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Guide to 3 D pattern fitting in coordinate metrology

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Guide

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1. Introduction

Physical gauges are very important for the inspection of products, as they were already used prior to the development of initial coordinate measuring machines (CMM). The inspection principle is called the Taylor principle. The gauge used – also called functional gauge [1] – consists of two parts. A go gauge determines the maximum permissible limit deviations for the shape of product features. For the inspection of a hole, this is, for example, a cylindrical gauge pin which is to be entirely inserted in the manufactured hole. If the pin gets jammed during insertion, the hole is a reject. The second feature of gauging is the no-go gauge. With this feature, local measures are inspected for permissibility, in contrast to the go gauge. For holes, this includes pins for the inspection of maximum permissible two-point measures of the hole's inner surface. In a similar way, snap gauges are used for the inspection of shafts. The inspection by means of functional gauges is not limited to individual geometric elements, but can also include patterns of several functional features of one product.

In modern productions, the components of several products are usually manufactured at different sites or by suppliers. The permissible shape deviations for workpieces are agreed upon by means of technical drawings with tolerance frames in accordance with the ISO standards 1101 [2], ASME Y14.5M [3] or other specific in-house standards. The ease of assembly of the components can be ensured later on, only if the tolerances in the technical drawings are interpreted in a consistent manner by the client and by the supplier. Therefore, a production which is adapted to the function indispensably requires the inspection of the fitting capability prior to the assembly. This problem is illustrated by the flange in Figure 1.

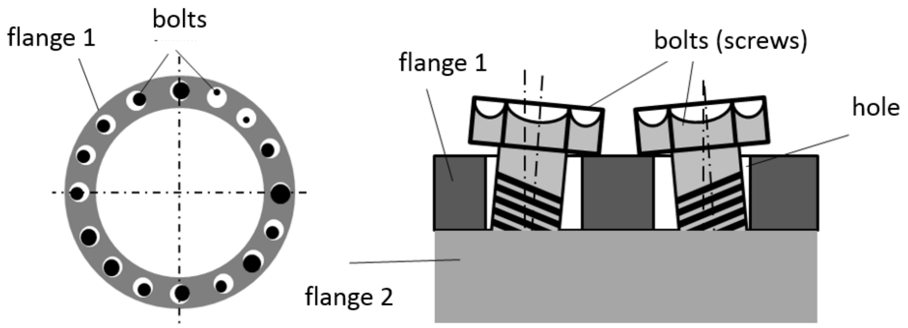

Figure 1 Outline of the assembly requirements for a flange connection with a bolt.

Figure 1 shows a flange with 16 equidistant holes. This flange is to be connected with a second flange via individual bolts in the form of screws. It is evident that the individual bolts of different diameters fit into the appurtenant holes. However, flange connections require all bolts to fit into the appurtenant holes at the same time. The right side of the figure makes the problem even more obvious, as holes and the appurtenant bolts can also be skew to each other. Generally, all shape deviations of the products have an influence on the fitting capability. This includes size, form and position deviations. If the requirements on accuracy are very high, influences by ripple and roughness of the workpiece surface at the mating surfaces must also be taken into account.

The implementation of gauging via CMMs is generally called "virtual" or arithmetical gauging. Here, the physical functional gauge is replaced by a CAD model. The CMM is also used to measure the workpiece surface by probing a finite number of points with the CMM. These data are also referred to as extracted workpiece geometry. Finally, the virtual gauging is implemented by the procedure of the 3D fitting of hole patterns between the extracted geometry and a CAD model of the gauge. Thereby, the term "hole pattern" refers to the 3D elements of size "cylinder" (hole, shaft) and pairs of parallel planes (slot, toung).

When fitting and associating single geometric elements, there is a wide range of standards for a uniform specification of procedures, robust reference algorithms and tests for industry ([4], [5]). In contrast to this, the geometric fitting with several elements in the form of hole patterns and the 3D fitting of hole patterns have hardly been uniformly documented and regulated by standards with regard to metrological and computing processes. Although drawing specifications are defined for the tolerancing of the 3D hole pattern fit in ISO standardization for geometric product specification (GPS), the necessary standard parts for the implementation in a uniform test procedure are lacking entirely. Guidelines on the correct use of different sensors for the metrological detection of measurement points are required, so that the highest points of the elements considered are very likely to be measured. They are significant for the calculation of the quantities and thus for the quality and reliability of product inspection. With this background, optical and CT measurement procedures are of particular importance for the 3D fitting of hole patterns via virtual gauging, as they are suitable to detect the entire surface of a product very rapidly. However, a large part of today's existing standards is only designed for tactile sensors which generally detect a product surface much more slowly.

Most of the inspection tasks which are dealt with by physical gauging or arithmetical gauging with several geometric elements refer to the tolerancing according to ISO 2691 [6]. This standard specifies the maximum material condition (MMC) and least material condition (LMC) for components. The guideline for the application of the standard is that a manufactured workpiece must be able to mate with a counterpart defined by drawing specifications. The implementation of the inspection by MMC and/or LMC in a simulation of assembly is not documented. Consequently, literature provides different approaches (e.g.: [7], [8], [9]) for arithmetical hole pattern fit. The diversity of the algorithms is also reflected in the measuring machine software. Solutions by various manufacturers are incompatible if different algorithms are used or if the procedures to be used have not been determined in the forefront. The general correct procedure is the fitting with all measurement points to the virtual counterpart. Furthermore, very large data sets occur in state-of-the-art coordinate measuring systems using multi-sensor technology, optical-tactile measuring sensors or also CT measuring systems. If, for reasons of efficiency, the software reaches its limits of economic benefit, filters are often used to reduce the measurement data prior to the evaluation. In addition to the fitting algorithm, they also have an influence on the measurement results during hole pattern fit. The task of this Guide to 3D hole pattern fitting is the user-oriented presentation of inspection processes for the arithmetical simulation of assembly with virtual gauging. Thereby, the fundamental requirements of classical physical gauging according to ISO 1101 are transferred to coordinate metrology and recommendations for a software-based implementation are presented. Various coordinate metrology systems with different sensors are considered. Product measurements with dimensional computer tomography (CT) are of special interest, as they allow the entire product surface to be efficiently extracted. Apart from the description of a general procedure in Section 2, the recommendations also comprise details for three special application examples. The first example is a flange which, e.g., has to be manufactured during the construction of wind engines. This flange is presented in Section 3. Based on a technical drawing in which several parallel holes have a common position tolerance with maximum material condition, a rotatory hole pattern fit via cylinders is described. In the sense of the ISO GPS standardization, a geometric ideal datum system is attributed to the flange which determines a rotation axis of the gauge for fitting.

In addition, Section 4 deals with tasks where miscellaneous geometric elements are fitted simultaneously. The example which is presented here is a conical disc with a slot consisting of parallel planes and several parallel holes. A datum is also available. The third application example in Section 5 specifies the fitting of a gauge made of cylinder elements in several holes in a cube whose axes are perpendicular to each other. Here, the case of a free fit will be considered, i.e. datum features which limit the fitting of the gauge geometry are not specified.

2. General Requirements

The 3D hole pattern fit is used for the inspection of specific form and position tolerances of functional product features. This mainly includes the inspection of complete geometries with arithmetical gauging.

In addition, there is a whole range of further applications where integral geometric elements, e.g. axes or median planes, can also be inspected according to the formal principles of 3D hole pattern fit. These principles are referred to in a later suitable section of this Guide. However, this will not be dealt with in greater detail as the necessary procedures and requirements for the inspection are sufficiently covered by the standardization or can simply be taken over by arithmetic gauging.

In Subsections 2.1, 2.2 and 2.3, the most important requirements placed on the hole pattern fit are presented which must be fulfilled when applying the ISO 1101 and ISO 2962 standards. The principle interpretation and supplement of hole pattern fitting for coordinate metrology are shown in Section 2.4. It also includes requirements placed on the evaluation software and relevant numerical procedures. Finally, Section 2.5 deals with the measurement deviations occurring during 3D hole pattern fit. These measurement deviations are significant for the consistency or inconsistency between arithmetical gauging and an inspection during real physical gauging.

2.1. Hole pattern fit in standardization

The subsequent considerations start with the technical drawings of products where the tolerancing according to the ISO 8015 [10] principle has been inserted. Hereby, the application of ISO 1101 and all attributed individual standards will be implied for the inspection of tolerances. The examples given in Figures 2-1 and 2-2 show a plate with two holes that are tolerated in four different ways. For each example, a 3D hole pattern fit will be used for inspection.

Figure 2-1 (left) (Example a) shows a position tolerancing of the median lines of the holes. DIN EN ISO 14660 Part 2 [11] regulates the procedure for the extraction of these lines for cylindrical geometric elements. The tolerance frame is used to define two cylindrical tolerance zones which are represented below the technical drawing. These cylindrical regions have an ideal shape. Their axes are parallel with a nominal distance of 20 mm. In addition, both cylinders are orthogonal to the datum plane A. When inspecting the plate, first the lateral surface A and the holes are extracted in order to attribute the datum surface and the median lines of the holes. During fitting, the tolerance zones are allowed to be shifted along the planes A and to be rotated orthogonally to A. The workpiece is permissible if both median lines are within the tolerance zones at the same time. As the median lines of the holes are constructed, i.e. are not available as directly measurable geometry at the manufactured product, the inspection can only be carried out by means of arithmetical hole pattern fit.

Example b) (Figure 2-1, on the right) inspects the position of the holes according to the maximum material condition. This has been specified with the aid of the drawing specification Ⓜ which is directly behind the value for the position's tolerance. Normatively, the requirements for the inspection via MMC are regulated in ISO 2962. Next, the plate must be fittable by means of a geometrically ideal counterpart which is shown below the engineering drawing. The counterpart is composed of a level contact plane. Two bolts are orthogonal to the contact plane. The axes of the bolts have the nominal distance of the holes and are parallel. The bolt diameter of 4.9 mm is calculated on the basis of the lower size limit of a 5.0 mm hole and the position tolerance. The inspection aims to verify whether the holes of the plate lie within the admissible tolerance. Physical gauging as well as arithmetical gauging can be applied. For the arithmetical hole pattern fit, the level contact surface of the gauging is aligned to the datum surface A of the plate and/or an ideal plane is assigned. At the same time, the two bolts of the gauge must fit in the holes of the plate.

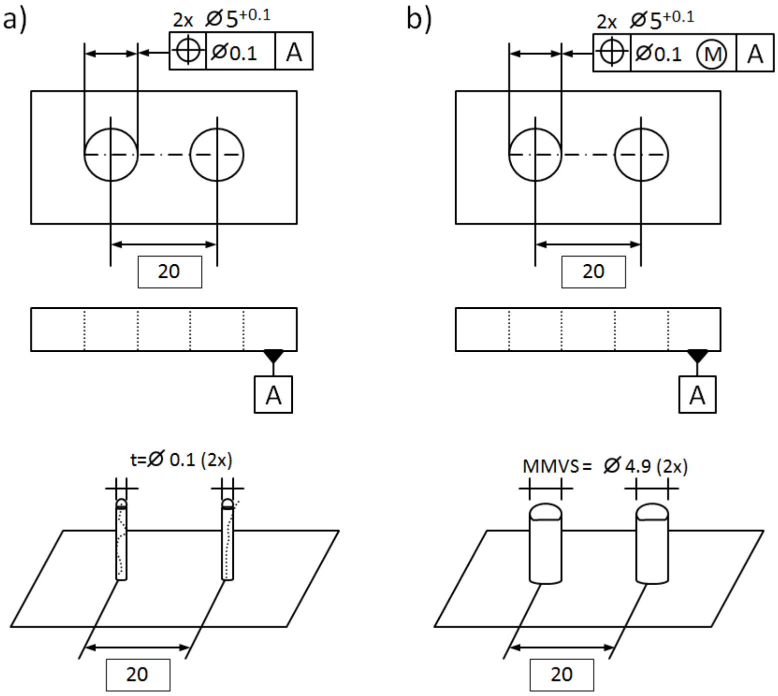

Figure 2-1 Two examples for tolerancing a plate with two holes. The virtual inspection of the tolerances presupposes the 3D hole pattern fit in both cases (Tolerance principle ISO 8015).

Several products have specific functional requirements. In these cases, datum elements with the specification Ⓜ for the MMC are also occasionally entered in drawings. Figure 2-2 shows two examples for this.

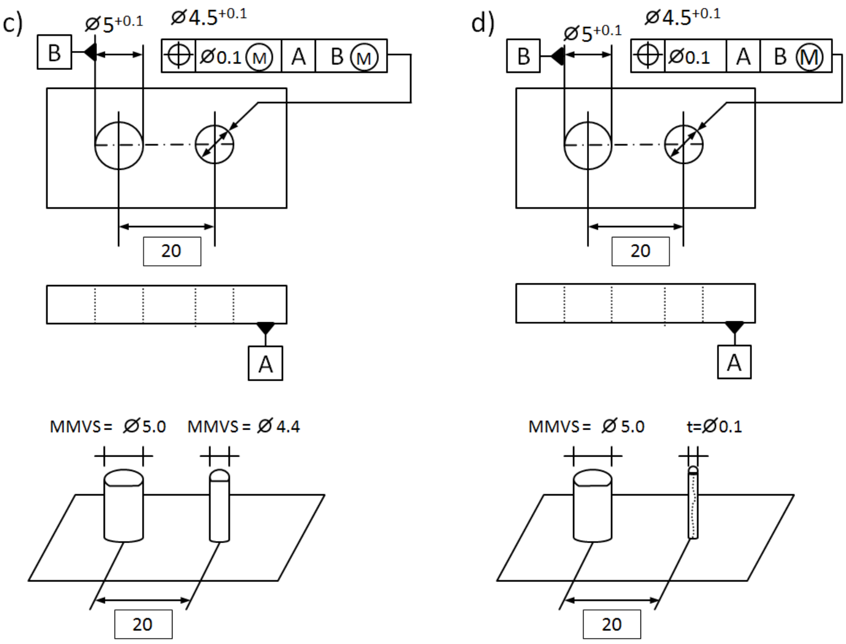

Figure 2-2 Tolerances with datum features including drawing specifications Ⓜ.

In example c), the contact surface A is the primary datum. The left hole is defined as secondary datum B. The right hole has a position tolerance with the specification Ⓜ for the tolerance zone and the datum B. The form of hole B or its orientation to A is not tolerated. For this reason, the quantities are neglected during the toleration. The compliance with the tolerance can be inspected again by classical physical gauging or arithmetical hole pattern fit. The nominal geometry of the go gauge assigned to it consists of a plane which is aligned to the datum surface A. On this datum surface, a gauge cylinder for the datum B is located having a diameter of 5.0 mm which is the lower size limit of the datum hole. The second gauge cylinder has a diameter of 4.4 mm which results from the lower limit of the hole minus the position tolerance. It stands for the position of the hole. Both cylinders are orthogonal to Plane A and their axes have the nominal distance of 20 mm. The classical inspection with physical gauging and the hole pattern fit in coordinate metrology can be applied in the example.

Example d) in Figure 2-2 shows that datum elements with the specification Ⓜ can be identified for the MMC, but that the tolerated element will not be inspected according to the MMC. These drawing specifications only occur in rare applications. The inspection of the tolerance requires an arithmetical 3D hole pattern fit as the extracted median line at the tolerated element does not exist at the real product as directly measurable geometry. A gauge cylinder in datum hole B and the cylindrical position tolerance zone to the median line will be fitted together for this purpose. The cylindrical tolerance zone has a diameter of 0.1 mm. The gauge pin has a diameter of 5.0 mm (the lower size limit of the datum hole). The bolts and the tolerance zone for the extracted axis are orthogonal on the datum plane A. The axes of gauge bolt and tolerance zone of the median line have a nominal distance of 20 mm.

In each of the four examples shown, the 3D hole pattern fit for tolerance inspection has three degrees of freedom. They include two orthogonal translations and one rotation within the datum surface A.

Further explanations refer to applications with the drawing specification Ⓜ for position tolerances and data. The following basic requirements apply for the correct application of drawing specifications according to ISO 2962.

- Only form and position tolerances can be supplemented by the specification Ⓜ.

