

Corinna Kroner

## Influencing factors on the performance of flow meters in liquid flow measurements

ISSN 2751-6598  
ISBN 978-3-944659-36-7

DOI 10.7795/110.20240926



# Physikalisch-Technische Bundesanstalt

Metrologie

PTB-M-6

Braunschweig, Oktober 2024

Corinna Kroner

## **Influencing factors on the performance of flow meters in liquid flow measurements**

ISSN 2751-6598

ISBN 978-3-944659-36-7

DOI 10.7795/110.20240926

<https://doi.org/10.7795/110.20240926>

## **Empfohlene Zitierweise/recommended citation**

Kroner, C., 2024.

*Influencing factors on the performance of  
flow meters in liquid flow measurements*

Braunschweig:

Physikalisch-Technische Bundesanstalt.

PTB-Bericht M-6. ISBN 978-3-944659-36-7.

Verfügbar unter: <https://doi.org/10.7795/110.20240926>

## **Herausgeber:**

Physikalisch-Technische Bundesanstalt

ISNI: 0000 0001 2186 1887

Presse und Öffentlichkeitsarbeit

Bundesallee 100

38116 Braunschweig

Telefon: (05 31) 592-93 21

Telefax: (05 31) 592-92 92

[www.ptb.de](http://www.ptb.de)



## **PTB-Report**

# **Influencing factors on the performance of flow meters in liquid flow measurements**

This project has received funding from the EMPIR programme co-financed by the Participating States and from the European Union's Horizon 2020 research and innovation programme.





## Table of Contents

1.	Introduction .....	1
1.1	Application of flow meters for fuel consumption measurements in the transport sector .....	3
1.2	Further considerations .....	3
2.	Stochastic measurement uncertainty component .....	5
3.	Systematic measurement uncertainty components .....	6
3.1	Uncertainty component related to the <i>k</i> -factor .....	6
3.2	Long-term stability of a flow meter .....	6
3.3	Temperature effects .....	7
3.4	Pressure effects .....	9
3.5	Influence of the test liquid .....	10
3.5.1	Reynolds number approach .....	10
3.5.2	Strouhal number .....	13
3.5.3	Uncertainty contribution associated with the Reynolds number .....	15
3.6	Performance assessments at dynamic flows versus stable flows .....	16
3.7	Other effects .....	21
4.	Combined measurement uncertainty .....	23
	Literature .....	25
	Annex I .....	27
	Annex II .....	28
	Annex III .....	29
	Annex IV .....	30
	Annex V .....	31

## **Acknowledgements**

This report contains results of investigations and measurements carried out by

Alexander Borchling, PTB

Oliver Bölker, RISE

Enrico Frahm, PTB

Andreas Hein, PTB

Torsten Jahn, PTB

Thomas Konnerth, PTB

Andrea Malengo, INRIM

Krister Stolt, formerly RISE

Heiko Warnecke, formerly PTB.

Their contributions are gratefully acknowledged.



## 1. Introduction

It is not possible to achieve error-free measurements. Accordingly, each measured value must be accompanied by a quality estimate – a measurement uncertainty. A distinction must be made between a stochastic uncertainty component (scatter of measured values, uncertainty type A, determined from the statistical analysis of a series of observations) and uncertainties evaluated by means other than the statistical analysis of a series of observations (uncertainty type B,). Basically, four situations can occur, as shown in Figure 1.

