring device and consists of the angular acceleration at the top and the output voltage of the transducer under test

$$H_{\text{top}}(i\omega) = \frac{\rho \cdot \Delta \phi_{\text{HB}}(i\omega)}{\phi_{\text{M}}(i\omega)},$$  

(8)

the other response function $H_{\text{bott}}(i\omega)$ represents the dynamic behaviour of the bottom part of the device under test giving

$$H_{\text{bott}}(i\omega) = \frac{\rho \cdot \Delta \phi_{\text{HB}}(i\omega)}{\phi_{\text{E}}(i\omega)}.$$  

(9)

The model parameters of the transducer under test can be determined from the two transfer functions derived from the ODE system of equation 6. For more details, cf. [7].

$H_{\text{top}}(i\omega)$ comprises the parameters $I_\text{H}, d_\text{r}, c_\text{r}$, and $\rho$, whereas $H_{\text{bott}}(i\omega)$ comprises all parameters of the DUT $I_\text{B}, I_\text{P}$, $d_\text{r}, c_\text{r}$, as well as $\rho$. Consequently, parameter identification is carried out that applies to both transfer functions at once.

The parameter identification can be carried out as a one-step non-linear regression using the frequency response functions, where some of the parameters are linear. Alternatively, it is possible to identify these linear parameters by a linear regression algorithm followed by a consecutive approximation of the non-linear parameters using non-linear methods [7].

9 Summary

Several industrial applications with highly dynamic torque excitation call for dynamic torque calibrations procedures. For this reason, research in this field was carried out as part of the joint research project IND09 of the EMRP. An existing dynamic torque measuring device was improved with a new exciter and a more capable air bearing. A mechanical model of the transducer and an extended model of the measuring device were developed. Three auxiliary measuring set-ups for the determination of the measuring device’s characteristics were developed, and the model parameters of the dynamic torque calibration device were determined. Based on measurement data, the model parameters of the transducer under test can now be determined in the next step. The paper briefly presents the procedures that will be applied to this model parameter identification and the data analysis which is involved.

Acknowledgements

The author would like to thank his colleagues Oliver Slanina (PTB) for the illustrations of the measuring device and the transducers and Barbora Arendacká (PTB) for the helpful recommendations and the support with system simulation and model parameter identification issues.

This work was part of the joint research project IND09 Traceable Dynamic Measurement of Mechanical Quantities of the European Metrology Research Programme (EMRP). The EMRP is jointly funded by the EMRP participating countries within EURAMET and the European Union.

References