- Tolerances and datum elements with the specification Ⓜ must refer to integral geometric elements from elements of size, such as the axis of a cylinder or of the median plane of a slot.

- Further elements of size include spheres and – in the two-dimensional case – circles and pairs of parallel lines. In this context, cones and wedges are not elements of size.

- Datum features can be supplemented by the specification Ⓜ. In the case of datum systems, individual datum elements can occur with and without the specification Ⓜ, as long as a Ⓜ element is not followed by an element which is not identified by Ⓜ at the evaluation sequence according to the drawing specification within the tolerance (justification according to ISO 5459 [12] for datum evaluation – uniqueness requirement).

2.2. Determination of the CAD parameters for gauging

The determination of go gauge geometric parameters is relevant for the hole pattern fitting. This means that the CAD model of a geometrically ideal counterpart must be constructed and manufactured as correctly as possible. When inspecting the fitting capability, an attempt is made to fit this part into the product without jamming. In Figure 2, these gauges are shown as sketches in the lower part of the picture, using the elementary example of the hole plate.

When defining geometrically ideal counterparts for gauging according to ISO 2692 in the case of drawing specifications numerous rules must be considered. The created ideal geometric elements which form a CAD model of a gauge are denoted as maximum material virtual condition (MMVC). The size of a geometric element of the gauge is referred to as maximum material virtual size (MMVS). Size, form and position tolerances can be equally integrated in the calculation of the MMVS. The following hand rules can be applied for the determination of the parameters required for the construction of the gauge:

- Gauge geometry for form and position tolerances with Ⓜ
  - External elements of size (shaft, toung):

    The MMVS of the gauge geometry will be formed as sum from the upper size limit of the tolerated geometric element Go and the tolerance for the form or likewise position deviation t.
    MMVS = Go + t
  - Internal elements of size (hole, slot):

    The MMVS of the gauge geometry will be formed as a difference from the lower limit of the tolerated geometric element Gu and the tolerance for form or likewise position deviation t.
    MMVS = Gu - t

- Gauge geometry for datum elements with Ⓜ
  - External elements of size (shaft, toung):
    - The MMVS of the gauge geometry is the upper size limit Go of the datum element for negligible form deviations.
      MMVS = Go
    - If the datum element has an additional form or position tolerance (tolerance value referred to as t) it must be taken into account for the MMVC of the datum element.
      MMVS = Go + t
  - Internal element of size (hole, slot):
    - The MMVS of the gauge geometry is the lower size limit Gu of the datum element for negligible form deviations.
      MMVS = Gu
    - If the datum element has an additional form or position tolerance (tolerance value referred to as t) it must be taken into account for the MMVC of the datum element.
      MMVS = Gu - t

The value for MMVS only determines the measures of the geometric elements which are plugged into or have to encircle the product during the gauging. As a gauge generally accounts for several of such coupling elements, the positions to each other and to a gauge coordinate system must be determined in the following.

The distances between centre points, axes and median planes of the gauge's geometric elements are clearly determined in the technical drawing by means of exact local measures. However, the assignment of a gauge coordinate system is not always obvious and the decision is left to the user. If datum elements are available according to ISO 5459 they can be used to determine the x, y and z axes of Cartesian coordinates, as local measures refer to the datum features in case of a correct drawing. If datum systems are incomplete or if no data are available, individual degrees of freedom remain available for the positioning of the gauge. For example, a single datum plane only defines the direction of one axis and one zero point on this axis. The user can then arbitrarily determine two further coordinate axes in the datum plane and their position. However, it must be ensured that – in the coordinate system provisions – the nominal distances between the gauge elements comply with the requirements in the technical drawing. Only then is it ensured that the fitting results between the different coordinate systems are still compatible with each other.

Further information for the use of the MMC including the drawing specification Ⓜ as well as the correct calculation of gauge complying with the standards are presented in Jorden [1], for example.

2.3. Degrees of freedom for fitting and datum features

So far, the question of how the term "fitting" ("coupling" or "plugging together") of a product and gauge can be formally interpreted has not been answered. For this purpose, the classical physical gauging is initially considered. A tester will try to plug the product together with the gauge. Thereby, he/she puts the product on the edges of the gauge and by simply "shaking" and "tilting" he causes both parts to slide into each other. If a product and a gauge can be plugged together by this procedure to such an extent that the requirements of the inspection are complied with, the product is within the tolerance which is referred to as complete fitting capability.

The CAD model of the virtual counterpart for arithmetical 3D gauging is available in the gauge coordinate system (x_G, y_G, z_G). The measurement points of the extracted product are given in the workpiece coordinate system(x_W, y_W, z_W). Both systems are Cartesian coordinates. The fitting of the gauge to the extracted geometry is described by a linear transformation.

$$T: (x_G, y_G, z_G) \longrightarrow (x_W, y_W, z_W)$$

This transformation maps the gauge geometry into the workpiece coordinate system. Overlapping or empty space can be quantified between the extracted product geometry and the transformed gauge. The virtual gauge and the extracted geometry are completely fitted if there is a transformation which represents a plugged state where there is still empty space between gauge geometry and the measurement points.

In the three-dimensional case, the transformation is defined by rotations – e.g. by means of the Euler angle – and translations towards the three coordinate axes x_G, y_G and z_G. Generally, six parameters are available for the transformation of gauge geometry. The unrestricted selection of a parameter value is referred to as degree of freedom for the fitting. If there are no restrictions for the parameter selection, they are referred to as full degrees of freedom to determine the transformation T.

In many cases, the transformation permitted for the fitting is limited by constraints. In the context of position tolerancing including hole pattern fit there are datum elements which block the degrees of freedom during transformation. If, for example, a datum axis of the workpiece is given, the workpiece coordinate system is initially oriented towards this. The rotation around and a translation along the datum axis then remain as degrees of freedom for the fitting. An exception for the restriction of the degrees of freedom is the datum elements labelled with the symbol Ⓜ.

2.4. Gauging with coordinate measurement systems

In Figure 3, eight individual steps are represented for the implementation of gauging or – in general – 3D hole pattern fit by means of coordinate measurement systems. They are split up in two groups. The first group is the workpiece measurement. It comprises the steps of extraction via the recording of measurement points at the functional surfaces of the product, partitioning and/or a reduction of the measurement points. The second group comprises all steps for the arithmetical evaluation of the hole pattern fit by means of given measurement points. Hereby, data and datum systems are initially set up which are required for the definition of product and gauge coordinate systems. By means of transformation of the measurement points into the nominal position of the gauge the procedure then leads to the actual arithmetical 3D hole pattern fit. In the following, details are described for the individual sub-steps within the scope of coordinate metrology. It should be pointed out that the influence of measurement deviations during product measurement on the arithmetical fit will be discussed in the next section.

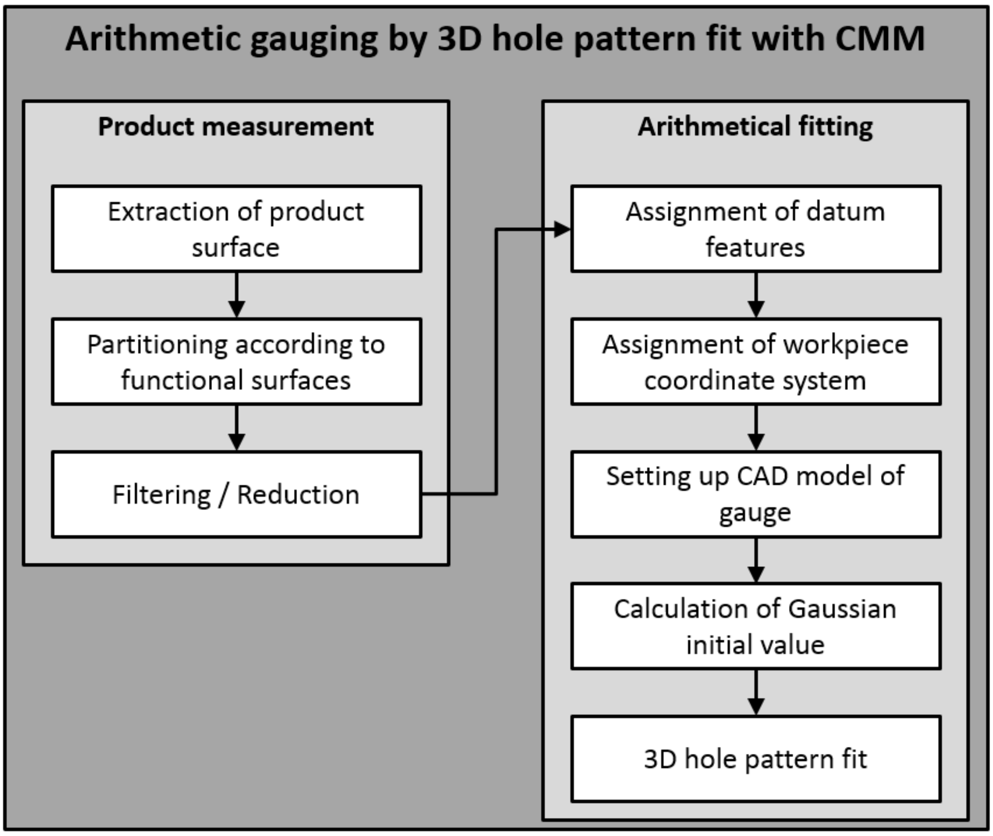

Figure 3 Principal procedure of the arithmetic gauging by means of coordinate measurement machines.

Extraction of the product surface: The measurement strategy has an essential influence on the result of the 3D hole pattern fit. In general, a user has different sensors at his disposal which can be used to extract a product [13]. This includes contact/non-contact probes, sensors that extract the product surface by means of single points, in lines or profiles as well as over the surface, or provide a volumetric image of the product, e.g. the dimensional computer tomography. The scope of application of the different sensors is based on – among other things – product dimensions, the accessibility, material properties, the specified tolerances and the economic aspects, such as costs and celerity.

Ideally, a measuring system should allow measurement points to be densely extracted on the product surface. The denser the measurement points lie together the greater the possibility to extract the surface points which are relevant for the hole pattern fit. They include, e.g. the tops of rippled surfaces. The ripple is thereby caused by the manufacturing procedure. In general, these points are also called "highest surface points". Experience has shown that during the mating of the product and the gauge they are almost adjacent to the gauge or even cause the jamming of gauge and product.

In practice, a user cannot arbitrarily select the fineness of measurement point density. In general, it is limited by the measuring system. Tactile measuring of single points is generally very accurate. During scanning measurements along a driving cycle of the probe, many measurement points with a small distance to each other can be included. However, for surface measurement with the same distance, many individual curves must be run. For this purpose, the time expenditure is very high. This generally makes a complete extraction of the surface uneconomic. Here, alternatives are specific iterative measuring strategies ([14], [15]). Their suitable use requires an additional inspection of the product surface. This includes systematic shape deviations caused by the manufacturing process. The barrel shape of ripples can be a possible deviation. Ripples and roughness are just as relevant.

For surface measuring, optical and CT systems, very large measurement point densities can be achieved as compared to tactile measuring. In case of a sufficient metrological spatial resolution of the sensor, the highest points of the workpiece with small measurement uncertainty will be included.

Partitioning according to functional surfaces: In case of partitioning, the whole extracted geometry of the product is fragmented into the relevant parts. Thereby, the individual measurement points are attributed to the associated datum elements and functional surfaces for 3D hole pattern fit. Different sensors and measurement strategies lead to different requirements made on partitioning.

For tactile-extraction products the partitioning is generally pre-defined by measurement planning, as different surface segments of a product are usually measured successively and the measurement points are assigned to exactly one workpiece feature. On the other hand, the measurement using a CT system initially provides an unpartitioned point cloud for the entire product. It is only in the course of the evaluation that individual measurement points are assigned to the subareas.

At places where product surfaces are merging, it is difficult to clearly determine which points must be assigned to which surface. For this purpose, the assignment of measurement points often dispenses with edges. During partitioning of the measurement points for hole pattern fit, this problem also arises. If a bolt, for example, protrudes vertically from a plane, the partitioning must ensure that no measurement points are added to the bolt from the plane. On the other hand, there are also products for which the partitioning of the edges is uncritical for the hole pattern fit. Here, one can imagine a clearance hole in a plate into which a bolt is fitted. Measurement points at the edges of the hole are then always in the direction of the material side of the hole and do not influence the fitting result. It is even suitable to assign measurement points of the plate's cover side at a small edge around the hole to the measurement points of the hole. It is thereby ensured that all relevant measurement points of the hole's surface are available for the fitting, including the edges.

Filtering and reduction: In some exceptional cases, the partitioned measurement points are not yet suited for further arithmetical evaluation. On the one hand, impurities of the product surface can cause gross measurement deviations. On the other hand, there are measurement errors of the extracted product surface, for example for measurements via CT, which do not exist in reality but which are artificially caused by physical effects during the measurement or product reconstructions. With suitable mathematical filter algorithms such deviations can be removed from the data. This includes both the manual visual inspection for simple products and the application of intelligent filter algorithms in general.

In this context, Gaussian filters are not suited for the elimination of undesired measurement deviations. The highest points are eliminated by Gaussian filters. Their application is thereby very likely to lead to false results in the case of 3D hole pattern fit.

If an evaluation programme can only process a limited number of measurement points in a time-efficient way, it may be necessary to use algorithms to reduce the measurement points in order to use the means of coordinate metrology in a cost-effective way. Here, there is also the risk that unsuitable procedures remove the important highest points and thus lead to imprecise measurement results.

Assignment of datum features: If datum elements are defined, all of them must be associated arithmetically. The procedures for datum evaluation are regulated in ISO 5459. Maximum circumscribed elements and maximum inscribed elements must be calculated as well as adjacent geometric elements according to the Chebyshev criterion. The datum elements determine special points, axes or planes at the extracted product. In connection with the hole pattern fit they later set constraints on the translation and rotation of the gauge. By selecting a suited workpiece and gauge coordinate system, these constraints can be implemented as inhibition of individual transformation parameters during fitting.

Determination of the workpiece coordinate system:
The measurement points of extracted workpiece geometry are considered in a workpiece coordinate system. In a general case, this is identical to the measuring machine coordinate system in which the points at the workpiece surface are included. However, for several applications with hole pattern fit it is suitable or even necessary to assign a coordinate system to the extracted workpiece which is independent of the measuring machine. The measurement point coordinates will then be transformed into the workpiece coordination system before the hole pattern fit takes place. This realization clearly simplifies the hole pattern fit.

In the application examples considered in the Guide, the position of the workpiece coordinate system is oriented towards the geometric datum elements which are assigned to the extracted geometry. If a datum does not clearly define the position and direction of the coordinate system – which is generally the case – it is up to the user to determine the missing degrees of freedom.

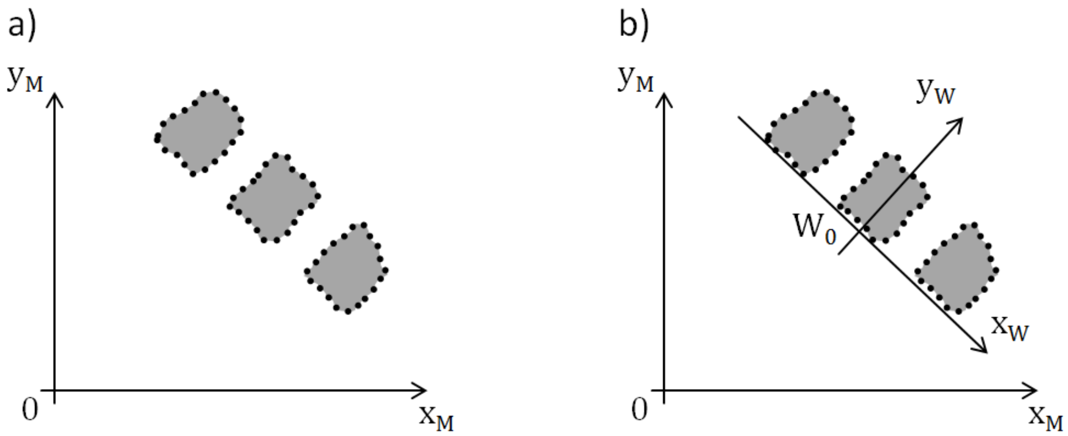

Figure 4 Workpiece datum systems: a) extracted workpiece geometry in measuring machine coordinates, b) Alignment of the workpiece coordinate system at the lower edge of the extracted product.