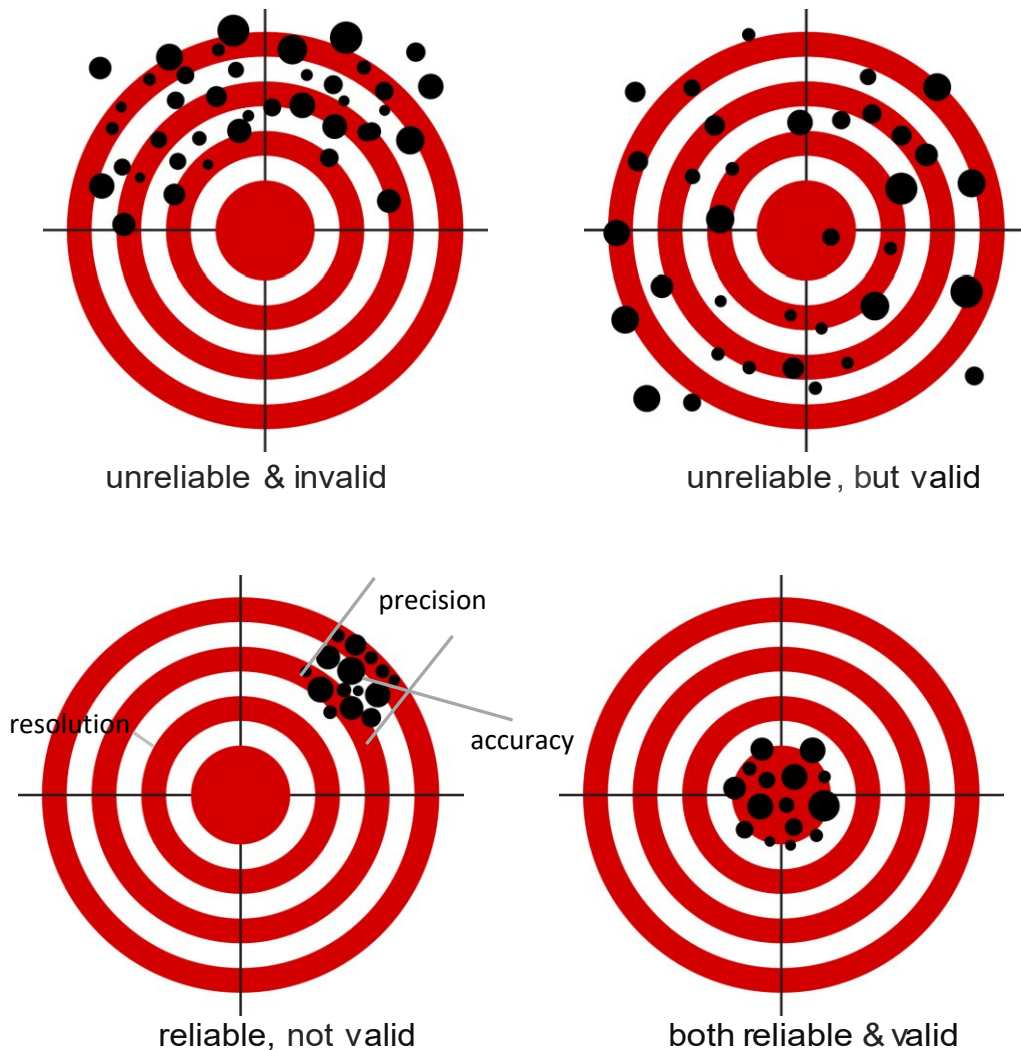


Fig. 1. Possible cases of occurrence of statistically distributed and systematic uncertainty components.

In addition to being expressed in SI units, a calibration factor or a calibration curve must always be accompanied by an uncertainty statement that includes both components.

The *k*-factor (e.g. pulses per volume) typically given for flow meters depends on the flow rate and on the pulse value of a flow meter. The pulse value is a device parameter that can be changed by the user and is independent of the flow rate. The value can depend on the measuring range, the smaller the range the higher the pulse value. This differs from meter manufacturer to meter manufacturer.

The higher the quality requirements of a measurement, the greater the need to understand the factors affecting the measurement accuracy of the flow meter used, and the greater the need to quantify these influences. The accuracy required by a calibrated flow meter for its subsequent application is essential. It must be taken into account here that any step down the calibration pyramid is inevitably associated with a further loss of accuracy.

The concept of uncertainty of measurement has become internationally established as a reliable metric for assessing the quality of a measurement. This is because the actual true value of any measurement quantity is difficult to find, whereas the uncertainty can almost always be quantified. A measurement result that includes a statement about the uncertainty and its confidence interval thus provides information about the reliability of a measured quantity. Internationally, the guidelines JCGM 100:2008 Evaluation of measurement data — Guide to the expression of uncertainty in measurement (GUM) resp. ILAC P14:09/2020 ILAC Policy for Measurement Uncertainty in Calibration are followed. It is common practice to specify the expanded measurement uncertainty. Its value defines an interval about the result of a measurement that can be expected to encompass a large fraction of the distribution of values that could reasonably be attributed to the measurand (cp. 4). This fracture corresponds to the confidence level (=coverage probability) of the interval.

