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The Role of the Center of Light for the Uncertainty of photogoniometric Measurements

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This document is an excerpt of the interactive Jupyter Notebook with accompanying code blocks in Python. The original notebook can be found here. If you do not have your own running Python or Jupyter Notebook environment, you can start a virtual server here.

Introduction to the Center of Light

The Center of Light is a concept that results from the way Luminous Intensity is defined. The quantity of luminous intensity is used to express the emitted light with a directional dependency-integrating over any spacial distribution of the light source. This is a desired simplification of reality. The Luminous Intensity itself is a model with the assumption, that the luminous object is a point source. The Center of Light is the link between a real luminous object and this model: It is the point that the object is reduced to. The Center of Light is therefore a property of the luminous object, that has to be taken into account when measuring luminous intensity.

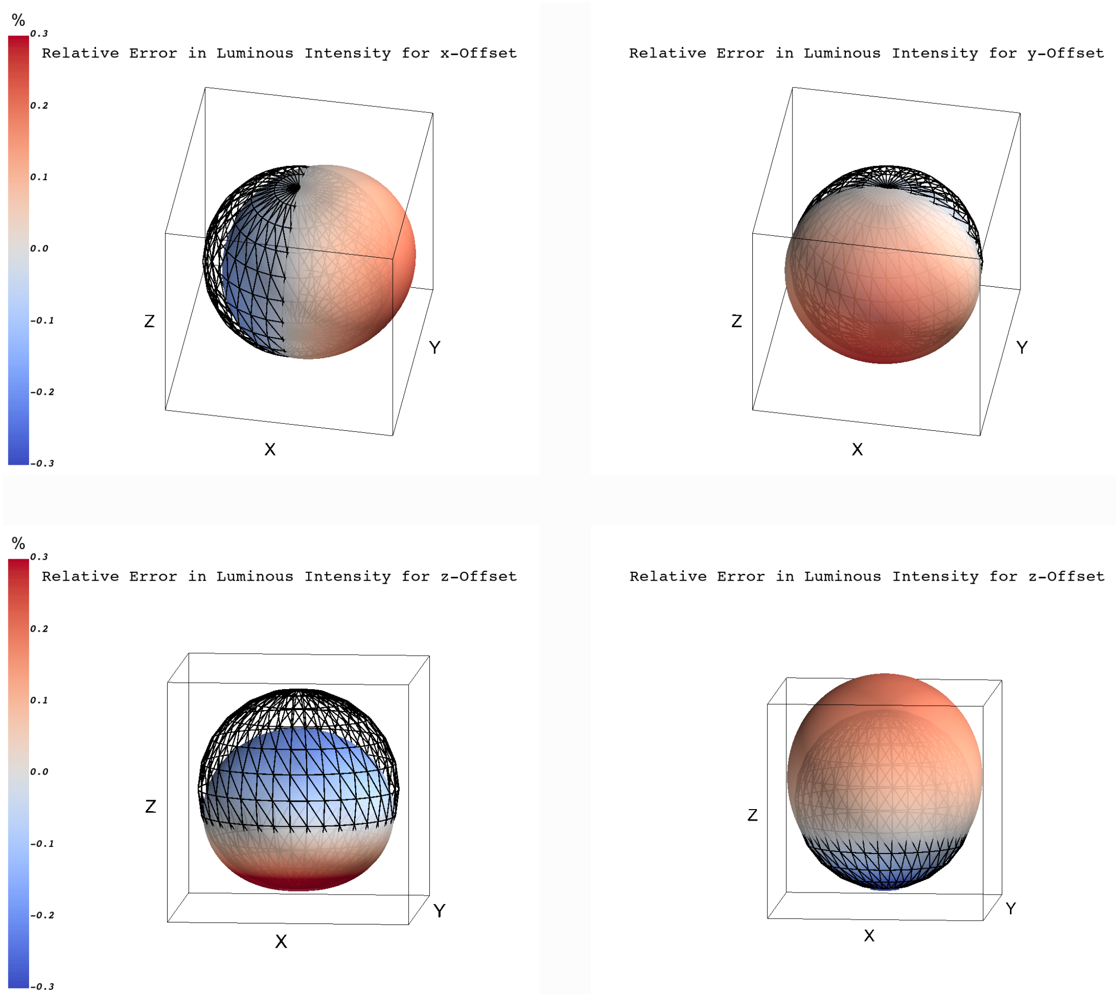
The standard setup for measuring luminous intensity is a farfield goniometer. It uses a detector revolving around the object or vice versa at a constant distance. A luminous intensity distribution (LID) is constructed from multiple measurements performed for different directions relative to the center of the machine. For every measured direction it samples the illuminance caused by the source at this distance and derives from it the luminous intensity. This measurement setup assumes a point source in the center of the machine. The Model is implicitly used, but the actual object position is up to the user. The Center of Light of the luminous object is generally unknown, this opens up the possibility to introduce an error from the placement of the object.