Figure 4 sketches the assignment of a workpiece coordinate system to a simple plate with two holes. The realization shows a section through the x-y-plane of the coordinate system with the available measurement points. On the left, there is the initial situation. The measurement point coordinate system is referred to as x_M and y_M. A datum plane is assigned to the lower edge of the extracted product. This defines the x direction x_w of the workpiece coordination system. The y direction can be an arbitrary vector which is orthogonal to x_w. In the figure on the right, y_W was selected as the symmetry line of the extracted geometry. The intersection W_0 between the axes of the workpiece coordinate system is the coordinate origin point.

In order to represent the measurement point coordinates in workpiece coordinates, a conversion of the measuring machine coordinate system into the workpiece coordinate system is necessary. This is realized by means of a linear transformation according to the following steps:

1. Translate all points by the coordinate origin W_0 into the zero point $N = (0,0,0)^T$.
2. Rotate all points around the zero point, so that the axes of the workpiece coordinate system are parallel to the Cartesian basis vectors $e_x = (1,0,0)^T$, $e_y = (0,1,0)^T$ and $e_z = (0,0,1)^T$.

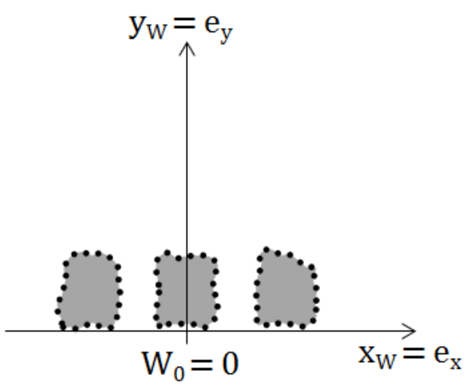

Figure 5 Conversion of the measurement point coordinates for the example from Figure 4.

The conversion of the measurement point coordinates is illustrated in Figure 5. The workpiece coordinate system has the same absolute position to the measurement points as in Figure 4 b). However, the point coordinates are converted so that the workpiece coordinate system is described by the zero point and the basis vectors of a Cartesian coordinate system. If only one single datum element is available, not all axes of the workpiece can be clearly determined. It is then the task of the user to define the remaining axes and/or the coordinate origin. Technically, the symmetry lines of the product and measurement point cloud centre of gravity projected on the datum elements component can be used for a decision. Furthermore, datum elements with the symbol Ⓜ do not define axes of the workpiece coordinate system.

Elaborating a CAD model of the virtual gauge: The CAD model of the geometrically ideal counterpart for hole pattern fit is also set up in a Cartesian coordinate system. Initially, the geometric elements fitted must be identified. From the tolerances, the virtual measures (MMVS) are calculated for each geometric element. Subsequently, the positioning of the elements takes place in accordance with the nominal positions. The absolute position and orientation of the elements towards the coordinate system is generally not clearly determined. Here, the user must decide where the elements lie. For example, the elements can be oriented towards symmetry lines of the product, centres of holes, median planes or lateral surfaces. Nominal datum elements in the engineering drawing used as a basis should absolutely be taken into account during positioning. Datum elements which are identified with Ⓜ belong to the gauge.

Often, the position points of geometric elements are not clearly determined. In the case of a cylinder, For example, each point can be selected on the axis to determine the position. A possible approach in case of an ambiguous definition is to transfer the centres of the geometric elements from the technical drawing.

Calculation of an initial value according to Gauss: At the beginning of the arithmetical fit, the workpiece coordinate system (with the extracted geometry) and the gauge coordinate system are superimposed. In this configuration, the gauge and the workpiece are strongly staggered and twisted. By means of an initial value, the gauge must be roughly oriented towards the extracted workpiece. This allows the arithmetical hole pattern fit to be reliably carried out later on.

The Gaussian method provides very good initial values. A variant is to calculate the transformation of the gauge for all gauge geometric elements and all measurement points at the same time, which minimizes the square sum of the orthogonal distances between points and the gauge. The disadvantage of this approach is that the calculation is very time-consuming for a large number of gauge elements or measurement points.

If the measurement points of the extracted geometry are uniformly and densely distributed on the product surface, a clearly simpler variant can be used for an initial value. This variant considers the centre of the ideal geometry in the technical drawing and the centre of gravity of the extracted geometry from the measurement data for each gauge element. For a uniform and dense product extraction it is expected that these points will lie close to each other for the hole pattern fit. Thereby, a procedure for the initial value is motivated where – in addition to all gauge elements – the sum of the distance squares between the ideal centre points and the centres of gravity are minimized. By reducing the fitting to the centre points, the calculation effort is clearly reduced in comparison to the first initial value variant. Technical details for the approach are provided for the application examples in the following sections.

3D hole pattern fit: In the last step, the numerical core procedure is carried out for the hole pattern fit. This procedure is a fitting application according to the Chebyshev criterion. At the beginning, the relevant measurement point set $P^{(k)}$is attributed to each gauge element k. Value a specifies a vector of transformation parameters for the translation and rotation of the virtual gauge for the extracted geometry. Furthermore, the orthogonal distances between the exterior gauge surfaces and the extracted measurement points are considered. They are called $f_{ki}(a)$. Index k is the geometry element number and index i specifies the measurement point from $P^{(k)}$. If the point is outside the material of the gauge, the distance is $f_{ki}(a) \leq 0$. At this point, the gauge has empty space to the product or touches it just at this point. If the point is in the interior of the gauge, the following applies: $f_{ki}(a) > 0$. The gauge overlaps with the product at this point. The Chebyshev fitting program

$$\min_{a} \max_{k,i} f_{ki}(a) \tag{1}$$

calculates the vector of transformation parameters which minimizes the maximum distance between the measurement points and the gauge.

If there is a transformation a^* with $F(a^*) \coloneqq \max_{k,i} f_{ki}(a^*) \leq 0$, then there is empty space between the gauge and the extracted product. For $F(a^*) = 0$ some measurement points touch the surface of the gauge exactly. In this case, the fitting capability of the extracted product with the CAD model of the gauge has been detected. Program (1) especially determines the transformation a with the biggest empty space $F(a) \leq 0$ and/or with the smallest overlap, if the solution is $F(a) > 0$. In the second case, there is no transformation where gauge and product are completely fittable.

For an arithmetical handling of Task (1), the following notes must be taken into account. These notes are especially aimed at developers of evaluation software for 3D hole pattern fits.

1. Task (1) is a minimax program and belongs to the field of non-linear and non-smooth optimization. The direct numerical handling can be carried out with the respective algorithms – e.g. bundle methods [16].
2. An equivalent formulation for (1) is the ordinary nonlinear optimization program.
$$\min_{a,s} s \text{ s.t. } f_{ki}(a) \leq s \text{ for all } k \text{ and } i \tag{2}$$
It is possible to solve this with the aid of special iterative methods like gradient method, penalty-barrier method or SQP procedure [16].
3. The initial value according to Gauss is recommended as the initial solution for the numerical algorithms to handle (1) or (2).
4. The numerical solver must be adapted in such a way that a calculation can be made in one calculation step with as little measurement points as possible. Thereby, linear equation

systems which must often be solved in subalgorithms must be kept of small dimensions. For example, it is suitable to apply active-set methods. These methods only calculate with the points which fulfil the constraints in (2) with equivalence.

5. The transformations of the gauge are calculated via trigonometric functions. To ensure that they run stable and that arithmetic rounding errors have lower influence on the solution accuracy, at least double floating point number precision should be used for the calculation. The standard IEEE 754 [17] defines the corresponding double precision format which is supported by default in modern computer hardware. In case of critical calculation with susceptibility to rounding errors, number formats and arithmetic with higher precision should be used, if possible.
6. The parallelisation of the solver is possible for the calculation with the distance values $f_{ij}(a)$ and their gradients. For very large measurement point amounts which especially occur for CT measurements, the calculation time can thus be clearly shortened. In the case of the 3D hole pattern fit with double arithmetic precision (IEEE 754 Double Precision) the CPU parallelisation is to be preferred to the GPU parallelisation on graphic boards. In principle, graphic processors are designed for the processing of large data amounts with simple arithmetic operations, however, most of the models available on the market only support a single arithmetic calculation precision. The software-based simulation of a calculation with double precision leads to a very high performance loss which can annul the advantages of the parallelisation.
7.

2.5. Influence of measurement uncertainties on the fitting result

Measurement uncertainties are a measure for the informative value of the 3D hole pattern fit results. According to the methodology in Figure 3, the uncertainty of the 3D hole pattern fit can be subdivided into one component for product extraction and one component for arithmetical fitting. If both uncertainty features are small, as compared to the calculated empty space between the gauge and the extracted product, there is a high probability that the real product lies within the specified tolerance. The same applies for overlapping which is considerably larger than the measurement uncertainty of the procedure. The product is then a reject, with very high probability.

The results of the 3D hole pattern fit – whose measurement uncertainty and empty space or overlapping $F(a)$ lie within the same order of magnitude – require particular attention. In this case, a reliable statement on the fitting capability is not given. It is possible that an arithmetically fittable component does not fit in practice. Vice versa, arithmetically non-fittable features may well be coupled with the counterpart. A safe classification of whether the component lies within the tolerance or is a reject is thus not possible.

For the economical use of the 3D hole pattern fit for the inspection of products, it is thus important to sufficiently know the measurement uncertainties for product extraction and the numerical evaluation, and, if necessary, reduce them by means of suitable procedures and methods to the required precision. As in the previous description of the product extraction – filtering and reduction – this is only possible by means of precise inspection of the product's characteristics and of the measurement process. What is relevant is the geometrical nature of the product surface, e.g. ripple and roughness. Furthermore, the system-specific measurement errors of the measuring machines must be taken into account. The software-based uncertainty can be detected by means of independent algorithm tests. There are appropriate approaches for the assignment of single geometric elements [18]. Tests for overall procedures during the 3D hole pattern fit are under development for this test platform.

3. Application example 1: flanges

The example of "assembly of a flange", as it occurs during the construction of wind engines shows that a seemingly simple geometrical product cannot always be treated in a trivial way. Characteristic for the flange at modern facilities are the large dimensions of more than 5 m of flange diameter as well as the high number of holes on the flange. The arithmetical simulation of assembly requires the extraction of almost the entire external surface of the flange. This is possible, for example with the aid of surface measuring optical sensors. Due to the geometry, the arising data amount is particularly large and too complex for a description of the hole pattern fit.

To show the core requirements for the arithmetical assembly simulation of the flange including 3D hole pattern fit we therefore resort to the simplified model in Figure 6. The principal set-up is equivalent to a flange for wind engines. The measures and the number of the holes are clearly smaller. Furthermore, different measuring systems can be used for the extraction of this type of products.

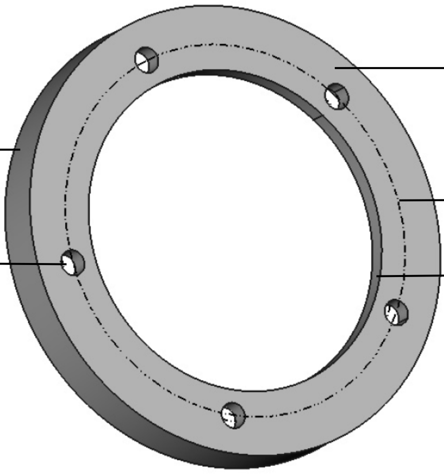

Figure 6 Application example of flange

The application example for the flange is shown in Figure 6. The top surface of the flange is described by two parallel planes at a distance of 8.0 mm. The external surface represents a cylinder with a diameter of 70.0 mm. The internal surface of the flange is described as cylinder with a diameter of 50.0 mm. Both cylinders are coaxial and perpendicular to both top surfaces. On the bolt circle with a diameter of 58.0 mm there are 5 individual holes. Each of them is staggered by 72°. Each hole has a diameter of 4.0 mm.

3.1. Inspection according to the standard

In order to be able to plug the flange, the contact surface (assembly surface) must be level. Furthermore, the holes are required to be orthogonal to the assembly surface. In addition, the bolt circle is concentrical to the axis of the cylinder of the external lateral surface. Thereby, there will be no eccentricity during operation later on which would lead to an increased wear of mobile parts for wind engines.

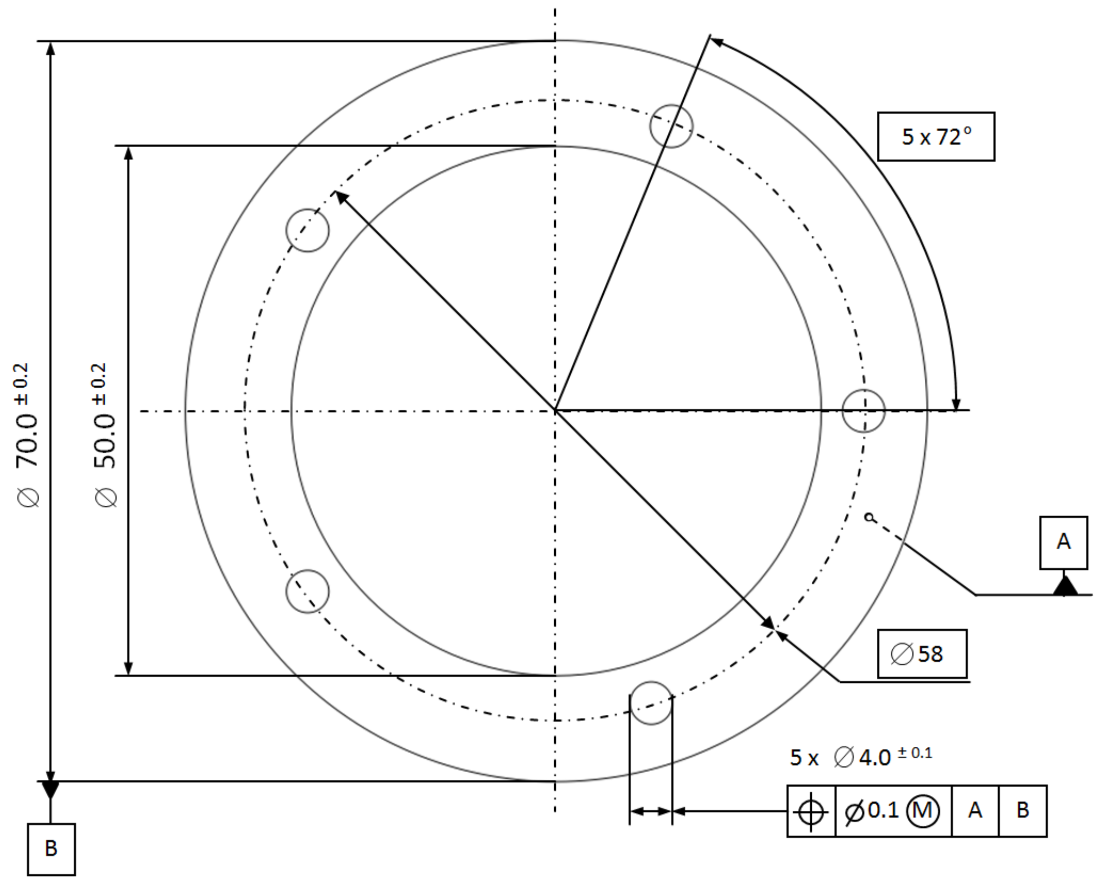

Figure 7 ISO 1101 compliant inspection of the flange.

According to these requirements, an unambiguous unique test task including tolerances of the manufactured flange can be derived. Figure 7 shows a technical drawing according to ISO 1101. The arithmetical inspection results in the following task: An ideal geometrical plane – identified as datum A – must be aligned to the real assembly surface at a minimum distance. Datum B specifies an ideal cylinder that is orthogonal to plane A and whose diameter is the smallest possible diameter where the external lateral surface of the product is completely enclosed, i.e. all measurement points lie within the assigned cylinder. The datum assignment is represented in Figure 8.