In the following, uncertainty contributions of Type A and Type B are considered in more detail and in a simplified way for the case of flow measurements of liquids. As a simplification, it is assumed that the values of a measurand are uniformly distributed (= rectangular distribution) or follow a good approximation of a normal distribution. It is also assumed that all measurands are uncorrelated. This refers to the correlation of the random variables, not the correlation of the physical variables. If the measurement uncertainty of one input quantity varies with the measurement uncertainty of another input quantity in the same or opposite way, this may be due to the fact, that for different input quantities, e.g.

- the same measuring instrument,
- the same standard,
- the same reference value,
- the same energy source,

was used. More detailed information on measurement uncertainty considerations or background information can be found, for example, in the documents above or in ISO 5168(2005).

Depending on the measuring principle, flow meters are affected differently by different influencing factors. There may even be differences between meters from the same manufacturer, or between outputs of the same meters, be it mass pulse output, volume pulse output or analog current output as different uncertainty components play a role. Therefore, as a general rule, each device needs be characterised individually. The characterisation should be carried out for the output that is to be used in the subsequent application. The scope of the characterisation depends on the accuracy requirements associated with the final application of the device.

This report provides an overview of common influencing factors observed in liquid flow measurements. The content is largely based on deliverables from the EMPIR project Safest (20IND13), but also goes beyond these in some cases. The contributions of the consortium partners are gratefully acknowledged. There is no claim to completeness.

## 1.1 Application of flow meters for fuel consumption measurements in the transport sector

Fuel flow measurements are performed in various areas in the transport sector. They are used among others to measure fuel consumption on engine test beds, in the development of fuel injection systems, or for onboard fuel consumption monitoring. Moreover, flow data is used to parameterise engine control units. Further applications comprise the use of flow meters as master meters on test benches for secondary calibrations and as transfer devices for comparing test benches for flow meter calibrations as part of quality management. When measuring in situ on a ship both the accumulated consumption from port to port and the continuous consumption measurement are relevant. The latter are needed as input to simulation models and for assistance systems, e.g. for route optimization to minimize consumption. High demands are placed on the provision of these measurement results with validated total measurement uncertainties with short measurement times.

Depending on the application area flow meters based on different measuring principles are typically used. In addition to requirements for measuring accuracy, factors such as measuring range and meter weight also play a role in the selection of the meter type. A distinction must also be made between bare flow meters and those that are installed in conditioned systems ensuring stable temperatures and pressures.

## 1.2 Further considerations

Typically, the volume is the relevant measurand in the applications considered here. From this follows that the temperature during the measurements is of particular importance, as it has a direct effect on the recorded volume and on the comparison with a reference of any kind. It may become necessary to convert flow measurements to values for a fixed temperature level. In general, *k*-factors should be provided with the information at which temperature, resp. for temperature range they apply. If mass is used as reference, the conversion of the measured reference value from mass to volume leads to an additional uncertainty component, because the density value needed for this has also an uncertainty. Another uncertainty component arises from the fact that the temperature of the liquid the meter under test is exposed to, is not identical for instance to the liquid temperature in the weighing vessel.

In an ideal world, flow meters would be calibrated with the liquid used later in operation. However, this is not always feasible or economic. Nevertheless, a calibration with a liquid of at least the same order of magnitude in terms of density and viscosity as the subsequent operating medium is desirable. There are flow meters on the market which already make flow data available which are corrected for temperature or liquid-related effects. Whether the quality of the correction is sufficient depends on the application.

In Figure 2 examples of temperature-dependent density and dynamic viscosity changes of some hydrocarbons and water are given. The coefficient of expansion of the various hydrocarbons is approximately  $1 \cdot 10^{-3}/K$ . The viscosity changes are more heterogeneous and are much greater. They significantly determine the flow behaviour of the liquid.

