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Dancing Coin

Investigation of temperature-dependent pressure dynamics in a temporarily open system

This article is on International Young Physicist's Tournament (IYPT)-problem three, Dancing Coin, of 2018. A coin is put on a strongly cooled bottle and after a period of time, movements of the coin and a sound can be observed. A theoretical model is developed to explain these occurrences by simplifying the heat flow into the system. The sound, on the other hand, is modelled as an open-end resonance phenomenon of the bottle with natural coin frequencies being damped by the tightening water.

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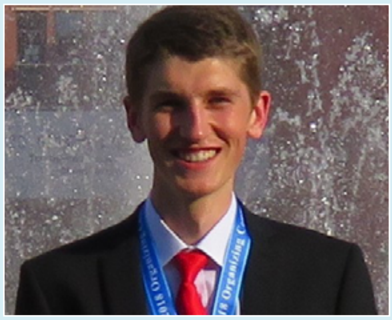

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Dancing Coin

Investigation of temperature-dependent pressure dynamics in a temporarily open system

Preliminaries

This article is based on problem number three of IYPT 2018, Dancing Coin. The IYPT is a physics competition for high school students. The problem statement reads as follows:

Take a strongly cooled bottle and put a coin on its neck. Over time you will hear a noise and see movements of the coin. Explain this phenomenon and investigate how the relevant parameters affect the dance.

1. Qualitative description

We consider the bottle—i.e. the bottle wall as well as the air that is sealed within it through the coin on top of the bottle neck—to be our system of concern. After cooling, a little water is added to the top of the bottle in order to seal the system when the coin is put on the bottle neck (without the water, no sealing of the system could be observed). At this point, the temperature within the bottle T_i is significantly lower than the ambient temperature T_a. At time $t = 0$, which is considered to be the moment the system is sealed, the pressure p within the system is equivalent to the ambient pressure p_a. There is a number of N air particles in the system. The temperature gradient $\vec{\nabla} T$ created by the temperature difference between T_i and T_a directs the flow of heat into the system according to Fourier's law of heat transfer. Thus, an increase in temperature of $+\Delta T$ occurs within the system. This increases the pressure by $+\Delta p$. As heat continues to flow into the system (at a slower rate with decreasing temperature difference), eventually a critical temperature increase $+\Delta_c T$ and along with it a critical pressure increase $+\Delta_c p$ is reached. At this point, the pressure within the bottle is enough to lift the coin, temporarily unsealing the system. This lift-off results in a sudden drop in pressure of precisely the critical pressure difference in a proper lift-off back to ambient pressure as well as a decrease of $-\Delta N$ air particles in the system due to the pressure adjustment of the system. This means that over time, there will be a net loss of air particles in the system, which makes sense as the cool air is denser at time $t = 0$. As the lift-off terminates, the coin comes crashing back onto the bottle neck and a sound can be perceived. In perfect experimental conditions, the system seals itself again as soon as the coin is back on the bottle neck, and the temperature in the system, which increases continuously with time, creates again a pressure difference that builds up to the critical temperature difference, eventually resulting in another lift-off. This process repeats until the pressure difference that can be reached is lower than the critical pressure difference required to lift the coin. Whether this pressure difference is sufficient or not is determined by the temperature difference between system and the surroundings.

On page 11 you find a table with description and unit of all symbols mentioned in this text.

2. Theoretical model

2.1 Temperature evolution

First consider the flow of heat into the system. A good approximation is given by Newton's law of cooling, which says that the rate of change of temperature $\dot{T}$ is proportional to the temperature difference ΔT, i.e.

$$\dot{T} = -k\,\Delta T = -k\left(T(t) - T_a\right) \qquad (2.1)$$

The general solution to this first-order differential equation is $T(t) = \chi e^{-kt} + T_a$, where, in our case, $\chi = T_i - T_a$, with the initial condition $T(0) = T_i$. The idea is to predict the warming coefficient k that is dependent on material properties. To do this, we use the notion that the heat flow rate $\dot{Q}$ is given by

$$\dot{Q} = \Delta T \sum_{i=1}^{n} U_i S_i \qquad (2.2)$$

where U is the thermal transmittance and S is the interaction surface area. We can see from this that we consider several parallel heat flows into the system—one through the coin and one through the bottle wall. The corresponding heat flows have different thermal resistances Ψ that are given through material properties and states of convection and other external conditions. Note that thermal transmittance is $U = \Psi^{-1}$. In addition to equation (2.2), we need the following equation

$$\dot{Q} = \dot{T} \sum_{i=1}^{n} \zeta_i m_i \qquad (2.3)$$

that describes the heat flow proportional to the rate of change of temperature of a system consisting of n bodies described by their masses m_i and their specific heat capacities ζ_i. Thus, k can be computed as

$$k = \frac{\sum_{i=1}^{n} U_i S_i}{\sum_{i=1}^{n} \zeta_i m_i} \qquad (2.4)$$

where

$$\sum_{i=1}^{n} U_i S_i = \frac{A_b}{\frac{1}{\alpha_{eb}} + \frac{d_b}{\xi_b} + \frac{1}{\alpha_{bi}}} + \frac{A_c}{\frac{1}{\alpha_{ec}} + \frac{d_c}{\xi_c} + \frac{1}{\alpha_{ci}}} \qquad (2.5)$$

The value of the warming coefficient k was found through fitting the model to the experimental data. It can, however, be concluded from equation (2.5) what the determining factors in the rate of change of temperature in the system must be. Radiation power is assumed to be entailed in the fitted value of k. [1]