There is an inherent error resulting from the size of the object. This has been explored under the topic of the limiting photometric distance. This Error has the same underlying root as the error from translated Center of Light: Light coming from places other than the assumed point location. The result of is a deviating measurement distance and angle for every measurement direction. The unknown luminance characteristic of the object leads to an uncertainty contribution that cannot be prevented. Instead, the measurement distance is increased to reduce uncertainty to an acceptable margin. There are good tools and rules of thumb in place to estimate this uncertainty. In contrast, the Center of Light is not taken into account at all. Therefore, the aim is to sensitize operators to the problems arising from Center of Light. Most importantly, this factor can be integrated into an uncertainty analysis by expressing the confidence in the position of the source.

Impact for a LID Measurement

How the object is placed and oriented can be described by a pose in the coordinate system of the goniometer, consisting of two components: rotation and translation. The rotation of the object is normally dictated by the measurement procedure. A desired orientation could be to align the optical axis of the object to a certain direction in goniometer coordinates. The effect of deviating rotation has already been investigated. Changing the rotation of the object in the goniometer in turn results in a rotated LID. As long as the region of interest is still inside the sampled solid angle, there is no loss of information. And a LID with the desired rotation can be calculated after the measurement. Another work examined the possibility to compare LID measurements of the same or similar objects with different rotations. This can be accomplished by aligning the two LIDs via correlation.

The effects of a translated source are more involved. A translation brings about a deviation of the measured luminous intensity that is different for every direction. This effect cannot be reversed from the measurement data alone. A LID can be thought of as a three-dimensional surface with the distance between origin and the surface translating to the luminous intensity in that direction. A measurement of a translated source results in a deformation of that surface. In the following example the LID of a lambert-source and its deformed counterpart are displayed. The mesh structure displays the expected sphere shape. The colored surface corresponds to the LID a goniometer measurement would produce for a translated source. The deformation is amplified considerably to make it visible. The surface color shows the actual deformation in %. In each case, the source was translated by the same value, along a single axis. The translation shown here amounts to 0.1% of the measurement distance. For a goniometer with 10m distance to the detector, this would mean a 10mm offset. Except for edge regions, where the LID approaches zero, the resulting error is small. There are multiple factors that can lead to an increase of this error, which are explored in the code sections of this notebook. The following images show simulations of this effect.



Determining the Center of Light

A definition for the Center of Light of an object can be constructed from its spatially distributed emission, more precise the model of light rays: The light emitted by the object consists of rays, that are modelled as lines. Using the point-source model forces the origin of all those rays to a single point, resulting in the error. The Center of Light is the best fitting origin for all rays of the actual object. It is important to note, that only the rays that exit the object are relevant for this approach. If rays originate from a light source within the object and then pass through a system of optics, only the diverted, exiting rays matter for the definition of the Center of Light. This is also the reason that the Center of Light does not need to coincide with the actual light source.

It is possible to generate a ray model of an object from a near-field goniometer measurement. The Center of Light can then be solved for. This problem can be described as error-minimization. Center of Light calculation is already an analysis feature, defining it as the point with the closest quadratic distance to all rays. This corresponds to the point, where the rays approximately coincide. The measurement can be modelled as unit rays or rays weighted with an intensity. The in the latter case, the weight has to be considered when estimating the center. Even tough the Center of Light can be measured, this has little practical use. It is infeasible to perform every photogoniometric measurement task with a near field goniometer. For most applications the Center of Light is unknown and has to be estimated. There are special cases, where this estimation is simple. For a source consisting of diffuse emitting surfaces, the Center of Light corresponds to the weighted geometric center of the light emitting surfaces. If the light source is constructed with a rotational symmetry, the Center of Light will lie on the rotation axis. Optics complicate the matter. Any element that either deflects, reflects or absorbs rays moves the Center of Light away from the actual light source. The Center of Light is not constrained to the dimensions of the object and can lie outside of it. A good intuition for the position of the Center of Light can in some cases be gained by looking into the source. In a setup, where a single light source is visible through the optical system, the source will appear roughly at the location of the Center of Light. This follows from the reversibility of optical paths. By tilting the object and tracking the depth of the apparent source is a good way to locate the plane, in which the Center of lIght lies. In any case the most important thing is to determine an uncertainty for the estimated point. This equivalent with to a confidence statement. This should be made conservatively, lowering the confidence for more complex sources.