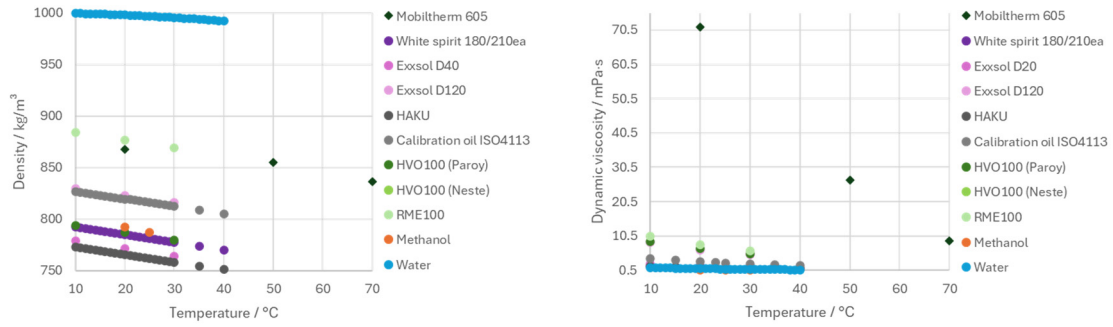


Fig. 2. Temperature-dependent change in density and viscosity of different hydrocarbon liquids and water at atmospheric pressure.

## 2. Stochastic measurement uncertainty component

To capture the stochastic measurement uncertainty component of a meter under test (MUT), repeated measurements and the subsequent observation of the mean value of the measurements as well as the standard deviation are typically used.

For repeated measurements, a distinction needs to be made between repeatability and reproducibility:

Repeatability refers to  $n$  consecutive measurements carried out without any changes in the measurement conditions. Reproducibility refers to  $n$  measurements with changes in the measurement conditions in between. For liquid flow measurements, this could mean taking a set of flow measurements in one day. Then the flow meter is removed and reinstalled in the measurement section on another day and a second set of flow measurements is performed. This provides insights into the uncertainty contributions due to e.g. handling. In general, the following applies with regard to the uncertainty components  $u$ :

$$u_{repeat} < u_{reprod} < u_{combined}.$$

To determine the best estimate of a measurand, the arithmetic mean  $\bar{x}$  of  $n$  observations  $x_i$  is considered:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i.$$

The standard deviation  $s$  is a measure of the scatter of the individual measurements of the same quantity

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}}.$$

One method to quantify  $u_{repeat}$  is to use the sample standard deviation  $\frac{s}{\sqrt{n}}$  of multiple measurements at each set point, weighted by the t-value (Student's distribution, see Annex II) at 95 % confidence for the number of points and then divide by 2 to give the standard uncertainty for the root sum of squares with other components.

This uncertainty component depends on the precision of the individual measurements and their number, i.e. by increasing the number of measurements, this measurement uncertainty component can be reduced. However, as the purpose is to capture the random variations in a process, the timescale for the data acquisition should reflect the presumed timescale for the variations. Collecting data at a millisecond interval for a process that fluctuates over several minutes will not characterize those variations adequately.

To estimate the reproducibility of the measurements, the standard deviation of the mean values measured e.g. on different days or after reinstallation of the MUT is taken as a basis.

It is important that the calculations are made individually for each flow point. It is good practise to carry out at least five repetitions per measuring point or test profile (section 3.7).

### 3. Systematic measurement uncertainty components

Systematic uncertainty components that need to be considered and, if necessary, accounted for in a flow measurement depend on various factors:

- the flow meter type
- the flow range of interest
- the demand for accuracy
- the deviations of the operating conditions from the calibration conditions.

In general, the systematic uncertainty contributions taking into account the maximum values are considered in the systematic uncertainty contributions rather than average values. This ensures that the actual measurement uncertainty is not underestimated in any case.

#### 3.1 Uncertainty component related to the $k$ -factor

The determination of a  $k$ -factors from the flow measurements is associated with a certain uncertainty (Fig. 3, 4). The maximum residual (= difference observation-fitting)  $r_+$  and  $r_-$  is used to determine the associated uncertainty contribution. All values between these limits are considered equally probable. This situation is described with a rectangular probability density. The uncertainty component  $u_{MUT}$  is calculated according to

$$u_{MUT} = \frac{|r_+| + |r_-|}{2\sqrt{3}}.$$

This consideration should be made depending on the flow rate.