2.2 Equation of state for lift-offs

Treating the air in the system as an ideal gas, it is possible to formulate an equation of state for the occurrence of a lift-off. This equation of state relates the pressure required to lift the coin to the temperature increment required to increase the pressure by this amount. The basis of this is the ideal gas law

$$pV = NRT \qquad (2.6)$$

that says that the pressure p of a gas times its volume V must be equivalent to its amount of substance N times its (absolute) temperature T times the ideal gas constant R. Pressure is the scalar quantity that relates the unit normal force acting on a unit area, i.e. $d\mathbf{F} = -p d\mathbf{A}$. Under the assumption that the force enacted by the warmed gas on the surrounding containment is homogeneous, this simplifies to $p = F / A$. The force is $F = m_c g + F_{st}$ where F_{st} is the surface tension force. To find it, consider the energy E_{st} required to build up the boundary layer between air and water. There are in fact two boundaries, one on the outside (i.e. the ambient air-water boundary) and on the inside (i.e. the water-inside air boundary). Thus, this energy E_{st} is composed of the energy to build the outer layer E_{so} and the inner layer E_{si}, i.e. $E_{st} = E_{so} + E_{si}$. This can be written as

$$E_{st} = E_{so} + E_{si} = v_o A_o + v_i A_i \qquad (2.7)$$

where v_o and v_i are the specific surface energies of the outer and the inner layer, respectively, and A_o and A_i are the outer and inner surface areas of the boundary layers. Assuming that the tightening layer of the water is very thin, the areas A_o and A_i become identical, as do the specific surface energies of the two boundaries—there's little reason they should be different, because the material properties are very nearly identical on both outside and inside of the system. The specific surface energy to be considered is the one of water that can be found in tabular works such as [2]. Thus, the surface tension energy becomes

$$E_{st} \approx 4\pi rhv \qquad (2.8)$$

The surface tension force can be found by taking the derivative of that energy with respect to the height of that layer:

$$F_{st} = \frac{dE_{st}}{dh} = 4\pi rv \qquad (2.9)$$

The pressure required to lift the coin is therefore

$$\Delta_c p = \frac{m_c g + 4\pi rv}{A_i} \qquad (2.10)$$

Applying the ideal gas law yields

$$\Delta_c pV = NR\Delta_c T \qquad (2.11)$$

where the amount of substance N can be rewritten as $N = \rho V M^{-1}$ where M is the molar mass of air and ρ is its density. The equation of state finally can be written as

$$\frac{m_c g + 4\pi rv}{A_i} = \frac{\rho}{M} R \Delta_c T \qquad (2.12)$$

2.3 Discrete lift-off model

We now seek to predict the time it takes for the n-th lift-off to occur. As every lift-off occurs after a discrete interval of time after the next one, finding up a progression that describes this process seems reasonable. Let us denote the density after the n-th lift-off as ρ_n and the time it takes to reach this lift-off as t_n. Then, from the gas law, ρ_n can be found as

$$\rho_n = \frac{p_a M}{RT(t_{n-1})} \qquad (2.13)$$

The initial value of the density is given by $T(0) = T_i$. We consider the temperature at time t_{n-1} rather than the temperature at time t_n. This is because the running index of the density is actually one too high, i.e. ρ_1 is before the first lift-off, while the density ρ_2 is just after the first lift-off. This small but important detail is necessary to set up an iterative process to predict how the system will evolve. Knowing the density in the system, the amount of substance in the bottle is given by

$$N_n = \frac{\rho_n V_b}{M} \qquad (2.14)$$

Knowing this, the critical temperature difference $\Delta_c T_n$ is computed by applying the ideal gas law once again:

$$\Delta_c T_n = \frac{\Delta_c p V_b}{N_n R} \qquad (2.15)$$

Thus, the temperature in the bottle after the n-th lift-off is given by the initial temperature and the sum of all the necessary temperature differences:

$$T_n = T_i + \sum_{j=1}^{n} \Delta_c T_j \qquad (2.16)$$

Now, it remains to determine the time that it will take for the system to warm up to the temperature T_n. To find this time t_n, the particular solution to equation (2.1) needs to be inverted:

$$t_n = -\frac{1}{k} \ln \frac{T_n - T_a}{\chi} \qquad (2.17)$$

Hereby, a set of equations is complete by which an iterative model can be set up. We can now bring our attention to the sources of the sound upon impact of the coin onto the bottle neck.

2.4 Sound model

At first it may seem improbable to accurately predict the source of the sound of the phenomenon, or its frequency. Keep in mind that the sources of the sound may be the bottle material oscillating, the coin oscillating, or the air in the bottle much like a Helmholtz resonator (an air-containing body with an opening where pressure discrepancies lead to the oscillation of a body of air at a constant frequency). First consider the oscillating bottle material. In a simplified approach, it will be assumed that there is a node where there is a significant change in curvature of the bottle, i.e. where the bottle neck ends and the cross-sectional area of the bottle increases. If only the bottle neck resonates, it can be stated that there are two possible ground modes of resonance: one closed-end ground mode and one open-end ground mode. Former has two nodes of the sound waves—one at either end of the bottle neck. The latter has one node at the end where the bottle neck curves into the body of the bottle and an anti-node at the opening of the bottle neck. Closed end-resonance is likely not to occur because the bottle can more easily oscillate in the open-end configuration as there is nothing but air giving resistance to an open-end oscillation. This is shown in figure 1, where h gives the length of the bottle neck. The length of the bottle neck can be expressed as

$$h = \frac{\lambda_0}{4} = \frac{3\lambda_1}{4} \qquad (2.18)$$

Using the identity of $f = c/\lambda$, where f is frequency, c is speed of sound and λ is wavelength, the following progression can be formulated which describes the upper modes of the open-end resonance in dependency of the ground mode:

$$f_n = (2n+1) f_0, \; f_0 = \frac{c}{4h} \qquad (2.19)$$

The modes for the coin resonance (both free- and clamped edge) can be found in literature, i.e. in [3]. With this, the Helmholtz resonance remains. The Helmholtz resonance frequency f_H, given by the frequency of the oscillator $0 = \ddot{y} + (A c^2 / Vl) y$ describing a Helmholtz resonator, is

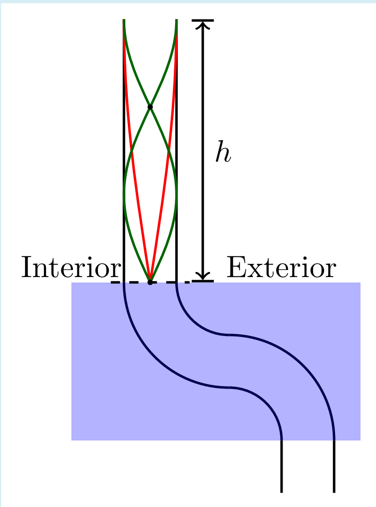

Fig. 1: Open end modes of resonance. The blue area shows the zone in which the curvature of the bottle leads to perturbations of sound waves travelling along the bottle neck and where thus a nodal point is assumed to be. Note that the ground mode λ_0 pertains to the red curve, while the first upper mode λ_1 pertains to the green curve.