A Model for the Influence of the Center of Light

The model is constructed in the context of a goniometer measurement. It defines a measurement setup based on a detector-based far-field goniometer. The results can also be applied to camera-based goniometers. The aim is to create a model that we can use to evaluate the effects of the center of light or equivalently the position of an object in the goniometer. Every other aspect of the measurement is treated as being ideal. To this purpose, the source is modelled as a point source. As outlined in the introduction, the influence of the object size and the Center of Light are closely linked. Nevertheless the Center of Light is investigated separately. Modeling both the source size and translation at the same time results in a very complex model. The effect of a translated source can also be separated reasonably well. Translating the source in one direction will impact the LID in a similar manner regardless of its size or shape.

The LID is generally calculated from the illuminance distribution in a certain distance from the photometric Center: A point light source with the distribution$I_s(\theta, \phi)$ placed in the center has an illuminance distribution according to the inverse-square-law.

$$E(\theta, \phi) = \frac{I_s(\theta, \phi)}{{r_c}^2} \cdot cos(\alpha_d)$$

With r_c being the measuring distance to the center and α_d being the incidence angle of the light striking the detector. Since the distance between source and detector is known, and the detector is oriented perpendicular to the source, the luminous intensity distribution can be calculated as follows:

$$I_m(\theta, \phi) = E(\theta, \phi) \cdot r_c^2 = I_s$$

The goniometer is described by a polar coordinate system. Since the source does no longer coincide with the goniometer origin, it gets its own polar coordinate system. All angle variables are marked according to the coordinate system or origin to which they are measured. θ_c and ϕ_c describe a direction originating from the photometric center and therefore correspond with the angles of the C-plane coordinate system. Likewise θ_s and ϕ_s describe angles relating to the light source. The measurement I_m is currently the same as the source LID I_s. If we now introduce a translation to this source, the measured LID deviates from the LID of the source. The aim of the model is to calculate the measured luminous intensity $I_m(\theta_c, \phi_c)$ for a given luminous intensity of the source $I_s(\theta_s, \phi_s)$ and a certain position of the source: $\mathbf{s} = (s_x, s_y, s_z)$. The whole model is first stated and subdivided into parts, that are explained separately.

$$I_m(\theta_c, \phi_c, \mathbf{s}) = \underbrace{I_s(\theta_s, \phi_s)}_{\text{direction}} \cdot \underbrace{\frac{r_c^2}{r_s^2}}_{\text{distance}} \cdot \underbrace{cos(\alpha_d)}_{\text{incidence}}$$

The influence of a translation can be separated in three different components. With the goniometer moved to a certain direction given by θ_c, ϕ_c the LID of the source is actually sampled at diverging angles: θ_s, ϕ_s. This component is designated as the **Direction Error**. Additionally, the distance between source and detector is now different for every direction, introducing a **Distance Error**. Lastly, the incidence angle of the light striking the detector changes, resulting in the **Angle of Incidence Error**.

1. **The Distance Error**

Because luminous intensity is not measured directly but instead inferred from illuminance, it requires the knowledge of the distance between source and detector. With the source offset from the photometric center, this distance r_s is different for every detector position. When converting to luminous intensity, the constant distance to the photometric center r_c is assumed instead, introducing an error. To describe this error, both source and detector are expressed as points in the cartesian coordinate system of the goniometer. The detector is modelled as revolving around the center and the source standing still. Position of the source: $\mathbf{s} = (s_x, s_y, s_z)$ Position of the detector for a particular measurement direction: $\mathbf{d}(\ _{\mathbf{c}},\ _{\mathbf{c}}) = (d_x, d_y, d_z)$ The measurement directions and the measurement distance define the detector position in polar coordinates. They are converted to cartesian coordinates as follows:

$$d_x(\theta_c, \phi_c) = r_c \cdot sin(\theta_c) \cdot cos(\phi_c)$$

$$d_y(\theta_c, \phi_c) = r_c \cdot sin(\theta_c) \cdot sin(\phi_c)$$

$$d_z(\theta_c, \phi_c) = r_c \cdot cos(\theta_c)$$

The actual distance between the source and particular detector position can then be calculated as $r_s(\theta_c, \phi_c) = ||\mathbf{sd}||$. Taking into account the actual distance for the illuminance calculation results in the factor $\frac{r_c^2}{r_s^2}$.