#### 3.2 Long-term stability of a flow meter

The characterisation of the measuring behaviour of a flow meter includes the indication of the device stability. In contrast to short-term reproducibility (section 2), the measuring performance over several years is considered here. Ideally, the flow meter should be calibrated at regular intervals on the same test rig at comparable conditions. An example of measurements carried out in different years is shown in Figure 3.

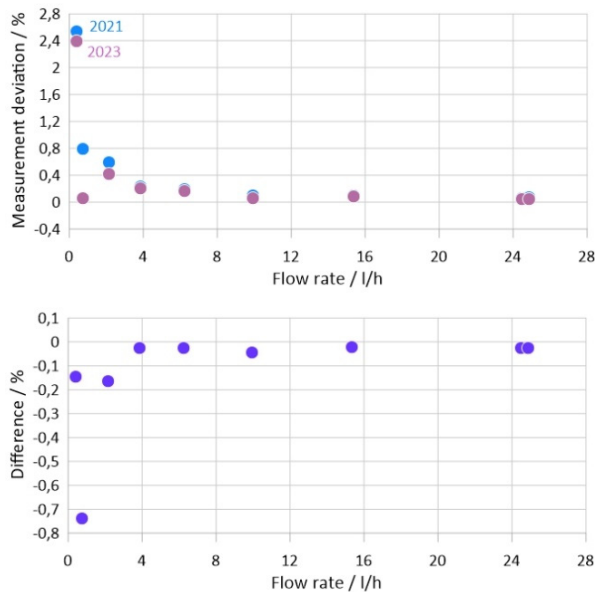


Fig. 3. Example of the drift of a flow meter; top: measurement deviations of the years 2021 and 2023, bottom: difference between the measurement deviations. Test liquid: water

For the estimation of the drift-related measurement uncertainty component  $u_{\text{drift}}$ , the upper  $a_+$  and lower limits  $a_-$  of the drift values are used, with all values in between considered equally probable. Thus, here again a rectangular probability density needs to be applied.  $u_{\text{drift}}$  is then obtained from

$$u_{\text{drift}} = \frac{|a_+ - a_-|}{2\sqrt{3}}$$

### 3.3 Temperature effects

In general, different temperature effects can affect the measurement of a flow meter and depending on the test medium and meter type, these can vary in order of magnitude and are of different relevance. Effects that can affect a flow measurement comprise, e. g.:

- the flow properties of a medium depend on viscosity, which in turn depends on the temperature;
- depending on the temperature (liquid or ambient), flow meters can slightly change their geometry and/or their elastic properties. This changes their measurement behaviour, which in turn affects the  $k$ -factor; examples of this are shown in Figure 7, 8 and 11;
- the volume of a liquid is temperature-dependent; therefore, a meter that measures volume directly will capture a different amount of liquid as the temperature changes and the volume measurement is not corrected for this effect.

The effects are partly non-linear and partially compensating. It is not uncommon for flow meters to have a built-in temperature compensation provided by the manufacturer. Whether this correction is sufficient depends on the application.

To illustrate the order of magnitude of temperature effects in liquids and blends thereof, Figure 4 shows an example of the change in density and thermal expansion coefficient as a function of temperature, but also as a function of the percentage of ethanol blended. The density and thermal expansion coefficient shift systematically as a function of the mixing ratio.

The temperature effect in the  $k$ -factor can be corrected for each individual flow rate if corresponding measurement data is available. In Figures 5 and 6 a fictive example is given for the