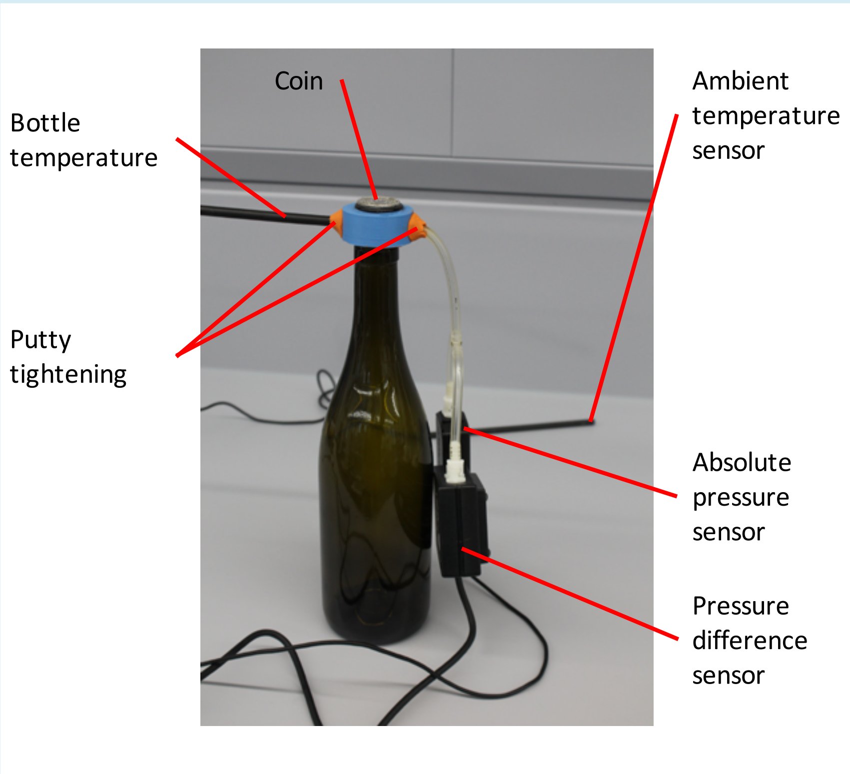

Fig. 2: Experimental setup. The tightening ring was latter omitted as it proofed to be redundant in later cap versions.

$$f_H = \frac{c}{2\pi}\sqrt{\frac{A}{lV}} \quad (2.20)$$

with

$$c = \sqrt{\frac{\kappa RT}{M}} \quad (2.21)$$

where c is the speed of sound, A is the cross-sectional area of the bottle neck opening, l is its length and V the volume of the entire bottle, κ is the heat capacity ratio, R the ideal gas constant, T temperature and M molar mass. It can be concluded, thus, that the speed of sound in air depends highly on temperature. Comparing, for instance, the Helmholtz frequencies at $\delta_1 = -10$°C and at $\delta_2 = 20$°C yields that

$$f_{\delta_1} \approx 0.944 f_{\delta_2} \quad (2.22)$$

3. Experimental setup

The experimental setup used constituted itself of several bottles of different materials and surface properties, all of whom influence the rate of change of temperature according to equation (2.5). Furthermore, a set of coins was investigated. Attention was paid to strict adherence to the task statement in that only coins in the sense of coins used for monetary purposes were used. This makes the experiment more difficult because these coins exhibit a rough surface, thereby considerably increasing the chance of a leak occurring in the tightening of the system, which prevents accurate measurement, and furthermore makes it more difficult to predict how surface tension forces between the different surfaces will impact the phenomenon. In order to keep the interaction area A_i between coin and the air in the system constant, a cap construction (visible as the blue piece in figure 2) was 3D-printed. This makes data more comparable. It, however, makes experimenting more challenging as well because there can more easily be a leak in the tightening. As can be gathered from figure 2, ambient temperature, system pressure and system temperature

Tab. 1: Data for the investigated coins used in figure 6a. The interaction area between coin and system air was held constant at $A = 2.10 \times 10^{-4} m^2$ throughout data collection through the application of the cap construction. The mean μ and the standard deviation σ pertain to the bell curve fits used on the histograms in figure 5. The mean μ gives the expected peak of the standard distribution and was used for fitting η.

Coin	Coin mass in g	Mean μ of the observed lift-off pressures (in Pa)	Standard deviation σ (in Pa)
CHF 0.1.–	3.00	130.4	25.4
CHF 0.2.–	4.00	167.7	8.1
CHF 1.–	4.39	201.5	8.5
CHF 2.–	8.81	387.2	114.4
Euro 0.02 €	3.06	154.0	9.9
Euro 0.1 €	4.10	186.7	9.2
Euro 0.2 €	5.74	252.0	41.7
Euro 1 €	7.53	339.6	9.3
Euro 2 €	8.53	336.2	58.1
NZD 0.1 $	3.30	181.2	17.2
SGPD 0.2 $	4.50	172.7	21.5
AUSD 0.2 $	11.3	311.4	29.5
NZD 0.5 $	5.00	216.9	47.4

were measured. Ambient pressure was measured once before the onset of the system measurement. During experiment, attention was paid to keeping external factors constant, i.e. not opening windows to change ambient temperature or to create breeze, both of which have an impact on equation (2.5). It is clear, also, that the temperature is not homogeneous in the system due to the parallel flows of heat according to equation (2.5). This was not accounted for, but could theoretically by applying Fourier's law of heat transfer. The temperature sensors are accurate within an uncertainty of ±0.5K, and the pressure sensors within an uncertainty of ±2%.