Finally, the size and position of the individual holes to the datum surface (datum point) and to the axes of the datum cylinder must be inspected. This is shown in Figure 9. The tolerance from Figure 7 with the symbol Ⓜ for the maximum material condition hereby defines 5 gauge cylinders with the diameter of 3.8 mm (MMVS). The axes of the cylinder are parallel to the datum axis B. They are regularly arranged on a bolt circle with a diameter of 58,0 mm at a distance of 72° segments. The bolt circle is parallel to the datum plane A. Its centre point lies on the datum axis B. A transformation is allowed for the fitting which rotates the gauge around the axis of the datum cylinder. One must check if there is a rotation angle where all gauge bolts are within the holes without overlapping with the material of the flange ring. In this case, the product is within the range of tolerance. On the other hand, the product cannot be assembled due to the inspection result.

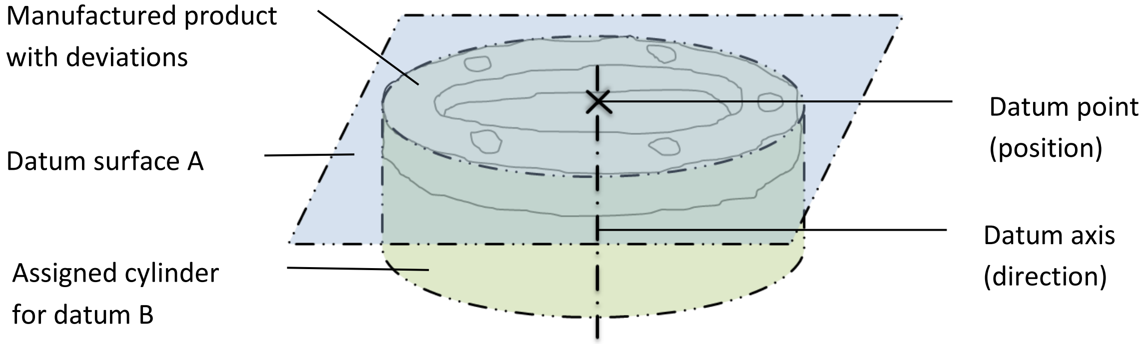

Figure 8 Datum assignment at the manufactured flange.

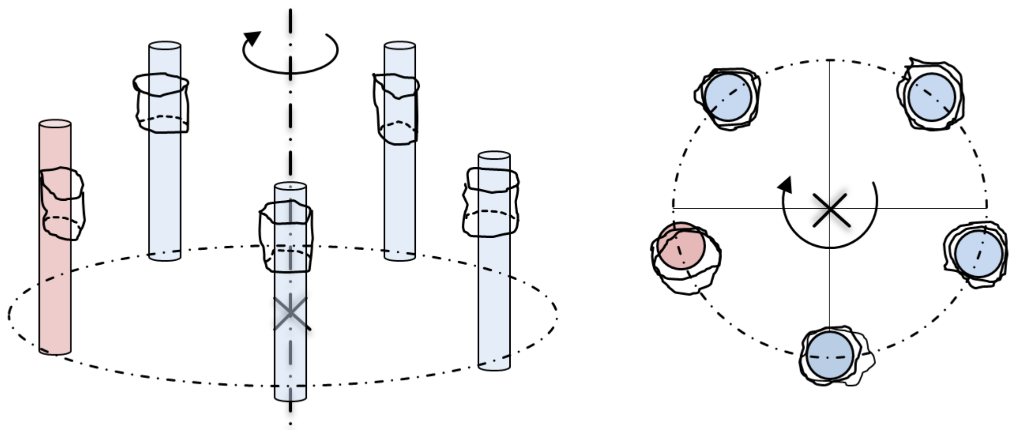

Figure 9 3D hole pattern fit for the flange (on the left: perspective view, on the right: top view); the gauge cylinder highlighted in red overlaps with the hole. By rotating all gauge cylinders around the datum axis, no position is found where all cylinders lie within the holes.

3.2. Mathematical modelling for datum features and fitting

In order to be able to carry out the 3D hole pattern fit arithmetically, a correct and reliable mathematical modelling is fundamental in order to meet the requirements and targets of the inspection. Figure 10 gives an overview at the process of the 3D hole pattern fit for the flange. Here, the calculation of measurands on the basis of the extracted data is shown. The 3D hole pattern fit is carried out in six consecutive steps. Every step is individual and requires an independent calculation method for the finalization.

The starting point is the calculation of the datum elements A and B. They include single geometric elements that are assigned to the extracted flange according to the Chebyshev and likewise Minimum circumscribed condition. A workpiece coordinate system is derived from the datum elements which – apart from a rotation at the workpiece – is uniquely determined. The lacking degree of freedom is determined by a special transformation of the measurement points at the initial position into the workpiece coordinate system. The modelling at the hole pattern fit includes the calculation of the gauge geometry (ideal counterpart for the hole pattern fit), the calculation of an initial solution of the fitting and, finally, the implementation of the hole pattern fit. The goal is to determine the position of the virtual gauge in such a way that maximum empty space and/or minimum overlapping is achieved between the gauge geometry and measurement points. By means of the Gaussian method for the calculation of the initial solution, this target is attained only roughly. The desired position at the gauge can only actually be determined by applying the Chebyshev criterion.

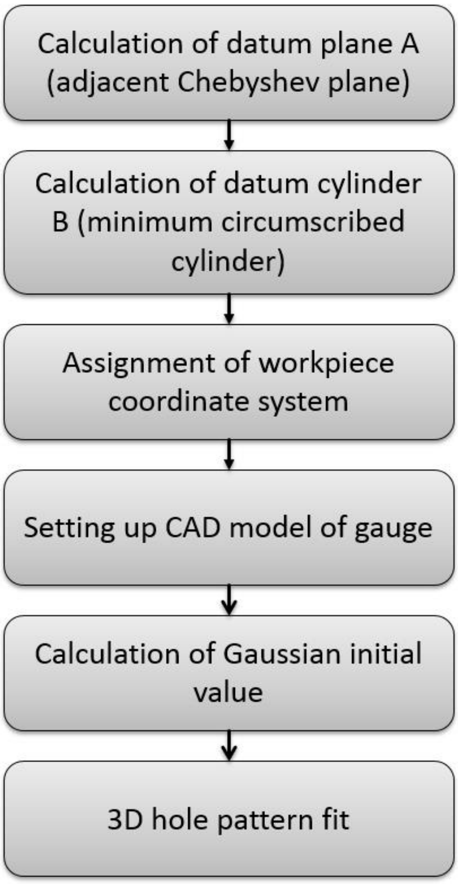

Figure 10 Flowchart of the 3D hole pattern fit for the flange.

The following subsections present details on the mathematical modelling and on the tasks to be solved for each of the 6 steps.

3.2.1. Assignment of the datum plane

The datum plane is assigned to the extracted top surface of the flange as an adjacent Chebyshev plane (minimum zone criterion). The input data for the calculation of the plane are the extracted points of the top surface.

$$P \coloneqq \{P_1, \dots, P_m\}, \;\; P_i \in \mathbb{R}^3$$

Every point has the form $P_i = (x_i, y_i, z_i)^T$. The coordinates x, y and z are specified in the measurement point coordinate system. The assigned plane has an ideal geometrical form. It has been parameterized via the normal vector

$$v = (v_x, v_y, v_z)^T \in \mathbb{R}^3$$

and a point on the plane.

$$C = (C_x, C_y, C_z)^T \in \mathbb{R}^3$$

With this definition, the representation of an ideal plane is not yet unambiguous. For example, for C, every arbitrary point of the plane is possible. Thus, further constraints are made on the parameters. On the one hand, the normal vector is supposed to have the length 1.

$$\langle v, v \rangle = 1. \tag{3}$$

On the other hand, the point of the plane must be located so that the distance to the centroid G of the data P is at a minimum. For this purpose, $v_1, v_2 \in \mathbb{R}^3$ shall be two vectors with a length of 1 which are orthogonal to the normal v. For those, $\langle v_1, v_2 \rangle = 0$ applies. Then, point C is the projected centroid, if

$$\begin{aligned} \langle G - C, v_1 \rangle &= 0 \\ \langle G - C, v_2 \rangle &= 0 \end{aligned} \tag{4}$$

is met. The centroid is calculated as an arithmetic mean.

$$G = \frac{1}{m} \sum_{i=1}^{m} P_i.$$

For the correct assignment of the datum plane the orthogonal distances between the ideal plane and the measurement points are considered. These are as follows:

$$f_i(C, v) = \langle P_i - C, v \rangle$$

The assignment finally takes place according to the following mathematical model.

Assignment task: Chebyshev plane as adjacent datum plane

If P is the extracted geometry of a plane surface. The assigned Chebyshev plane has the parameters C and v which solve

$$\min_{C,v} \max_{i} |f_i(C, v)| \tag{5}$$

and meet the constraints (3) *and* (4)*. For $s = \max_{i} |f_i(C, v)|$, the adjacent Chebyshev datum plane with the parameters $\hat{C}, v$ is calculated by means of the translation*

$$\hat{C} = C + s \cdot v. \tag{6}$$

The sign of v must be determined in such a way that this vector points away from the material side of the extracted geometry for the calculation of (6).

3.2.2. Assignment of the datum cylinder

The input data for the cylinder calculation are the points measured at the outer flange surface. In the case of the planes, they are simply called P. The assigned cylinder has a geometrically ideal form. It is parameterized by a parameter for the direction of the cylinder axis

$$v = (v_x, v_y, v_z)^T \in \mathbb{R}^3,$$

a parameter for the position of the cylinder axis

$$C = (C_x, C_y, C_z)^T \in \mathbb{R}^3$$

and the radius of the cylinder lateral surface

$$r > 0$$

As the cylinder is assigned as secondary datum element, the following constraints apply for the parameters. The direction of the cylinder axis corresponds to the normal vector of the datum plane previously determined. For the unambiguous calculation of the position, a further criterion for the position of the point on the cylinder axis must be given. This point can be the intersection between the axis and the datum plane.

The assignment of the cylinder parameters is carried out according to the minimum circumscribed element criterion. For its mathematical formulation the orthogonal distances

$$f_i(C, v, r) = \|(P_i - C) \times v\| - r$$

between the measurement points from P and the lateral surface of the ideal cylinder are considered. The following task must be solved for the correct calculation of the cylinder parameters.

Assignment task: envelope cylinder as secondary datum vertically to a plane:

Determine the point C and the radius r in such a way that

$$\min_{C,r} r \qquad s.t.\ f_i(C, v, r) \le 0 \ \ for\ 1 \le i \le m \tag{7}$$

is obtained.

3.2.3. Workpiece coordinate system and virtual gauge

In the case of the flange considered here, the virtual gauge for the hole pattern fit must only be rotated around the axis of the datum cylinder. As the axis can generally be tilted in space, this can only be solved with great technical effort. By suitably assigning a workpiece coordinate system and the suitable geometry of the virtual gauge, the fitting can be technically realized more easily.

Assignment of the workpiece coordinate system

The axis of the envelope cylinder (7) is the basis for the definition of the workpiece coordinate system. This example specifies that the direction vector v defines the z axis z_W of the workpiece. Likewise, point C from (7) is defined as the centre point of the workpiece coordinate system. The remaining axes x_W and y_W are not clearly determined by the datum system. It is only during the transformation of the extracted geometry into the workpiece coordinate system – which is still incomplete at that point in time – that they are specified. The transformation shown here is a possible variant for this. There are also other approaches which lead to the same fitting result later on, however, they will not be discussed here.

The conversion of point coordinates $P_i \in \mathbb{R}^3$ into the workpiece coordinate system is implemented by the linear transformation

$$\hat{P}_i = R(P_i - C).$$

The 3x3 matrix R defines a rotation which map the direction vector v of the datum cylinder on the basis vector $e_z = (0,0,1)^T$. As v corresponds to the z axis of the coordinate system, the following must be valid

$$Rv = e_z.$$

By means of the Tait-Bryan angles for rotations around the x and/or y axis

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) \\ 0 & \sin(\alpha) & \cos(\alpha) \end{pmatrix} \text{ and } R_y = \begin{pmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{pmatrix}$$

$$R := R_y^T R_x^T$$

is created. The point coordinates $\hat{P}_i$ then lie in the workpiece coordinate system with the origin point $W_0 = (0,0,0)^T$, x axis $x_w = (1,0,0)^T$, y axis $y_w = (0,1,0)^T$ and z axis $z_w = (0,0,1)^T$.

The following algorithm provides a numerically more stable procedure for the calculation of the two Euler rotation angles α and β.

Algorithms for the calculation of the rotation angles for the coordinate transformation

Step 0: *Set a positive accuracy* $\varepsilon \ll 1$.

Step 1: *Calculate* $t = \sqrt{v_y^2 + v_z^2}$

If $t < \varepsilon$, *set* $cos(\alpha) = 1$ *and* $\sin(\alpha) = 0$.

If not, set $cos(\alpha) = \frac{v_z}{t}$, $\sin(\alpha) = \frac{v_y}{t}$ *and* $v_z' = \text{t}$.

Step 2: Calculate $t = \sqrt{v_x^2 + v'^2_z}$

If $t < \varepsilon$, *set* $cos(\beta) = 1$ *and* $\sin(\beta) = 0$.

If not, set $cos(\beta) = \frac{v'_z}{t}$ *and* $\sin(\beta) = \frac{v_x}{t}$.

The limit for the precision ε determines when a component of a vector v already lies close enough to the direction achieved of the z axis. In this case, no further rotation of the data is done.

The transformation must be implemented for all measurement points which have been assigned to the cylindrical holes on the flange. Each hole is available with its own set of measurement points.

$$P^{(1)} = \left\{P_1^{(1)}, \dots, P_{m_1}^{(1)}\right\},$$

$$P^{(2)} = \left\{P_1^{(2)}, \dots, P_{m_2}^{(2)}\right\},$$

$$P^{(3)} = \left\{P_1^{(3)}, \dots, P_{m_3}^{(3)}\right\},$$

$$P^{(4)} = \left\{P_1^{(4)}, \dots, P_{m_4}^{(4)}\right\},$$

$$P^{(5)} = \left\{P_1^{(5)}, \dots, P_{m_5}^{(5)}\right\}$$

For simplification reasons, the index k with $1 \le k \le 5$ will be introduced for the assignment of the measurement point sets to the individual holes in the following. It is written as $P^{(k)}$ and/or $P_i^{(k)}$ with the coordinate values $P_i^{(k)} = (x_{ki}, y_{ki}, z_{ki})^T$. Furthermore, the values m_k stand for the number of points of the extracted geometric element with the index k. The transformation of the measurement data is $\hat{P}_i^{(k)} = R\left(P_i^{(k)} - C\right)$. The extracted geometry is outlined with the workpiece coordinate system in Figure 11.

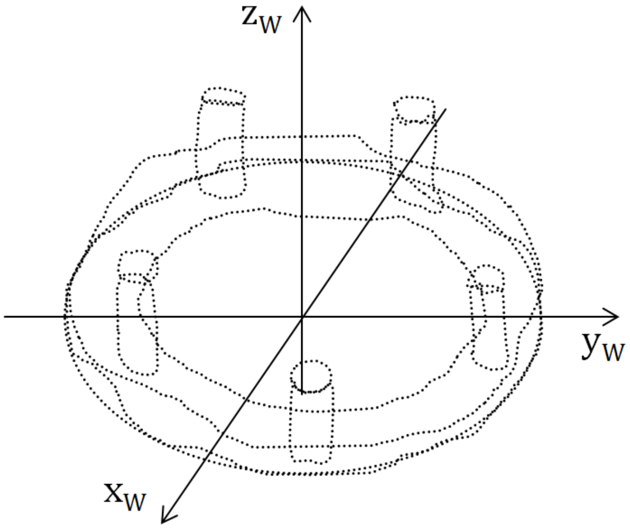

Figure 11 Workpiece coordinate system and extracted geometry at the flange.

To simplify things, the further notation $P_i^{(k)}$ is used instead of $\hat{P}_i^{(k)}$ for the point coordinates in the workpiece coordinate system referring to the respective context.