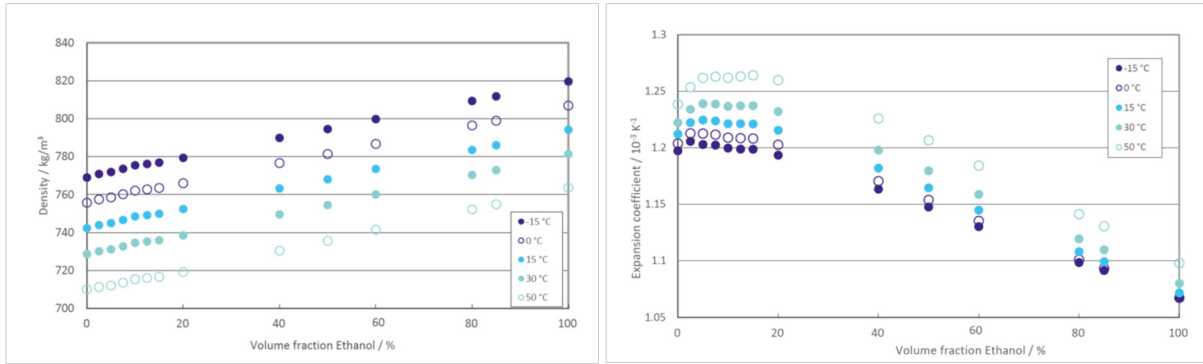


Fig. 4. Dependence of density and coefficient of thermal expansion on the mixing ratio for ethanol-gasoline blends. Summer quality gasoline (after Wolf, 2014).

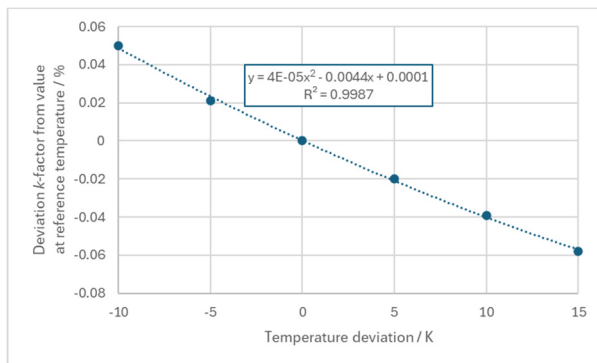


Fig. 5. Fictive example of the deviation of the  $k$ -factor from the value at a given reference liquid temperature versus deviation of measurement temperature from reference temperature.

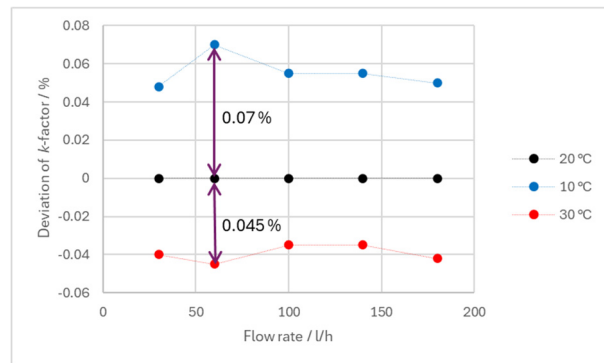


Fig. 6. Fictive example of systematic change in  $k$ -factor from value at reference temperature depending on liquid temperature; maximum change of  $k$ -factor 0.07 % for  $\Delta T = -10$  K.

deviation of the  $k$ -factor from the value at a reference temperature versus the difference of the temperature during the measurements from the reference temperature. Correspondingly, it is possible to adapt the  $k$ -factor at a given reference temperature to the measurement temperature if needed.

If there is a significant temperature dependence of the  $k$ -factor and this is not taken into account, the maximum expected effect must be considered as a measurement uncertainty component for the flow rate of interest. This means that by correcting for this effect, this uncertainty component can be reduced. The changes can be in the range of some tenths of a percent.

If a flow meter has a mass output and a volume output, it cannot automatically be assumed that both outputs will provide data with identical effects. The example in Figure 7 illustrates this situation. While the data of the mass output shows a temperature-related spread of 0.08 %, with the value of the  $k$ -factor decreasing with increasing temperature, the data of the volume output do not show such a noticeable dependence. The example also shows that the  $k$ -factor is slightly different for the two outputs. This temperature dependence of the  $k$ -factor of the flow meter of Figure 7 is also of a similar order of magnitude when the test liquid and test bench are changed (Figure 8). This means this is a temperature effect in the flow meter data.