4. Data

In figure 3, it can be seen that the pressure evolution adheres to the qualitative model in that it builds up until it reaches a critical pressure, after which it drops again. It is interesting that the black bottle shows a faster change in temperature, which is reasonable due to its surface being more alike to a black body radiator than the shiny metal bottle (fig 4). Figure 6a shows the assumed linear dependency of the critical lift-off pressure from coin mass when keeping other parameters, such as air-coin interaction area and surface tension forces constant. As the model overpredicts, the necessary

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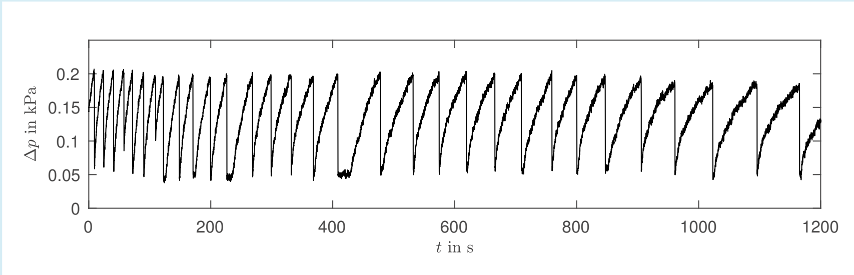


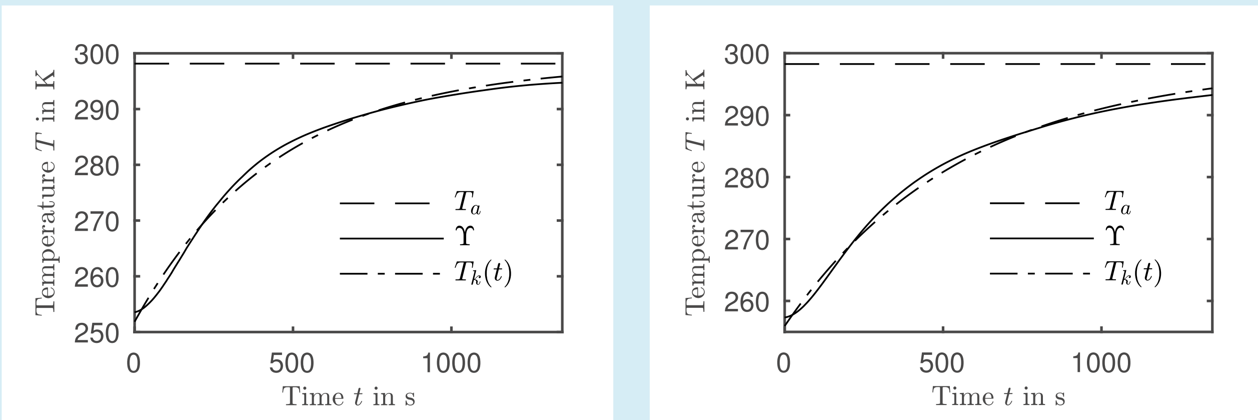
Fig. 3: Pressure difference with time (Optimal data). Particularly interesting is a data pattern precisely as suggested by the qualitative model: after having reached a critical pressure difference, the system pressure very rapidly drops to ambient pressure with the intervals between the lift-offs increasing in length.

force to lift the coins must be lower. Therefore, a linear fit was laid through the peaks of the normally distributed lift-off pressures that gives a factor η which affects the necessary force required for a lift-off (i.e. η is a factor by which $\Delta_c p$ is multiplied). The linear regression was done with one parameter, namely η. It was found to be $\eta \approx 0.71$. However, as the regression gives a root mean squared error (RMSE) of 40.8 (see fig. 6b), it will be subject to further discussion whether the factor η gives a credible fit. The fit was laid through the peaks (the mean μ) of the normal distribution of the observed lift-off pressures as shown in figure 5. Those peaks were found by fitting a bell curve through the histogram of the observed lift-off pressures. The values thus used for fitting and, therefore, to determine η can be taken from table 1. An overprediction is also observable in figure 7 where the predicted $t(n)$, without the fitted factor η, overpredicts significantly, while this is not so dramatic when the factor η is taken into account, i.e. in the line $t(n)_\eta$.

With this, turn your attention now to the sound model. Figure 8 shows the predictions made from the simplified open-end neck resonance model. In figure 9, a possible explanation of the minor peaks is presented as a result of upper modes of the Helmholtz resonance frequencies of the bottle. It shows the same coin and the same bottle at different temperatures. Note that the black line in the left plot is at f_2 = 1848Hz and the one in the right at f_1 = 1727Hz. The ratio of those two frequencies is f_1/f_2 = 0.935. Comparing this value to the predicted ratio value of the Helmholtz resonances according to equation (2.22), this data is evidence that the frequency discrepancy may well be explained through upper modes of Helmholtz resonance as the deviation between theory and data is less than 1%.

5. Interpretation and Discussion

The experimental setup used in this experiment overall gave satisfactory results: it was possible to monitor the relevant physical entities relatively easily using the cap construction outlined in section 3, and it allowed for data from many coins to be collected under similar conditions, making it more comparable. The approach to fit all the relevant measuring devices into a single cap construction allowed for a high efficiency and degree of organisation. However, there were some drawbacks to the design as well. Firstly and most significantly, the cap construction was



a) Temperature evolution over time of a black metal bottle.

b) Temperature evolution over time of a shiny metal bottle.

Fig. 4: Two different temperature evolutions of bottles with different surfaces. T_a is the ambient temperature, Y is the measured temperature and $T_k(t)$ is the model equation with the fitted warming coefficient k.