Specification of the virtual gauge:
The virtual gauge consists of 5 cylinders with an ideal geometrical form. All cylinder axes are parallel with the common direction vector $v = (0,0,1)^T$. Likewise, every cylinder has the same radius $r = 1.9$ mm. However, the positions of the individual cylinder axes

$$C_1, \dots, C_5$$

are different. They are also referred to as C_k $(k = 1, \dots, 5)$. The nominal position is carried out by means of the specified bolt circle with the radius $r_L = 29$ mm and the angular distance $\tau = 72°$ to

$$C_k = \begin{pmatrix} C_{kx} \\ C_{ky} \\ C_{kz} \end{pmatrix} = \begin{pmatrix} r_L \cos(k\tau) \\ r_L \sin(k\tau) \\ 0 \end{pmatrix}$$

The following matrix formulation is suitable for the storage of the geometric parameters of the gauge.

$$M = \begin{pmatrix} C_{1x} & C_{1y} & C_{1z} & v_x & v_y & v_z & r \\ C_{2x} & C_{2y} & C_{2z} & v_x & v_y & v_z & r \\ C_{3x} & C_{3y} & C_{3z} & v_x & v_y & v_z & r \\ C_{4x} & C_{4y} & C_{4z} & v_x & v_y & v_z & r \\ C_{5x} & C_{5y} & C_{5z} & v_x & v_y & v_z & r \end{pmatrix} \tag{8}$$

Note: The formulation of (8) requires that the normal vector of the datum plane A points towards the material side of the flange. However, if the datum direction is turned, i.e. if the normal vector points away from the material, then the sequence of the entries in the parameter matrix changes. Then, C_5 to C_1 must be entered instead of C_1 to C_5.

3.2.4. Initial value and 3D hole pattern fit for the flange

The starting point for the modelling of the 3D hole pattern fit for the flange are the measurement points of the holes $P^{(1)}, \dots, P^{(5)}$ in the workpiece coordinate system and the matrix with the parameters of the gauge geometry M.

The free parameter of the fitting is the rotational angle φ which turns the gauge or, likewise, the gauge parameter around the z axis in the coordinate origin of the coordinate system. The angle determines a special rotary matrix H.

$$M(\varphi) = M \cdot H(\varphi)$$

There is

$$H(\varphi) = \begin{pmatrix} co & si & 0 & 0 & 0 & 0 & 0 \\ -si & co & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

with the values $si = \sin(\varphi)$ and $co = \cos(\varphi)$. In addition,

$$C_k(\varphi) = \left(C_{kx}, C_{ky}, C_{kz}\right)^T \cdot \begin{pmatrix} co & si & 0 \\ -si & co & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is used for the rotated position points of the gauge cylinders in order to simplify the notation for the mathematical modelling of the fitting task.

In the following subsections, the mathematical models are described for a consistent 3D hole pattern fit. The procedure starts with a Gaussian-type coarse fitting which is used as the starting solution for the exact fit according to the Chebyshev criterion (1).

Starting solution with a Gaussian fitting

In a first step, the centroid $Q_k = \left(q_{kx}, q_{ky}, 0\right)^T$ is calculated for every point set $P^{(k)}$. Thereby the following applies:

$$q_{kx} = \frac{1}{m_k} \sum_{i=1}^{m_k} x_{ki}$$

and

$$q_{ky} = \frac{1}{m_k} \sum_{i=1}^{m_k} y_{ki}.$$

The calculation is also efficient for large data sets. In order to determine the starting position of the gauge, the angle φ_0 is calculated by means of a best fitting which solves the minimization program

$$\min_{\varphi_0} \ \tfrac{1}{2} \textstyle\sum_{k=1}^{5} \| C_k(\varphi_0) - Q_k \|^2 \tag{10}$$

The sum of the distance squares between the centre point of the gauge cylinder axes and the point cloud centroids is hereby minimized. This coarse fitting is shown in Figure 12.

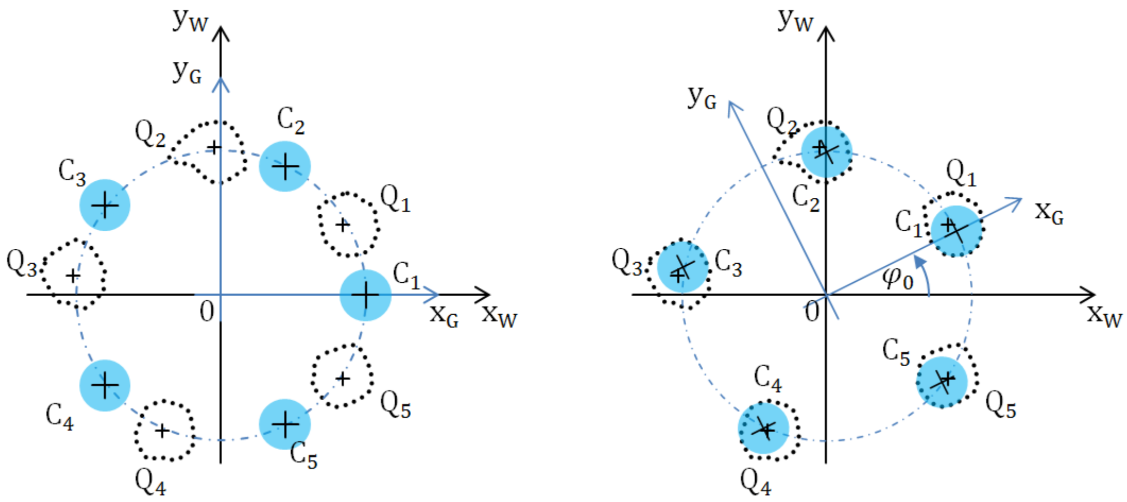

Figure 12 Coarse fitting at the flange

On the left, the initial situation where the workpiece coordinate system and the gauge are superimposed is shown. Measurement points and gauge cylinders are shown in the top view on the x-y-plane. The gauge cylinders coloured in blue are clearly staggered to the black dotted extracted holes. On the right side of the figure, the distances between the gauge centre points and the point cloud centroids were minimized. Thereby, the gauge coordinate system was rotated around the angle φ_0.

Calculation of the 3D hole pattern fit

The measurement points at the 5 individual holes describe cylinders which – due to measurement and manufacturing deviations – do not have an ideal geometric shape. In general, there are deviations of measure, shape and position. They influence the result of the previous Gaussian fit. The angle φ_0 calculated from the initial value (10) is not yet the angle with the smallest possible overlap or largest possible empty space between the gauge cylinders and the measurement points available.

For the calculation of the 3D hole pattern fit, the orthogonal distances between the gauge cylinders and the measurement points of the holes are defined as follows:

$$f_{ki}\big(M(\varphi)\big) = r - \left\| \left(P_i^{(k)} - C_k(\varphi) \right) \times v \right\|$$

Index k is the number of the hole and index i specifies the number of the measurement points to the hole k. The norm is the Euclidean standard norm in the $\mathbb{R}^3$.

The application of the general fitting program (2) from Chapter 2.4 thereby provides the 3D hole pattern fit for the flange (11).

$$\min_{\varphi\in\mathbb{R}, s\in\mathbb{R}} s \qquad s.t. \quad f_{ki}(\varphi) \le s \ \ \forall\, k = 1,..,5 \ \ and \ \forall\, i = 1, \dots, m_k \tag{11}$$

The rotation angle φ calculated from this program and the maximum distance s is the determination of the quantity searched – minimum overlapping or maximum empty space – and easy to realize. The following statements are valid

- If $s > 0$, there is an overlap between the gauge and the measurement points. It has the value s.
- If $s < 0$, there is empty space between the gauge and the measurement points. It has the value s.

- If $s = 0$, the gauge is adjacent to the measurement points. There is neither empty space nor overlap.

Further comments:

(a) The procedure (11) for 3D hole pattern fit including the Chebyshev criterion does not lead to a correct solution for arbitrary starting angles φ_0.

(b) By means of the individual values of the distances

$$s_k := \max_{\mathrm{i}=1,..,\mathrm{m_k}} f_{ki}(\varphi)$$

it can be individually realized which holes, if necessary, must be processed further, and where and how, so that the product is within the tolerance.

3.4 Generalization of the test task for the simulation of assembly

A further application of 3D hole pattern fitting exists for the simulation of assembly of two parts. The flange according to Figure 4 and the counterpart in Figure 13 are given.

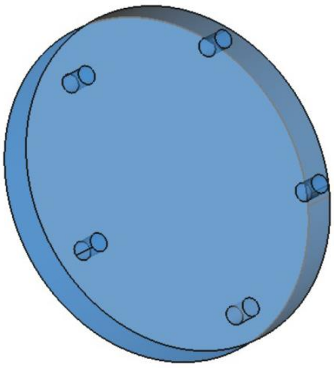

Figure 13 Counterpart for flange assembly.

The counterpart for the flange is defined by a cylindrical disc with a diameter of 70.0 mm and a height of 10.0 mm. 5 bolts with a diameter of 3.8 mm are arranged perpendicularly to the surface of the disc on a circle with a diameter of 58.0 mm. The radial distances between the holes are 72°, respectively. The circle is concentrical to the axis of the disc.

The basic idea of the inspection procedure is to extract the manufactured product in a first step and to assign ideal geometric elements to it in a suitable manner. These elements then form a virtual gauge. In a second step, the arithmetical 3D hole pattern fit is used to inspect whether these virtual gauges and the extracted flange can be fitted to each other.

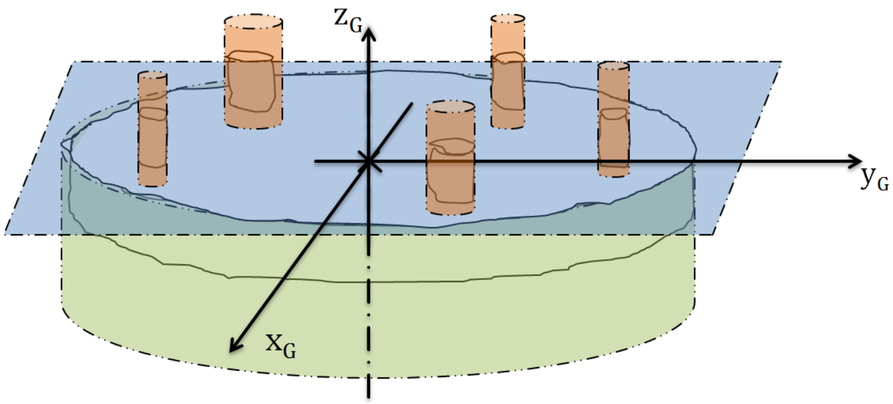

Figure 14 Assignment of the virtual gauge to the counterpart.

Figure 14 illustrates the assignment of the gauge to the product. The measured features of the counterpart are used to construct a virtual gauge. At the surface of the counterpart disc, an adjacent plane is assigned according to the Chebyshev criterion. The latter is marked in blue. To determine the centre of the gauge – which is required as a position for the later fitting – a minimum circumscribed cylinder is calculated for the lateral surface of the disc whose axis runs orthogonally to the assigned plane. This cylinder is marked green. In a last step of the gauge calculation, a surrounding minimum circumscribed cylinder is assigned to each extracted bolt (yellow). The axes are perpendicular to the adjacent blue plane in the form of a constraint. The gauge coordinate system (x_G, y_G, z_G) is applied parallel to the axis of the external cylinder. The point of origin is located in the assigned plane.

With the gauge geometry measured from the counterpart, the hole pattern fitting to the flange is carried out as described at the beginning of this chapter. As the position and the measurements of the gauge cylinder can vary, the arithmetical counterpart generally no longer has rotation symmetry. Therefore, 5 different starting positions must be inspected for the virtual gauge. This is achieved by means of cyclically exchanging the assignment between measurement points of the holes and the gauge cylinders.

Generally, the procedure can also be transferred to the assembly of two flanges including bolts. Thereby, one of the flanges is treated like the gauge part. Instead of circumscribed cylinders, inscribed cylinders are assigned to the holes. Each of these cylinders must have a minimum diameter in order to push the bolts through. The second step of the arithmetical assembly simulation is carried out by means of 3D hole pattern fit.

4. Application example 2: conical discs

The conical disc describes the fitting of several groups of geometric elements including different sizes and shapes. Generally, the arrangement of the elements is not rotation-symmetrical. The application example is shown in Figure 15.

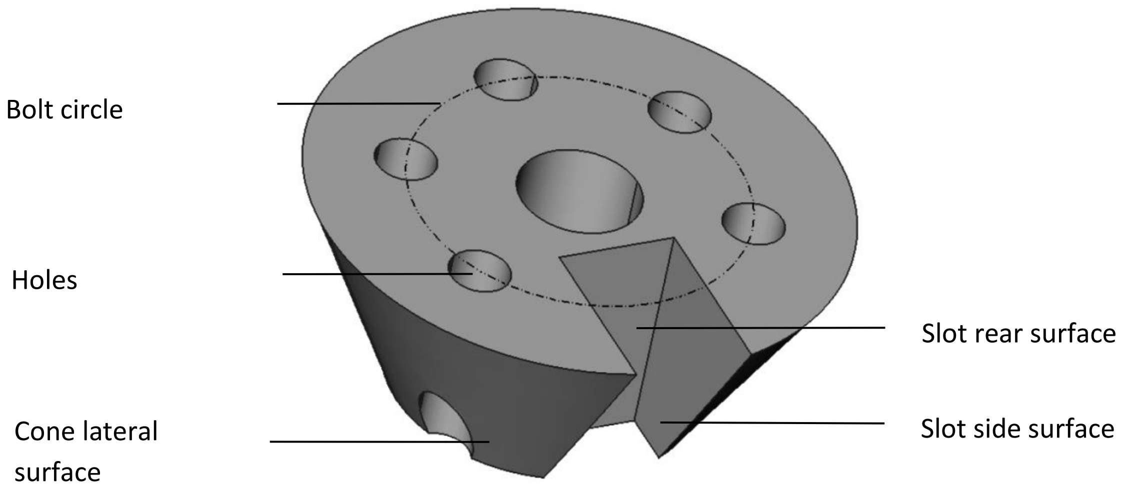

Figure 15 Example of a conical disc

A truncated cone forms the base geometry. The diameters at the two circular ends are 70.0 mm and 40.0 mm. The height of the cone is 30.0 mm. Parallel to the cone axis, five cylindrical holes have been integrated. Each hole has a diameter of 8.0 mm. They are situated on a bolt circle. This bolt circle is concentrical to the conical axis and has a diameter of 44.0 mm. Each axis of the five holes is arranged around 60° segments of the bolt circle. Vis-à-vis of the holes is a slot. The internal surfaces of the slot include a rear and two side surfaces. They are parallel to the conical axis. The width of the slot is 16 mm. Furthermore, the lateral surfaces are oriented symmetrically to the conical axis. The rear surface has a distance of 11 mm to the conical axis. The model corresponds to the typical industrial applications. The cone is a specific kind of mating part. The tapering cone shape leads to a centring when plugged on a similarly shaped counterpart. For example, such fits are used for drill chucks for assembly via a transmission shaft. By means of the slot, an orientation of the conical part is possible. The holes have other functional tasks. For example, they can be used as clearance holes for screws or for running lines through.

4.1. Inspection according to the standards

A simple inspection task on the conical disk is shown in Figure 16.

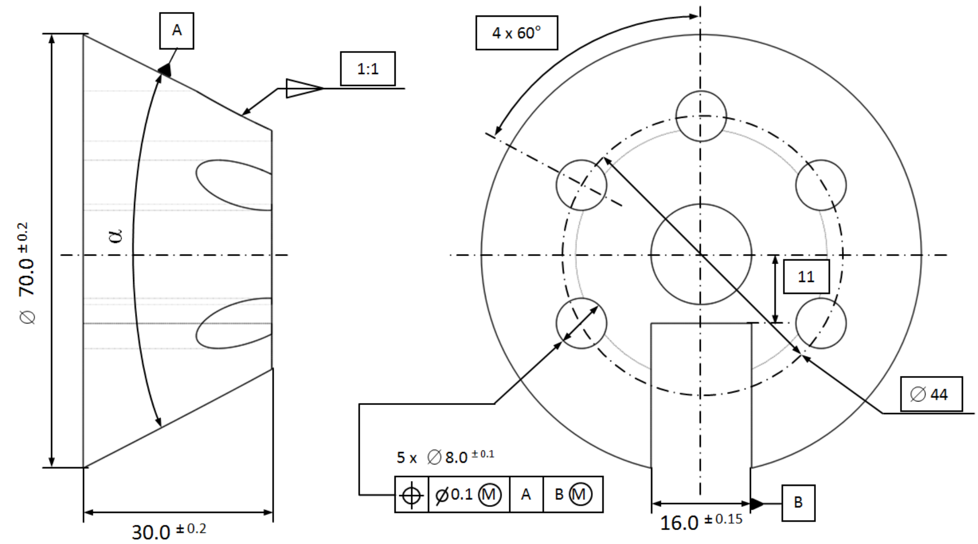

Figure 16 Tolerancing for the conical disc (basis ISO 1101).