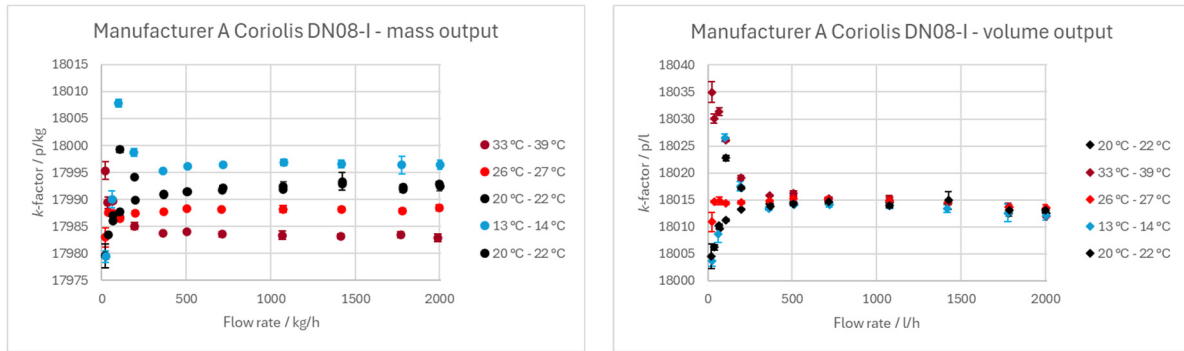


Fig. 7.  $k$ -factors obtained for mass and volume output of a Coriolis flow meter (pulse value: 18000 p/kg resp. p/l) at different liquid temperatures. Test liquid: water,  $u_{\text{test bench}} (k=2)$ : 0.05 %.

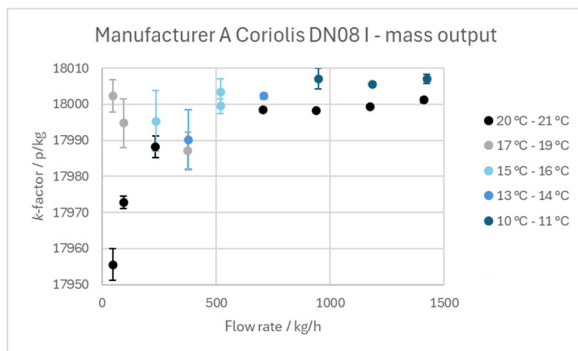


Fig. 8.  $k$ -factors obtained for the mass output of a Coriolis flow meter (pulse value: 18000 p/kg) at different liquid temperatures. Test liquid: white spirit,  $u_{\text{test bench}} (k=2)$ : 0.1 % - 0.05 %.

### 3.4 Pressure effects

In principle, there are two types of pressure effects prevalent in a flow measurement:

- liquid pressure affecting the flow meter; high pressure can cause an increase in the flow meter diameter;
- pressure affecting the liquid itself; an increase in pressure results in a decrease in the liquid volume; for typical liquids at standard testing conditions the liquid can be assumed to be incompressible; this assumption must be reconsidered if the liquid contains gas.

An example for the impact of line pressure on the performance of a Coriolis flow meter is shown in Figure 9. Often flow meters have built-in pressure compensation provided by the manufacturer. The sufficiency of this correction depends on the application. Liquid pressure can also affect the elasticity of the measurement tubes of a Coriolis flow meter. Due to a pressure increase the tubes become more rigid which leads to a decrease in Coriolis forces and an under-registration of the mass flow. For some designs, the so-called Bourdon Effect occurs – the curved tubes stiffen with increasing liquid pressure and attempt to straighten to their original tube form (Wang & Hussain, 2010).

Dependencies can be determined according to the procedure described in section 3.3 for the effect of temperature. Figure 10 shows an example of the  $k$ -factor of a servo-controlled PD-

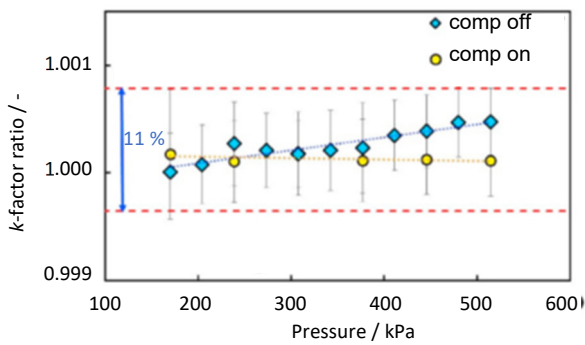


Fig. 9. Example of effect of inline pressure on the  $k$ -factor of a Coriolis flow meter (after Costa et al., 2020); ratio of calibration factor determined with and without pressure correction and manufacturer's meter calibration factor, flow rate: 23.5 t/h.