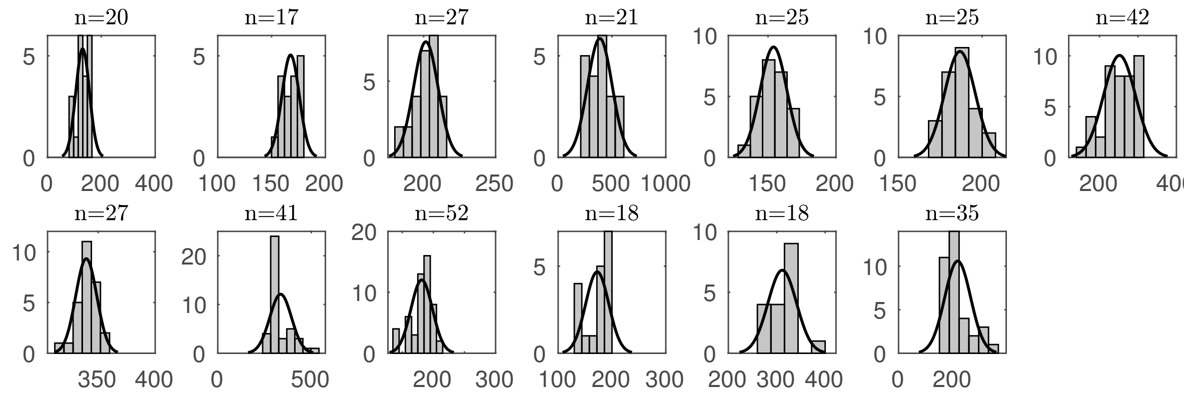
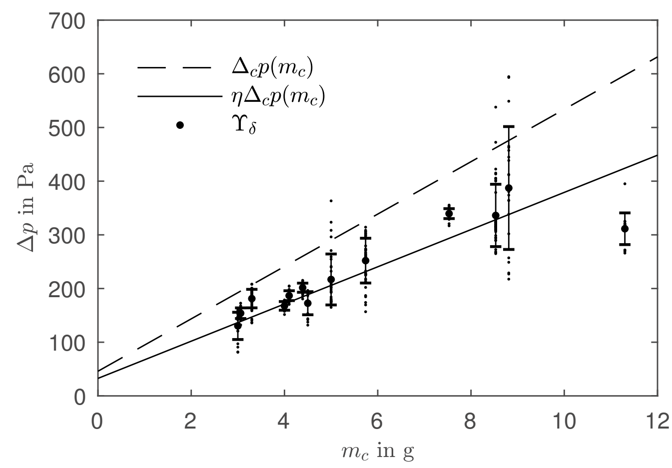


Fig. 5: Histograms of the observed lift-off pressures for the following coins (reading from left to right and from top to bottom): CHF 0.1.–, CHF 0.2.–, CHF 1.–, CHF 2.–, 0.02 Euro, 0.1 Euro, 0.2 Euro, 1 Euro, 2 Euro, 0.1 NZD, 0.2 SGPD, 0.2 AUSD, 0.5 NZD. The horizontal axis gives the observed pressure p at a lift-off in Pa and the vertical axis gives the observed quantity q. The number and width of the bins was chosen according to how widespread and how frequent the lift-offs in a certain range were. Through the histogram is laid a normal distribution function $p \rightarrow q$ where p is the pressure and q is the observed quantity according to $q(p) = (2\pi\sigma)^{-0.5} \exp(-(p-\mu)^2/(2\sigma^2))$ where σ is the standard deviation and μ is the mean (i.e. the expectation or the local maximum of the standard distribution). The values of those entities can be taken from table 1 for each coin investigated.

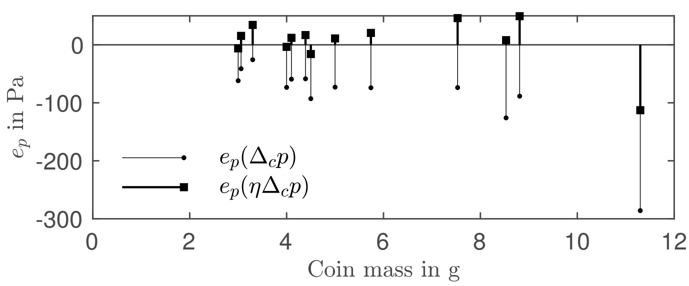
prone to air leaks, which made accurate measurements more difficult. Secondly, it was time-intensive to design, print and set up and lastly, it only proofed to be effective in a certain range of coins, which, however, may have to do with the mechanics of the coin motion as well. This will be discussed at a later stage.

The temperature prediction using Newton's law of cooling gives an outlook over the relevant parameters that impact the temperature evolution with a high degree of certitude. However, the fact that the evolution cannot be modelled deterministically since a lot of material properties would have to be measured with high accuracy is a limitation to the model's predictive power. While the approach of fitting the temperature curve and then using the fitted value gives a good result, it therefore leaves a bit to be desired in terms of predictive power. Furthermore, Newton's law of cooling only predicts the evolution of temperature in the time-domain, while more fundamental equations of heat flow such as Fourier's law of heat transfer would give an overview over the temperature change in both time- and the spatial domain. This may have given an indication of how temperature fluctuations within the system impact the pressure fluctuations in the bottle. Ultimately, this may have given a hint, both numerically and qualitatively, as to how the critical pressure required for a lift-off may have to be altered by the factor η.