The position of the five holes is inspected on the common bolt circle according to MMC. In addition, there is reference datum system available. The conical lateral surface forms the primary datum A. This corresponds to a centring conical fit during the assembly. The drawing specifications for a cone have been implemented according to DIN EN ISO 3040 [18]. The nominal cone angle $\alpha = 2 * arctan\left(\frac{c}{2}\right) \cong 26.565°$ is calculated by the taper ratio $c = 1:1$.

The assignment of a geometrically ideal cone for the external lateral surface in order to form the datum A is shown in Figure 17. The cone angle α has been specified for the assignment. It is used to determine the cone parameters in such a way as to minimize the largest distance between the manufactured cone lateral surface and the ideal cone lateral surface. It is externally adjacent to the product. The assignment is specified as Chebyshev assignment within the scope of the Guide.

The slot forms the secondary datum B. The additional specification Ⓜ following the datum A in the tolerance frame indicates that B must be considered as part of the gauge for the inspection of the product. The gauge required for the inspection of the product is sketched in Figure 18. The left part of the figure shows the extracted geometry and the gauge in a perspective view. The lateral surfaces of the gauge prism for the slot is marked in green. Their theoretical exact width is 15.85 mm. The blue cylinders are gauge counterparts for the holes. They have a nominal exact diameter of 7.8 mm. Both the lateral surfaces of the slot and the cylinders are parallel to the axis of the cone. The extent of the surfaces and the length of the cylinders are theoretically unlimited. However, they are represented in truncated form in the figure. The manufactured product only shows the functional surfaces that are relevant for the assembly. They include the inner surfaces of the holes and the lateral surfaces of the slot. The degree of freedom for the fit is the rotation of the whole gauge geometry around the axis of the datum cone. A shift along the axis is not permitted. This degree of freedom is blocked via the datum point of the cone apex. The given example sketches a position of the gauge to the product geometry with the complete fitting capability (empty space between the product and the gauge).

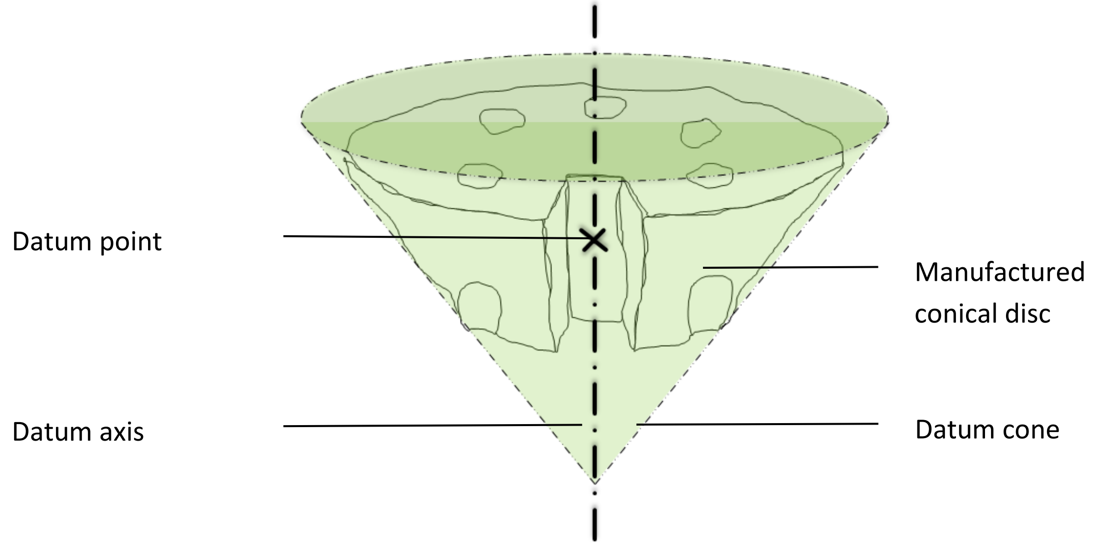

Figure 17 Datum assignment at the manufactured conical disc.

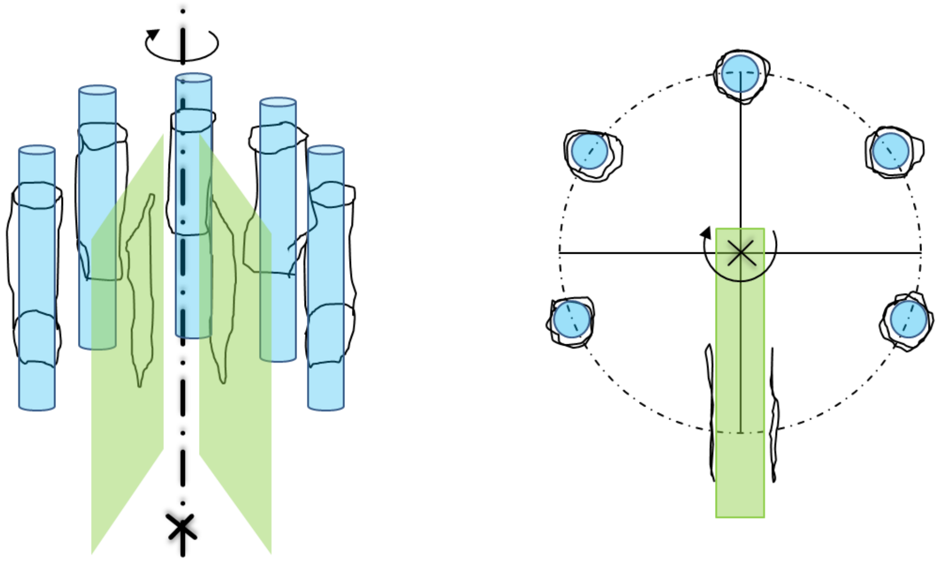

Figure 18 3D hole pattern fit for the conical disc.

Additional form and position tolerances are not available for the conical disc and the slot. Hereby, an elementary tolerancing of the product is considered. For practical applications, further tolerances must be taken into account for datum creation. See Section 2.3.

4.2. Mathematical model of datum creation and fit

In this section, the precise procedure of arithmetical 3D hole pattern fit is described for the application example 2, the conical disc. The starting point is the extracted product geometry which, for example, has been determined by means of a coordinate measurement system. To implement the 3D hole pattern fit, the steps shown in Figure 19 must be completed.

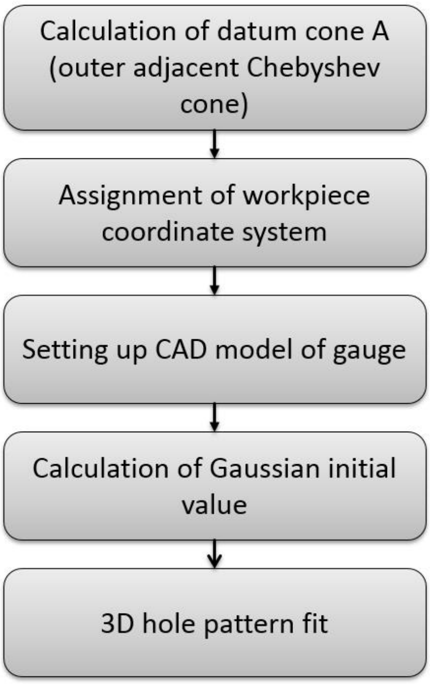

Figure 19 Flowchart for 3D hole pattern fit for a conical disk.

The following subsections clarify details on the individual steps of the arithmetical fit.

4.2.1. Assignment of the datum cone

The cone as a datum element must be fitted with its nominal value of the cone angle according to ISO 5459. The assignment is implemented according to the Chebyshev criterion. The latter aims at the minimization of the largest distance between the product's surface and the assigned cone's lateral surface. The lateral surface of the conical disc is extracted for the fit. The $m \in \mathbb{N}$ measurement points should be available, whereby m≥6 is assumed. The measurement points are referred to as follows:

$$P = \{P_1, \dots, P_m\}$$

The individual points have the coordinates $P_i = (x_i, y_i, z_i)^T$ for all $i \in I := \{1, \dots, m\}$. The geometry of the assigned cone is described by three parameters. The first parameter is the direction of the cone axis.

$$v = \left(v_x, v_y, v_z\right)^T.$$

It points towards the cone apex. The definition requires

$$v^T v = 1 \tag{12}$$

i.e., the direction vector is standardized to the length 1. The second parameter is the position of a point on the cone axis, specified by the coordinates.

$$C = \left(C_x, C_y, C_z\right)^T.$$

As the position of the point can be arbitrarily situated on the cone axis, a further constraint was introduced which exactly determines the position. The centroid G of the point cloud projected on the cone axis is thus especially robust as a definition of C. It can be calculated from the condition

$$\langle G - C, v \rangle = 0 \tag{13}$$

whereby

$$G = \frac{1}{m}\sum_{i=1}^{m} P_i.$$

The third parameter is the cone radius in the point C. The latter is designated by the symbol

$$r > 0.$$

The parameter is the radius of the circular cross section orthogonal to the cone axis. The circular cross section has the centre C. Furthermore, the cone geometry is described by the cone angle of $0 < \alpha < 180$.

Next, the orthogonal distances between the lateral cone surfaces and the extracted measurement points are considered. Their definition is

$$f_i(C, v, r) := (e_i - r)\cos\left(\frac{\alpha}{2}\right) - d_i \sin\left(\frac{\alpha}{2}\right)$$

with $e_i := \|(P_i - C)\times v\|$ and $d_i := \langle P_i - C, v\rangle$. The values are defined for all $i \in I$. Finally, the following mathematical assignment task results for the datum cone:

Chebyshev assignment for an adjacent cone:
Determine the parameters $C \in \mathbb{R}^3, v \in \mathbb{R}^3$ and $r > 0$ with (12) *and* (13) *which* solve

$$\min_{C,v,r} \max_{i\in I} |f_i(C, v, r)| \tag{14}$$

The externally adjacent Chebyshev cone is calculated from the solution of (14) *by the translation*

$$\hat{C} = C + \gamma \cdot v \tag{15}$$

The value for the translation factor is

$$\gamma = \frac{\max_{i\in I} |f_i(C,v,r)|}{\sin\left(\frac{\alpha}{2}\right)}. \tag{16}$$

4.2.2. Workpiece coordinate system and definition of the gauge geometry

In case of the flange, the separate determination of a workpiece coordination system leads to the simplification of the fitting model. A similar situation is the example of the cone disc. Here, the hole pattern fit has the constraint that the virtual counterpart (gauge) can only be rotated around the axis of the datum cone.

Determination of the workpiece coordinate system

The z axis of the workpiece coordinate system is determined as the direction of the cone axis v from (14). The coordinate origin point $W_0$is the projected centroid C on the cone axis. The remaining axes x_W and y_W are not clearly defined by the datum element. Again, they are caused by the transformation of the measurement points of the extracted geometry in the incomplete workpiece coordinate system.

The transformation of the measurement point coordinates is $\hat{P} = R \cdot (P - C)$. As in case of the flange, R is a rotary matrix which turns the direction vector v to the basis vector $(0,0,1)^T$. For the translated measurement point coordinates, $x_W = (1,0,0)^T$ and $y_W = (0,1,0)^T$ are the determination for the undefined axes of the workpiece coordinate system. The calculation of the transformation parameter is identical to the procedure in section 3 (algorithm for the calculation of the rotation angle for the coordinate transformation). In the case of the conical disc, the geometry parameter C and v are derived from the datum cone axis. The assigned coordinate system is sketched in Figure 20.

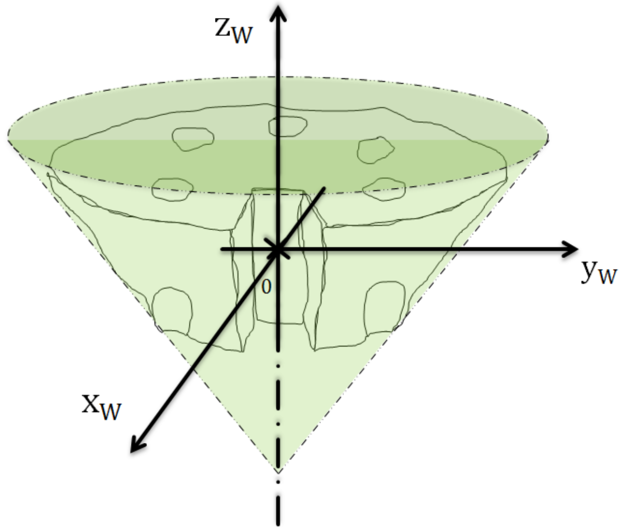

Figure 20 Workpiece coordinate system for the conical disc

For the conical disc, special measurement point amounts are available

$$P^{(1)} = \left\{P_1^{(1)}, \dots, P_{m_1}^{(1)}\right\},$$

$$P^{(2)} = \left\{P_1^{(2)}, \dots, P_{m_2}^{(2)}\right\},$$

$$P^{(3)} = \left\{P_1^{(3)}, \dots, P_{m_3}^{(3)}\right\},$$

$$P^{(4)} = \left\{P_1^{(4)}, \dots, P_{m_4}^{(4)}\right\},$$

$$P^{(5)} = \left\{P_1^{(5)}, \dots, P_{m_5}^{(5)}\right\}$$

for the five holes. Furthermore,

$$P^{(6)} = \left\{P_1^{(6)}, \dots, P_{m_6}^{(6)}\right\}$$

and

$$P^{(7)} = \left\{P_1^{(7)}, \dots, P_{m_7}^{(7)}\right\}$$

stand for the lateral surfaces situated opposite of the slot. When defining $P^{(k)}$, m_k is the respective number of points per data set. The transformation of the measurement points into the workpiece coordinate system is formally calculated by $\hat{P}_i^{(k)} = R\left(P_i^{(k)} - C\right)$. In the following, the designation $P_i^{(k)}$ is used for the points in the workpiece coordinate system to simplify the notation.

Specification of the virtual gauge

The virtual gauge consists of five cylinders and a pair of parallel planes with an ideal geometrical form. All cylinder axes are parallel to the common direction vector $v = (0,0,1)^T$. Likewise, every cylinder has the radius $r = 3.9$ mm (diameter 7.8 mm). However, the positions of the individual cylinder axes are different:

$$C_1, \dots, C_5$$

They are also referred to as C_k $(k = 1, \dots, 5)$. The calculation of the nominal position is carried out by means of the specified bolt circle with the radius $r_L = 22\ \mathrm{mm}, \tau_0 = -30°_0$ and the angular distance $\tau = 60°$

$$C_k = \begin{pmatrix} C_{kx} \\ C_{ky} \\ C_{kz} \end{pmatrix} = \begin{pmatrix} r_L \cos(\tau_0 + k\tau) \\ r_L \sin(\tau_0 + k\tau) \\ 0 \end{pmatrix}.$$

The pair of parallel planes for the slot is initially defined by the normal vector $n = (1,0,0)^T$ in its orientation. Thus, the planes are parallel to the conical axis and symmetrical to the five gauge cylinders. For each plane, the sign of the normal is selected in such a way that it points away from the theoretical material side in the technical drawing and/or at the real product. Furthermore, the position is defined by a point which is situated centrally between the two planes. For $k = 6$ and $k = 7$ this point is

$$C_k = \begin{pmatrix} C_{kx} \\ C_{ky} \\ C_{kz} \end{pmatrix} = \begin{pmatrix} 0 \\ -22 \\ 0 \end{pmatrix}.$$

This is exactly the intersection point of the theoretically exact bolt circle and the y axis of the gauge coordinate system which is situated centrally in the slot. The orthogonal distance of the lateral planes to the median plane via point C_k with the normal vector n is $d = 7.925$ mm. The matrix formulation for the storing of the gauge geometry parameters is

$$M = \begin{pmatrix} C_{1x} & C_{1y} & C_{1z} & v_x & v_y & v_z & 0 & 0 & 0 & r \\ C_{2x} & C_{2y} & C_{2z} & v_x & v_y & v_z & 0 & 0 & 0 & r \\ C_{3x} & C_{3y} & C_{3z} & v_x & v_y & v_z & 0 & 0 & 0 & r \\ C_{4x} & C_{4y} & C_{4z} & v_x & v_y & v_z & 0 & 0 & 0 & r \\ C_{5x} & C_{5y} & C_{5z} & v_x & v_y & v_z & 0 & 0 & 0 & r \\ C_{6x} & C_{6y} & C_{6z} & 0 & 0 & 0 & -n_x & -n_y & -n_z & d \\ C_{7x} & C_{7y} & C_{7z} & 0 & 0 & 0 & n_x & n_y & n_z & d \end{pmatrix}. \tag{17}$$

The first five lines are the parameters of the gauge bolts for the holes. The two remaining lines provide the parameters of the pair of planes for the fitting of the slot.