$$I_{m1} = I_s \cdot \frac{r_c^2}{{r_s}^2}$$

2. **The Directional Error**

The angles θ_s, ϕ_s describe the direction at which the detector is pointed at the source. To calculate these angles, the detector positions are required in the coordinate system of the source position. The coordinate transform is done by substracting the cartesian coordinates of source and detectors: $\mathbf{sd} = \mathbf{d} - \mathbf{s} = (sd_x, sd_y, sd_z)$ These cartesian coordinates are then converted back into the polar-coordinates as follows:

$$\theta_s(\theta_c, \phi_c, \mathbf{s}) = cos^{-1}(\frac{sd_z}{r_s})$$

$$\phi_s(\theta_c, \phi_c, \mathbf{s}) = tan^{-1}(\frac{sd_y}{sd_x})$$

These angles can now be inserted into the model function.

$$I_{m2} = I_s(\theta_s, \phi_s) \cdot \frac{r_c^2}{{r_s}^2}$$

Unlike the other two error components, the directional error is not only defined by the geometry of the measurement setup but also depends on the LID of the source that is measured. In the simplest case, when the LID is isotropic, this error does not occur, because the luminous intensity $I_s(\theta_s, \phi_s)$ is the same as $I_s(\theta_c, \phi_c)$. The magnitude of this error is defined by the difference in luminous intensity between the assumed and the actual measurement direction. It therefore scales with the gradient of the object's LID and is larger for directions with a large gradient in luminous Intensity.

4. **The Angle of Incidence Error**

Lastly, there is an error because a translation of the source changes the incidence angle with which the light hits the detector. This angle is denoted as α_d and can be obtained as the angle between two vectors. One vector being **cd** between photometric center to the detector and the other vector being **sd**. between the source and the detector. It is calculated as:

$$cos(\alpha) = \frac{\langle \mathbf{cd}, \mathbf{sd} \rangle}{||\mathbf{cd}|| \cdot ||\mathbf{sd}||}$$

This factor reduces the measured illumninace for an increasing incidence angle. This assumes a cosine characteristic of the detector, which is very reasonable for the typically small incidence angles. With the addition of this factor the final model function becomes:

$$I_m(\theta_c, \phi_c, \mathbf{s}) = I_s(\theta_s, \phi_s) \cdot \frac{r_c^2}{r_s^2} \cdot cos(\alpha_d)$$

Since all parts of the error are invariant to scaling, this model can also be used with relative units. For the following investigations all distances are expressed relative to the measurement distance r_c. This allows for generalized insights, that are applicable for all goniometer setups.

Uncertainty Analysis

A Luminous Intensity Distribution is constructed of multiple measurements. Each of these measurements is individually afflicted with an uncertainty. The uncertainty comprises various contributions. They stem from the mechanical measurement setup, the sensor and are also effected by the object to be measured. It is an ongoing effort to model the uncertainty of photogoniometric measurements. Currently most measurements lack an uncertainty evaluation entirely, which complicates a comparison of measurements. The uncertainty contribution of the sensor, either photometer or camera-based, can already be modelled. The same is true for the uncertainty contribution from the manipulator, that is positioning and orienting the source or detector. Since the location and orientation of the object on the goniometer is unknown, the uncertainty of the manipulator pose cannot be propagated to the source and its luminous intensity distribution and further through the sensor system. The Center of Light is therefore a missing link in the uncertainty chain, that prevents a holistic uncertainty evaluation.

What is necessary to close this gap is to define a position and its uncertainty. This uncertainty contains both the confidence in the Center of Light and the confidence in the mounting accuracy. It will need to be rules of thumb on how to assign this uncertainty. One such rule could be to take the dimensions of the object as a first estimate for the region in which the Center of Light is assumed to be in.