The prediction of the critical lift-off-pressure in dependency of the coin mass uses the equations presented in the theoretical model. It needs to be stated that, as can be seen in figure 6b, there are some large fluctuations in the required lift-off pressures that generally seem to increase in heavier coins. This may be explained as follows: as the pressure increases, the air eventually is forced out of the bottle. However, the pressure needs to compensate both the surface tension force of the water as well as the coin weight. As the coins get heavier, the "way of least resistance" for the air is to overcome the surface tension of the water, rather than lifting the coin, as can be seen in figure 10. Therefore, not the entire pressure as predicted by theory will build up. Likewise, there may be higher pressures if the forces to be overcome are larger than surface tension and weight. Reasons for this may be inhomogeneous water distribution between coin and cap, affecting surface tension force, or the surface texture of cap or coin, again affecting surface tension. Those factors may contribute to the factor η. The discrepancy between theoretical prediction and observed data is thus significant, as is visible in figure 6a and especially in figure 7 where a systematic discrepancy between theory, factor-amended theory and data is clearly visible. In figure 6a, there is a visible tendency that standard deviations increase with increasing coin mass in data. This may be due to the fact that there is literally more space for water affecting surface tension to be distributed over the coin-cap interaction area that gives rise to more fluctuations in observed lift-off pressures as the required force to lift the coins can vary more greatly. In figure 7, the difference between theory and theory including η becomes very large as the number of lift-offs increases, and there seems to be a systematic deviation between the theory fitted with η and the data. This is problematic as the coherence of data and theory may not be interpreted unambiguously. With increasing number of lift-offs, the temperature in the system increases gradually towards ambient temperature. Consequently,



a) Lift-off pressure with changing coin mass. The measured pressure differences Y_δ are the critical lift-off pressures for the pertaining coin. It was found by taking the peak of the normal distribution curve of a histogram that bins all the observed lift-offs of that particular coin (shown as the small black dots). The line $\Delta_c p(m_c)$ is a function of the coin mass according to equation 2.10. The line $\eta \Delta_c p(m_c)$ was obtained by taking a linear fit through Y_δ that corresponds to the mean μ of each coin in table 1.



b) The residuals e_p of figure 6a that are given by a measured value Y and a predicted value P according to $e_p = Y - P$. There is no trend observable that would suggest the presence of a systematic deviation, but the residuals nicely show how the discrepancies in the model without the correcting factor η grow very large. The RMSE-value of the fitted line is 40.8.

Fig. 6: Theoretical predictions and data of coin mass and pertaining lift-off pressure that is given by equation (2.10).

it will take longer for the subsequent lift-off to occur (which is precisely shown in the model), potentially offering a higher probability for the water tightening to be disrupted. The systematic overprediction may be due to the fact (which was not accounted for in the model) that with increasing time interval and coin mass, it may be that the pressure will find the water tightening to offer less resistance than lifting the coin would. As overcoming surface tension forces requires less pressure, less time would be required to lead to a drop of pressure, which was the physical process actually measured to determine the time of a lift-off-event. The systematic discrepancy between (fitted) theory and data begs the conclusion that there may be further factors affecting the time and pressure of a lift-off. The nature of those factors is subject to speculation, though tightening of the setup, surface tension models and the interaction area between the coin and the pressurised air within the system are amongst the most plausible uncertainty reasons. This is particularly interesting as there is no such systematic deviation visible in figure 6a, which indicates that there was either a systematic error during the measurement, i.e. a wrongly calibrated sensor, or there is an aspect that influences the time evolution of the system that was not accounted for. It seems that, therefore, the model to predict the required lift-off pressure works well but would need more refined experimental data to verify it, especially paying attention to how the water is spread on the coin and how large the interaction area is. The discrete model to predict the time interval, however, would require more refinement in terms of considering further physical aspects that may shorten the predicted time for the n-th lift-off to occur. The RMSE value of the regression that gives the factor η is fairly high at 40.8. The reason for this is very likely linked to the fact that the pressure values of the observed coins have a high uncertainty (as can be taken from table 1). The reason that the observed mean μ of the required pressure for a particular coin may be lower is outlined above. The effect of this is that the data is fairly widespread, which then leads to the high RMSE value. Once again, a higher correlation may well be achieved by refining the experimental setup.

Considering the sound model, the approach via the modes of resonance is rather promising. However, there is a number of factors that have a massive impact on the Eigenfrequencies of the bottle, foremostly its geometry. Thus, given a distinct bottle shape where perturbations most likely can occur at a particular zone between bottle neck and bottle body, the approach using the open-end resonance modes may produce good predictions. As

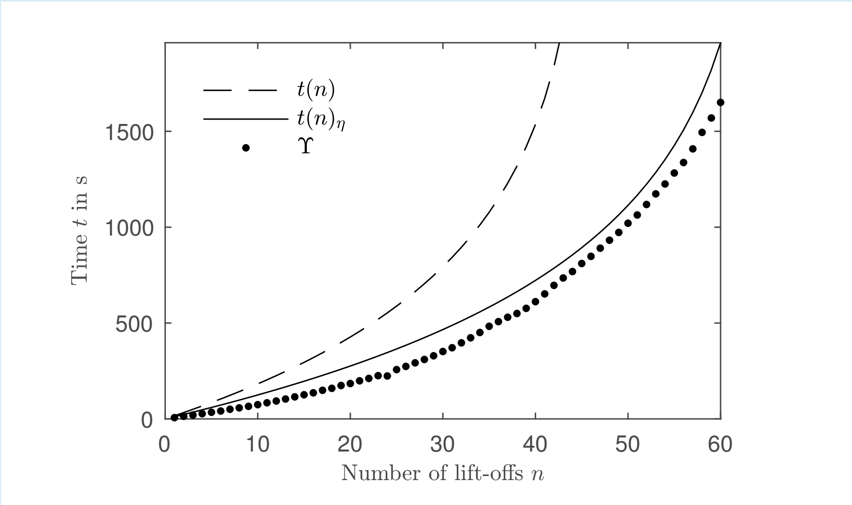


Fig. 7: Lift-off time with lift-off index. The theoretical prediction t(n) was found using the iterative model presented in section 2.3, as was the prediction including η (this was achieved by multiplying $\Delta_c p$ by this factor). The measurement Υ in this experiment was made with a small glass bottle and a Swiss 2-franc-coin.

soon, however, as the geometry of the bottle is such that the transitional zone between neck and body is not easily distinguishable, the simplified approach using the Eigenfrequencies based on a nodal point where the cross-sectional area of the bottle increases becomes insufficient. To improve this, the wave equation for the particular geometry would have to be solved. This is therefore a limitation to the predictive power of the model. Interesting in particular is that the frequency peaks of the spectra nicely correlate with theory. This is strong evidence that the explanation of the main peaks using the approach with the Eigenfrequencies described above holds at least for the bottle used during experiment—one, where the transitional zone between neck and body is very clear. No clear statement, however, can be given when considering different bottles, even though their main peaks should also correlate to an Eigenfrequency. Their value is the issue, as it cannot easily be determined in more complex geometries. Note that the model does not consider the coin to oscillate. The reason for this is quite palpable: Oscillations of the coin could occur at the moment the coin comes crashing back onto the bottle neck. As it then generally flatly lies on the latter, and is furthermore in contact with the sealing water, it is assumed that any oscillations of the coin would be damped. Once again, therefore, does the explanation of the major peaks in the frequency spectra seem to be plausible.