4.2.3. Calculation of the initial value and the 3D hole pattern fit

The starting point for the modelling of the 3D hole pattern fit of the conical disc are the measurement points transformed into the workpiece coordinate system of the holes $P^{(1)}, \dots, P^{(7)}$ and the matrix with the parameters of the gauge geometry M.

The free parameter of the fitting is the angle φ which rotates the gauge in the coordinate origin around the z axis of the workpiece coordinate system. The angle determines a special rotary matrix H which helps to calculate the parameters of the rotated gauge via

$$M(\varphi) = M \cdot H(\varphi).$$

Hereby,

$$H(\varphi) = \begin{pmatrix} co & si & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -si & co & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & co & si & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -si & co & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (18)$$

with the values $si = \sin(\varphi)$ and $co = \cos(\varphi)$. In $M(\varphi)$, only the position entries (the x and y coordinates of the gauge bolts, plane centre) and the normal vectors of the pair of planes change. For a simplified notation, these entries are thus also referred to as

$$C_k(\varphi) = (C_{kx}, C_{ky}, C_{kz}) \begin{pmatrix} co & si & 0 \\ -si & co & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and

$$n_k(\varphi) = (n_{kx}, n_{ky}, n_{kz}) \begin{pmatrix} co & si & 0 \\ -si & co & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

4.2.3 Initial value and 3D hole pattern fit for the conical disc

In this subsection, the mathematical models are described for the 3D hole pattern fit. In turn, a coarse fitting starts according to a suitable Gaussian criterion. Subsequently, the 3 D hole pattern fit is implemented according to the Chebyshev criterion.

Starting solution with a Gaussian fitting

As in the case of the flange, the calculation of a coarse fitting considered here is oriented towards the centres of the ideal gauge geometries and the measurement points. In a first step, the centroid is calculated for each extracted hole $P^{(k)}$ with $k = 1, \dots, 5$. The z component of the centroid is set to 0, as it must not influence the fitting result. The other components are

$$q_{kx} = \frac{1}{m_k} \sum_{i=1}^{m_k} x_{ki}$$

and

$$q_{ky} = \frac{1}{m_k} \sum_{i=1}^{m_k} y_{ki}.$$

The calculation is efficient for large data volumes. Likewise, a centroid is calculated for the extracted pair of parallel planes of the slot. This centroid is $Q_6 = (q_{6x}, q_{6y}, 0)^T$ with

$$q_{6x} = \frac{1}{m_6 + m_7} \left(\sum_{i=1}^{m_6} x_{6i} + \sum_{i=1}^{m_7} x_{7i} \right)$$

and

$$q_{6y} = \frac{1}{m_6 + m_7}\left(\sum_{i=1}^{m_6} y_{6i} + \sum_{i=1}^{m_7} y_{7i}\right).$$

In order to determine the starting position of the gauge, the angle φ_0 is calculated in a second step which solves the minimization task

$$\min_{\varphi_0} \ \tfrac{1}{2}\textstyle\sum_{k=1}^{6}\|C_k(\varphi_0) - Q_k\|^2. \tag{19}$$

Problem (19) is a Gaussian fitting. It minimizes the sum of the distance squares between the centroids of the point clouds and the arithmetically ideal centres of the gauge elements. The distances are only calculated in the x-y-plane of the workpiece coordinate system. The situation is shown in Figure 21 for a better understanding.

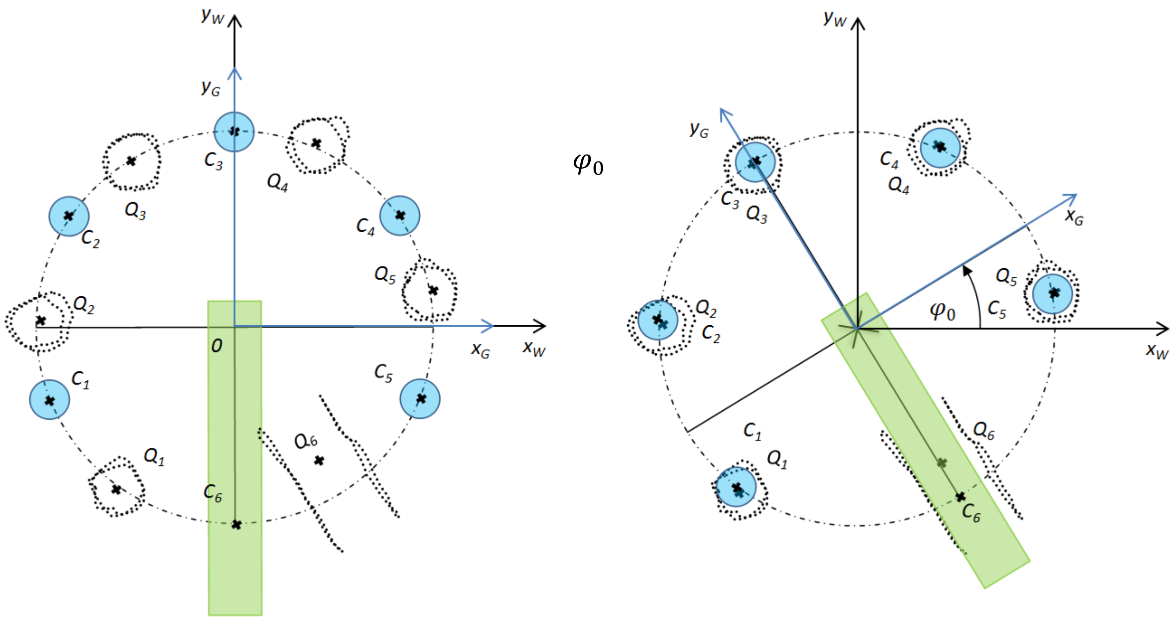

Figure 21 Initial value of the conical disc fitting.

In the top view, the gauge geometry is shown in the direction of the median axis (conical axis). The gauge geometry comprises the cylindrical bolts marked in blue and the slot area marked in green. The extracted geometry of the holes and slot lateral surfaces is sketched using black points. The gauge geometry and the extracted geometry are twisted towards each other prior to the fitting. This is shown in the left side of the figure. The ideal centres of the gauge C_k and the centroids of the extracted geometry Q_k defined for the Gaussian fitting are clearly visible. After the rotation of the gauge by the calculated angle φ_0, the initial fit is available which is shown on the right side of the figure.

Calculation of the 3D hole pattern fit

The solution angle φ_0 from the initial value (19) is not yet the angle with the smallest possible overlapping and/or largest possible empty space between the gauge geometry and the measurement points. In order to apply the general Chebyshev fitting (2) from chapter 2.4, suitable distance functions for the gauge cylinders and the slot must be defined. As in case of the flange, the following definition is valid for the gauge cylinder with $k = 1, \ldots, 5$

$$f_{ki}\big(M(\varphi)\big) = r - \left\|\left(P_i^{(k)} - C_k(\varphi)\right)\times v\right\|.$$

In case of the slot, a distinction is made between the left and the right lateral surface. This is realized by a respective assignment of the point sets to the planes of the gauge geometry and the opposite determination of the normal direction. In this application example, $P^{(6)}$ is the extracted geometry of the left lateral surface and $P^{(7)}$ is the extracted geometry of the right lateral surface. For this, the following is valid:

$$f_{ki}\big(M(\varphi)\big) = d - \langle P_i^{(k)} - C_k(\varphi), n_k(\varphi)\rangle.$$

The following fitting procedure is set up.

$$\min_{\varphi\in\mathbb{R}, s\in\mathbb{R}} s \qquad s.t. \quad f_{ki}(\varphi) \leq s \ \ \forall\, k = 1,..,7 \ \ and\ \forall\, i = 1, \dots, m_k \tag{20}$$

By means of the rotation angle φ calculated from this fitting program and the maximum distance s the determination of the desired quantity for a minimum overlap or a maximum empty space is easy to realize. The following statements are valid:

- If $s > 0$, there is an overlap between the gauge and the measurement points. It has the value s.
- If $s < 0$, there is empty space between the gauge and the measurement points. It has the value s.
- If $s = 0$, the gauge is adjacent to the measurement points. There is neither empty space nor overlap.

To be able to evaluate for which of the gauge geometry elements there is an overlap where $s > 0$, the distances to the locally assigned measurement points can be inspected for each individual element. These are as follows:

$$s_k \coloneqq \max_{\mathrm{i=1,..,m_k}} f_{ki}\big(M(\varphi)\big)$$

for all $k = 1, \dots, 7$. If $s_k > 0$, there is an overlapping with the product.

5. Application example 3: cubes

In the case of the application examples of flanges and conical discs considered above, the gauge elements are parallel. The inspection using a physical gauge is possible by means of a one-sided plugging of the gauge with the test specimen. In contrast to this, the application example for cubes considers the assembly of products on top of holes or bolts that are twisted toward each other. I.e., the geometry elements of the gauge are no longer parallel. As a result, the inspection by means of physical gauges that are made of only one part are no longer possible. Specifically in this case, virtual gauging by means of CMM and 3D hole pattern fit is a suitable means of testing with regard to feasibility. For the application, the cube sketched in Figure 22 is considered.

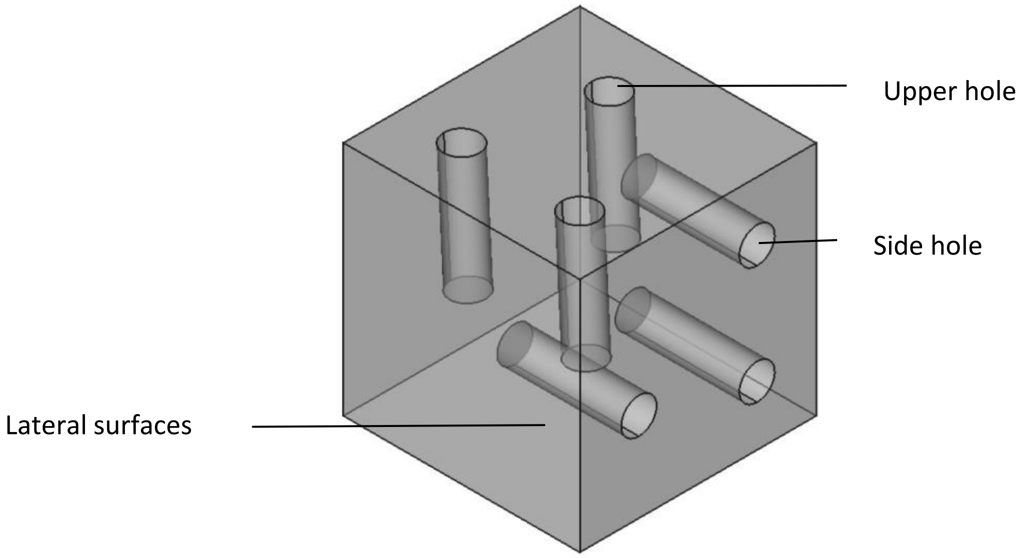

Figure 22 Application example for a cube

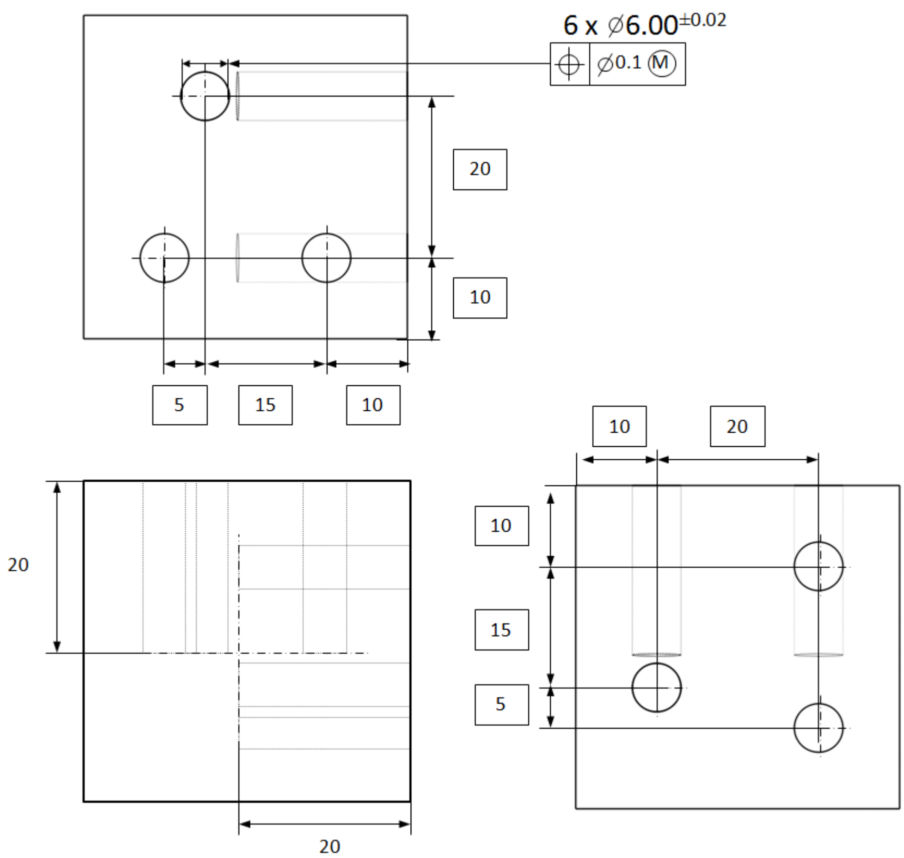

Figure 23 ISO 1101 compliant inspection task for the cube.

The cube has an edge length of 40.0 mm. In two of the six lateral surfaces, three holes respectively, have been integrated. The depth of all holes is 20.0 mm each. The diameters are 6.0 mm. All hole axes are vertical to the respective lateral surfaces. In order to better identify the position of the holes, the external surface of the cube was drawn as transparent (Fig. 22).

5.1. Inspection according to the standard

No specific function is attributed to the inspection of the cube. Therefore, a common inspection task is described here. This task is shown in Figure 23.

When selecting the inspection task, a datum or datum system was totally dispensed with. Only the position of the six holes is tolerated. This leads to a gauging or 3D hole pattern fit with a maximum number of six degrees of freedom. At the same time, it places the highest demands on numerical stability and efficiency of mathematical procedures for an arithmetical fit.

5.2. Mathematical model of the 3D hole pattern fit

The arithmetical 3D hole pattern fit is carried out by the four steps shown in Figure 24.

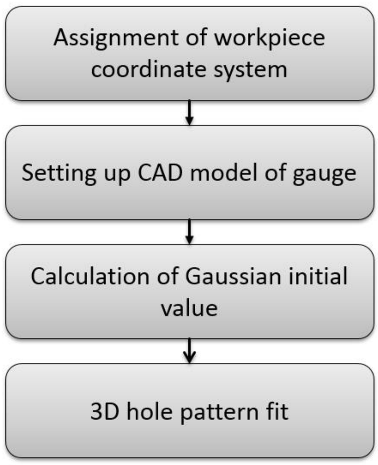

Figure 24 Flowchart on the 3D hole pattern fit for the cube.