Modelling the minor peaks has proofed fairly difficult. With a high degree of certitude, there are some higher modes of resonance of the bottle and perhaps even of the coin that can be perceived apart from the main peak explained above. Another approach is an explanation using Helmholtz resonance. This approach is justified by the fact that a number of air molecules leave the bottle, thereby possibly creating the pressure difference necessary for a Helmholtz oscillation phenomenon to occur. A part of the theory is strongly reflected in data, namely the high degree of correlation of the frequency ratios at varying temperature even though simple coincidence cannot safely be excluded. This being said, it is not clear how exactly these frequencies are to be interpreted: they are assumed to be higher modes of Helmholtz resonance. It is neither clear, however, if they exist, and if yes, how they arise. Regardless of whether or not it is upper modes of Helmholtz resonance that cause some of the minor peaks, the very accurate prediction of the Helmholtz resonance frequency ratio and the frequency ratio of minor peaks in data strongly suggests that at least part of the minor peaks in the spectra are the result of a form of air oscillation. The temperature-dependent differences in the speed of sound of air, therefore, perhaps form the basis of a more holistic approach to explaining the perceived sound.

6. Conclusion

The task has been to take a strongly cooled bottle and to put a coin on its neck, and then to explain the subsequent lifting-like motion of the coin and the sound that follows it, as well as to investigate the relevant parameters. Given that a temperature difference of 40 K is considered

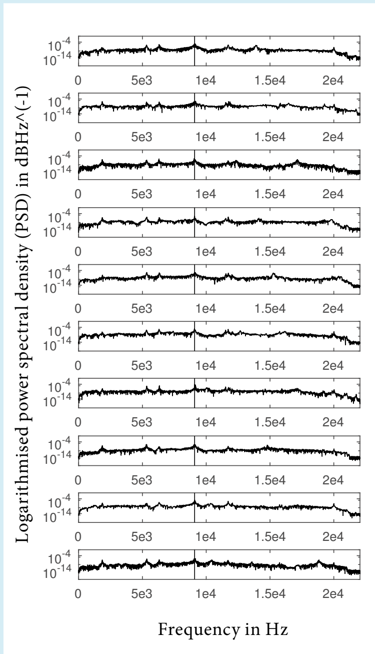


Fig. 8: The peaks of the sound spectra superimpose perfectly with the predicted ground mode frequencies of the open-end neck resonance ground mode (black lines in the spectra). Data was taken from ten different coins. From top to bottom, the coins are: 1 Euro, 0.2 AUD, 0.2 CHF, 1 CHF, 2 CHF, 5 CHF, 2 Euro, 0.1 NZD, 0.5 NZD, 0.2 SGPD.

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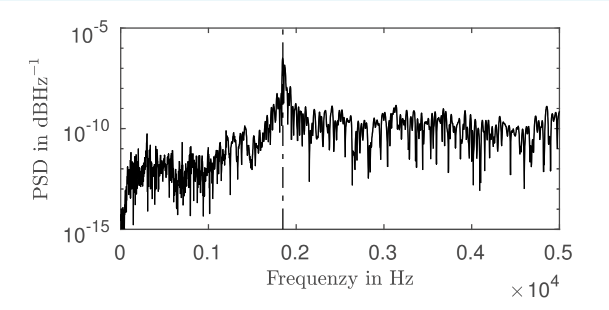
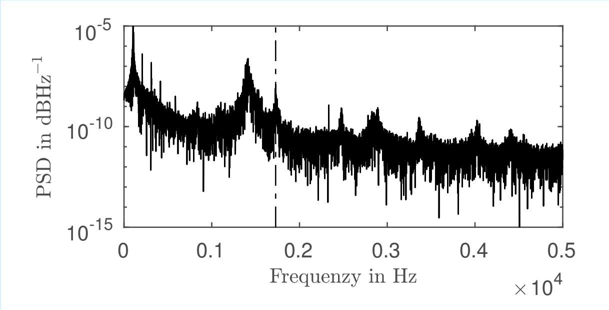


a) The frequency spectrum of a CHF 0.2.– coin dropping onto the bottle (a larger spectrum of the same event is presented in the third spectrum from the top in figure 8) at a temperature of 293K. The black dashed line indicates the frequency with the highest spectral density at f_2 = 1848 Hz.



b) The frequency spectrum of the same bottle at a temperature of 263 K with enforced Helmholtz resonance. The lower peak indicated by the dashed line is at a frequency of f_1 =1727Hz.

Fig. 9: As the resonance frequency of the bottle (as well as of the coin) is dependent on temperature according to equation (2.20), the lower peaks in the spectra of the resonating bottle may be explained as upper modes of Helmholtz resonance.

significant enough, the lifting of the coin was found to be due to pressure dynamics in the system. The sound, on the other hand, cannot be fully explained, even though some promising approaches are presented. The relevant parameters, which are coin mass, the presence of absence of water tightening and bottle parameters influencing the rate of change of temperature in the system when considering the lift-off, and bottle geometry and material when considering the sound, respectively, have been investigated and modelled. Subject to further research may exemplary be the prediction of the precise motion of the coin, such as the angle of lift in dependency of coin mass and temperature and more aspects. Further investigation would also have to be done into the influence of coin surface texture and other factors possibly influencing the required pressure for a lift-off. Very interesting would also be to investigate the proposed limitation to the lift-off-model by the lift-off-pressure discrepancy that could be explained by heavier coins.