At the beginning, the workpiece coordinate system and the gauge parameters were assigned. Subsequently, the discussion of a suitable initial value took place. The procedure is much more time-consuming than for the previous applications. Furthermore, a general 3D transformation of the gauge geometry must be defined. Finally, the formal specification of the Chebyshev 3D hole pattern fit is carried out.

Workpiece coordinate system and gauge parameters

As we have the case of a general fit for the cube (no constraints due to datum elements), basically every Cartesian coordinate system can be used as a workpiece coordinate system. In the example, the Cartesian measurement point coordinate system is selected for the workpiece, therefore, we do not need to convert the point coordinates in this case.

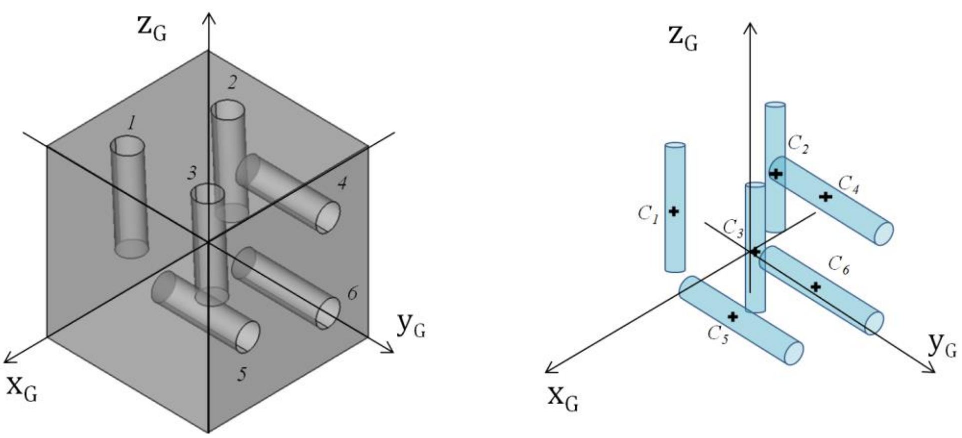

Figure 25 Coordinate system and model of the virtual gauge for the cube.

The gauge consists of six ideal cylinders. These cylinders are assigned to each hole of the cube. They have a radius of $r = 2.94$ mm (corresponding to a diameter of MMVS = 5.88 mm). The position and direction of the gauge cylinders is presented in Figure 25.

On the left side of the figure, the gauge coordinate system of the cube is shown. Furthermore, the holes are clearly numbered by the indices of $k = 1,..,6$. In the figure on the right, the gauge geometry (the counterpart to the fitting) is shown. The centres $C_1, \dots, C_6$ of the geometrically ideal cylinders have been plotted. Each of these cylinders is located on the axis of the gauge cylinder. The position is the centre between the ends of each hole in the technical drawing of the product. In the following, the parameter values are provided.

$$C_1 = \left(C_{1x}, C_{1y}, C_{1z}\right)^T = (-10, -30, -10)^T, \quad C_4 = \left(C_{4x}, C_{4y}, C_{4z}\right)^T = (-30, -10, -10)^T$$

$$C_2 = \left(C_{2x}, C_{2y}, C_{2z}\right)^T = (-30, -25, -10)^T, \quad C_5 = \left(C_{5x}, C_{5y}, C_{5z}\right)^T = (-10, -10, -25)^T$$

$$C_3 = \left(C_{3x}, C_{3y}, C_{3z}\right)^T = (-10, -10, -10)^T, \quad C_6 = \left(C_{6x}, C_{6y}, C_{6z}\right)^T = (-30, -10, -30)^T$$

The direction vector of the cylinders 1 to 3 is

$$v_1 = \left(v_{1x}, v_{1y}, v_{1z}\right)^T = (0,0,1)^T.$$

The cylinders 4 to 6 have the direction vector

$$v_2 = \left(v_{2x}, v_{2y}, v_{2z}\right)^T = (0,1,0)^T.$$

Here, the parameter values are also summarized in a joint matrix M.

$$M = \begin{pmatrix} C_{1x} & C_{1y} & C_{1z} & 1 & v_{1x} & v_{1y} & v_{1z} & r \\ C_{2x} & C_{2y} & C_{2z} & 1 & v_{1x} & v_{1y} & v_{1z} & r \\ C_{3x} & C_{3y} & C_{3z} & 1 & v_{1x} & v_{1y} & v_{1z} & r \\ C_{4x} & C_{4y} & C_{4z} & 1 & v_{2x} & v_{2y} & v_{2z} & r \\ C_{5x} & C_{5y} & C_{5z} & 1 & v_{2x} & v_{2y} & v_{2z} & r \\ C_{6x} & C_{6y} & C_{6z} & 1 & v_{2x} & v_{2y} & v_{2z} & r \end{pmatrix} \tag{21}$$

The column with the numerical values of "1" is of special importance for the transformation of the parameter matrix. This will be explained in the following.

Specification of the transformation operator for the fitting

The transformation comprises six different parameters. Initially, three translations of the gauge along the workpiece coordinate system are possible. These translations are denoted by the parameter.

$$T = \left(t_x, t_y, t_z\right)^T$$

The components indicate the respective fraction of the translation towards the coordinate axis with an identical index. Furthermore, three rotations around the workpiece coordinate axes are possible. This is realized here with the aid of the Euler rotation angle.

$$\varphi = \left(\varphi_x, \varphi_y, \varphi_z\right)^T$$

Each component defines a rotation matrix. These are

$$R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\varphi_x) & -\sin(\varphi_x) \\ 0 & \sin(\varphi_x) & \cos(\varphi_x) \end{pmatrix}^T,$$

$$R_y = \begin{pmatrix} \cos(\varphi_y) & 0 & \sin(\varphi_y) \\ 0 & 1 & 0 \\ -\sin(\varphi_y) & 0 & \cos(\varphi_y) \end{pmatrix}^T$$

and

$$R_z = \begin{pmatrix} \cos(\varphi_z) & -\sin(\varphi_z) & 0 \\ \sin(\varphi_z) & \cos(\varphi_z) & 0 \\ 0 & 0 & 1 \end{pmatrix}^T.$$

From the individual rotation matrices, the following matrix is formed by means of multiplication:

$$R = R_x R_y R_z = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

Thereby, the matrix $H(T,\varphi)$ for the transformation of the gauge geometry M in (21) is

$$H(T,\varphi) = \begin{pmatrix} r_{11} & r_{21} & r_{31} & 0 & 0 & 0 & 0 & 0 \\ r_{12} & r_{22} & r_{32} & 0 & 0 & 0 & 0 & 0 \\ r_{13} & r_{23} & r_{33} & 0 & 0 & 0 & 0 & 0 \\ t_x & t_y & t_z & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r_{11} & r_{21} & r_{31} & 0 \\ 0 & 0 & 0 & 0 & r_{12} & r_{22} & r_{32} & 0 \\ 0 & 0 & 0 & 0 & r_{13} & r_{23} & r_{33} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{22}$$

The transformed parameter matrix is

$$M(T,\varphi) := M \cdot H(T,\varphi).$$

During the calculation, the points indicating the position of the gauge cylinders are rotated and subsequently shifted. The calculation formula is $C_k(T,\varphi) = C_k R + T$. The direction vectors of the gauge cylinders are exclusively twisted. The rotated vectors are designated with $v_l(\varphi) = v_l R$. These transformation provisions for the gauge parameters are used for the specification of the fitting task later on.

Formulation of an initial value according to the Gaussian criterion

Again, a favourable initial orientation of the gauge is being sought for via a suitable Gaussian fitting. Initially,

$$P^{(k)} = \left\{P_1^{(k)}, \dots, P_{m_k}^{(k)}\right\}$$

with $k = 1, \dots, 6$ shall be the point clouds of the measured holes of the cube. The indication corresponds to the specification from Figure 25. The arithmetical centroid is assigned to each point cloud:

$$Q_k = \frac{1}{m_k} \sum_{i=1}^{m_k} P_i^{(k)}$$

The initial value is then the solution of the Gaussian program

$$\min_{T_0,\varphi_0} \frac{1}{2} \sum_{k=1}^{6} \left\| C_k(T_0,\varphi_0) - Q_k \right\|^2 . \tag{23}$$

In Figure 26, the initial value is illustrated once again. The upper part shows the gauge and the extracted holes in the starting position. The measured holes are shown in simplified manner by means of black dotted contours. The transformation operators are illustrated at the axes of the coordinate system. By means of the Gaussian fitting, the median gauge points C_k are shifted as near as possible to the centroids of the point clouds. This is shown by the lower half of the figure.

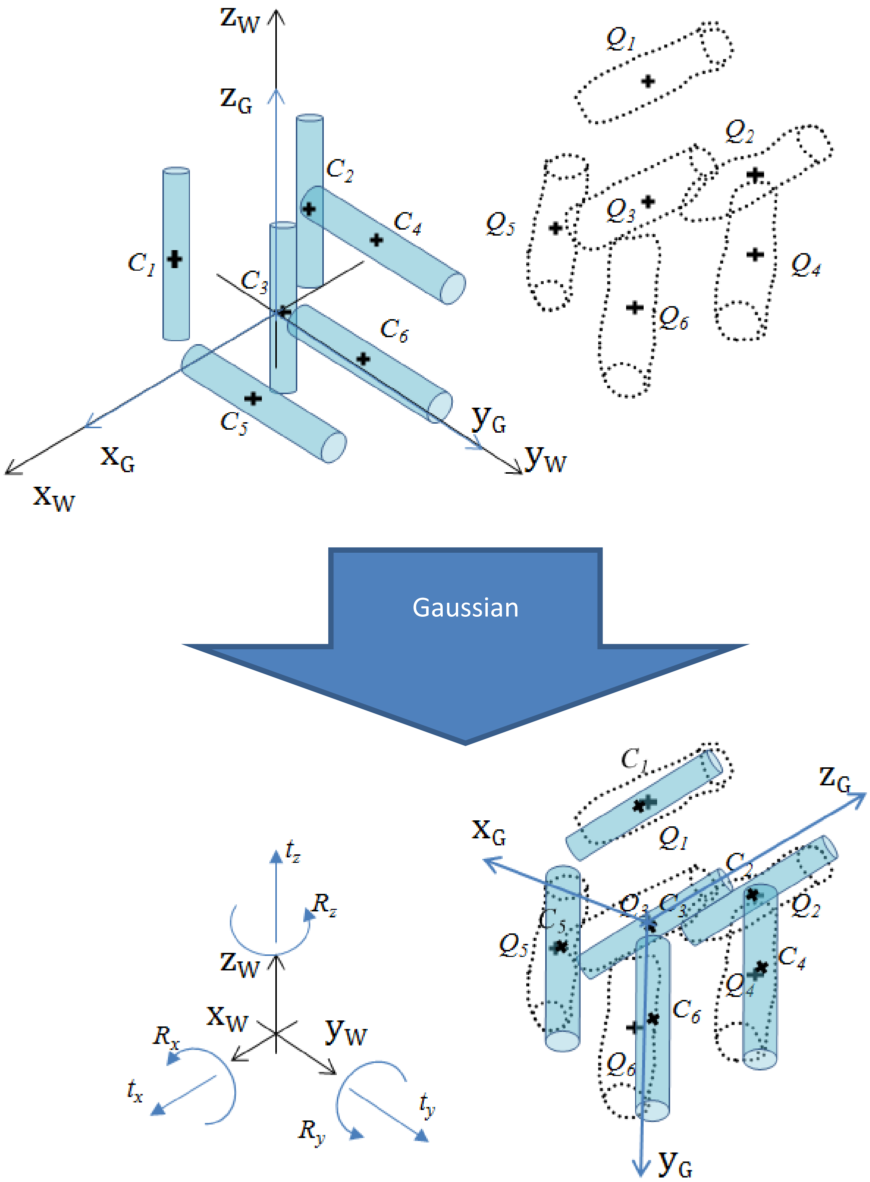

Figure 26 Initial value for the cube fitting

Definition of the 3D hole pattern fit for the cube

Also in this application example, cylindrical gauge elements are available. Therefore, the local orthogonal distances between gauge and the measured points near the elements $k = 1, \ldots, 3$ are defined by

$$f_{ki}\big(M(T,\varphi)\big) = r - \left\| \left(P_i^{(k)} - C_k(T,\varphi) \right) \times v_1(\varphi) \right\|$$

and for $k = 4, \ldots, 6$ by

$$f_{ki}\big(M(T,\varphi)\big) = r - \left\| \left(P_i^{(k)} - C_k(T,\varphi)\right) \times v_2(\varphi) \right\|.$$

The following 3D hole pattern fit procedure is thus formulated as follows:

$$\min_{T\in\mathbb{R}^3,\varphi\in\mathbb{R}^3,s\in\mathbb{R}} s \qquad s.t. \quad f_{ki}(T,\varphi) \le s \;\; \forall\, k = 1,..,6 \;\; and \; \forall\, i = 1,\dots,m_k \qquad (24)$$

The following statements for the solution of (24) are valid:

- If $s > 0$, there is an overlap between the gauge and the measurement points. It has the value s.
- If $s < 0$, there is empty space between the gauge and the measurement points. It has the value s.
- If $s = 0$, the gauge is adjacent to the measurement points. There is neither empty space nor overlapping.

To be able to evaluate in the case of $s > 0$ which gauge geometry element shows an overlap, the distances to the locally assigned measurement points can be inspected for each individual element of the gauge. These are as follows:

$$s_k := \max_{i=1,\dots,m_k} f_{ki}\big(M(T,\varphi)\big)$$

for all $k = 1,\dots,6$. If $s_k > 0$, the hole shows an overlap of the gauge with the material of the extracted geometry.

5.3. Generalization of the inspection task for the simulation of assembly

Finally, an application is discussed that uses the measurement of a real counterpart during assembly. In turn, the geometry data obtained are used as a measured virtual gauge for the inspection of the holes of the cube. Figure 27 shows the example of a simple counterpart to the hole pattern cube.

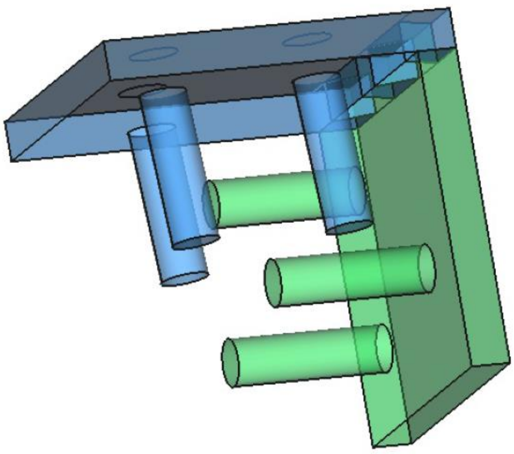

Figure 27 Counterpart for the cube

The counterpart consists of two individual components. One of them is marked in green, the other is marked in blue. Each component has 3 bolts as counterpart for the holes in the cube. Each bolt is 18.0 mm long. This is 2.0 mm shorter than the nominal depth of the holes in the cube. Furthermore, the cylinder radius is 2.94 mm as in case of the virtual gauge. Both components can be assembled to each other interlockingly.

To set up the virtual gauge, only the lateral surfaces of the 6 bolts are extracted. A geometrically ideal enveloping cylinder is assigned to each lateral surface. These cylinders are to be calculated in such a way that they have the smallest possible radiuses where all measurement points of a bolt are still enclosed. As centres, the orthogonal projections of the point cloud centroids are adjusted to the respective assigned cylinder axis.

The gauge parameters measured and arithmetically evaluated in this way can also be stored in a matrix as in (21). Hereby, every cylinder has different direction vectors and radiuses. In the further steps, the initial value according to Gauss and finally the 3D hole pattern fit are inspected by means of the extracted holes of the cube. This must be implemented twice. The reason for this is the permutability of the assignment of gauge elements and holes. This can be seen from the two components of the counterpart. These components are interchangeable due to a simple symmetry of the hole positions.

The inspection allows us to arithmetically determine pairs of cubes and counterparts which can still be assembled in spite of inadmissible deviations according to the inspection which complies with the standard.

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