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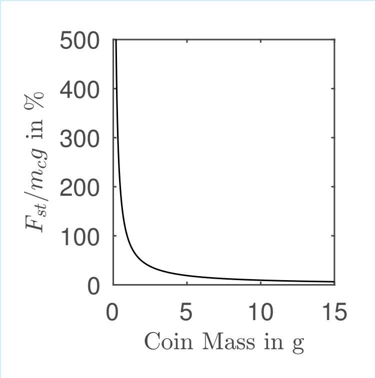

Fig. 10: Ratio between surface tension and coin weight with constant radius of roughly r = 1 cm. For heavier coins, a bigger part of the force needed to lift it is its weight. It is therefore plausible that the pressure does not build up to the full pressure required by equation (2.10), but that the water tightening starts leaking before that due to the pressure, possibly explaining the large discrepancies observed in figure 6a and 6b.

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Symbols

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Sign	Description	Unit
T_i	Initial temperature in the bottle	K
T_a	Ambient temperature of the surroundings	K
p_a	Ambient pressure of the surroundings	Pa
$\Delta_c T$	Necessary temperature increase after a lift-off for the next one to occur	K
$\Delta_c p$	Necessary pressure increase from ambient pressure for a lift-off to occur	Pa
k	Warming coefficient that determines how quickly the system warms up	Hz
U	Thermal transmittance, the reciprocal of thermal resistance	$\mathrm{W\ m^{-2}\ K^{-1}}$
Ψ	Thermal resistance, a measure of how much resistance a body offers to the flow of heat	$\mathrm{m^2\ K\ W^{-1}}$
ζ	Heat capacity of a body, a measure of how much energy that body takes up to change its temperature	$\mathrm{J\ kg^{-1}\ K^{-1}}$
A_b	Surface area of the bottle	$\mathrm{m^2}$
A_c	Surface area of the coin	$\mathrm{m^2}$
α_{eb}	Heat transfer coefficient from the exterior air to the bottle	$\mathrm{W\ m^{-2}\ K^{-1}}$
α_{ec}	Heat transfer coefficient from the exterior air to the coin	$\mathrm{W\ m^{-2}\ K^{-1}}$
α_{bi}	Heat transfer coefficient from the bottle wall to the interior air of the system	$\mathrm{W\ m^{-2}\ K^{-1}}$
α_{ci}	Heat transfer coefficient from the coin to the interior air of the system	$\mathrm{W\ m^{-2}\ K^{-1}}$
d_b	Thickness of the bottle wall	m
d_c	Thickness of the coin	m
ξ_b	Thermal conductivity of the bottle wall material	$\mathrm{W\ m^{-1}\ K^{-1}}$
ξ_c	Thermal conductivity of the coin material	$\mathrm{W\ m^{-1}\ K^{-1}}$
V_b	Volume of the bottle	$\mathrm{m^3}$
A_i	Interaction area between system air and coin	$\mathrm{m^2}$
ν	Specific surface energy	$\mathrm{J\ m^{-2}}$
M	Molar mass	$\mathrm{kg\ mol^{-1}}$

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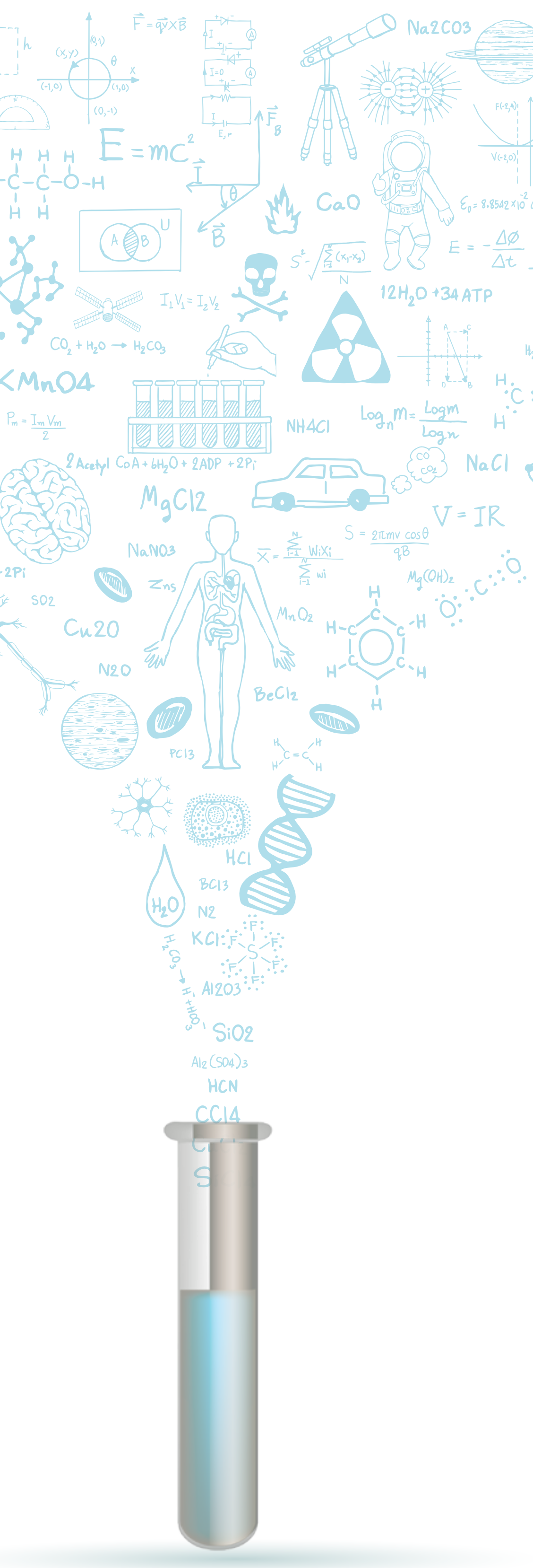

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