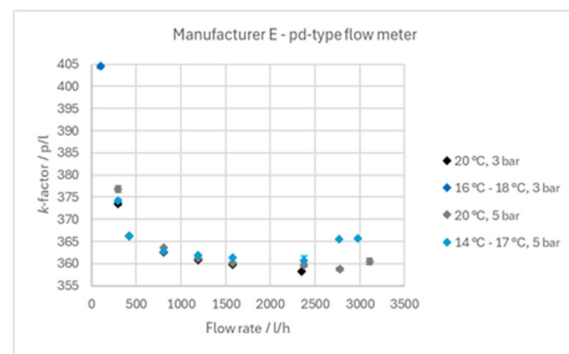


Fig. 10. Example of a servo-controlled PD-type flow meter without effects due to liquid temperature or liquid pressure in the  $k$ -factor without any corrections applied. Test liquid: white spirit,  $U_{\text{test bench}} (k=2)$ : 0.1 % - 0.05 %.

type flow meter intended for maritime applications, where the factor is not affected by liquid temperature or liquid pressure. The  $k$ -factor changes significantly non-linearly with the flow rate, which should be taken into account when using the meter.

### 3.5 Influence of the test liquid

#### 3.5.1 Reynolds number approach

Flow meters are often calibrated with water, and in selected cases with a few other liquids, usually hydrocarbon-based. It is neither feasible nor economical to calibrate flow meters for a wider range of liquids. Therefore, an approach is needed to transfer the calibration for one liquid to another one. The transport properties of liquids which are relevant for the flow patterns are determined primarily by their viscosity and secondarily by their density. The Reynolds number  $Re$  with:

$$Re = \frac{\rho v d}{\eta} = \frac{4 Q_m}{\pi \eta d}$$

$\rho$ : liquid density

$v$ : liquid velocity

$\eta$ : dynamic viscosity of the liquid

$d$ : pipe diameter MUT

$Q_m$ : mass flow

takes these dependencies into account. It is therefore generally used when converting the calibration of a flow meter to other viscosities and densities, assuming that a similar Reynolds number corresponds to similar flow behaviour.

In Figure 11 an example of the measurement deviations of a Coriolis flow meter for different test liquids and at different liquid temperatures is given. The associated Reynolds numbers are also shown. The measurements with water at different temperatures (diagrams below) illustrate that at flow rates below ~50 l/h temperature-related auto-zero effects occur. The measurement errors in this flow range with other test liquids are partly significantly greater than with water, so a combination of influences must be assumed here. This effect can also be seen to some extent in the variation of the Reynolds numbers.



Fig. 11. Measurement deviations and Reynolds numbers of a Coriolis flow meter (pulse value: 60000 p/kg resp. 60000 p/l) for different types of liquids and at different liquid temperatures. Partly different scaling of x- and y-axes. Top: Temperature-dependent dynamic viscosity and density of the hydro-carbon test liquids.

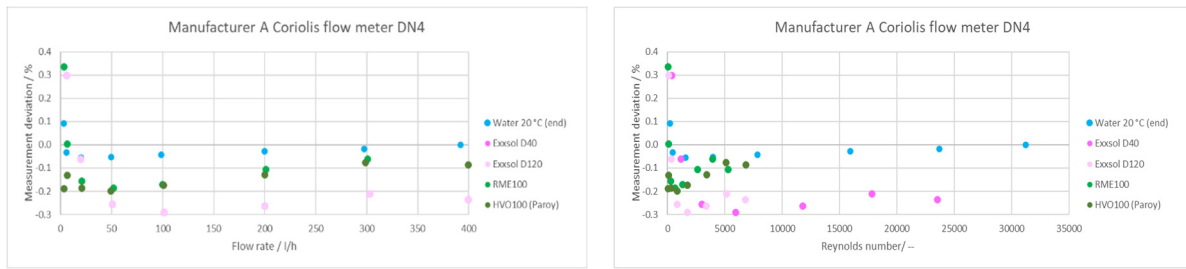


Fig. 12. Measurement deviations and Reynolds numbers of a Coriolis flow meter (pulse value: 60000 p/kg resp. 60000 p/l) for different types of liquids at a liquid temperature of 20 °C.

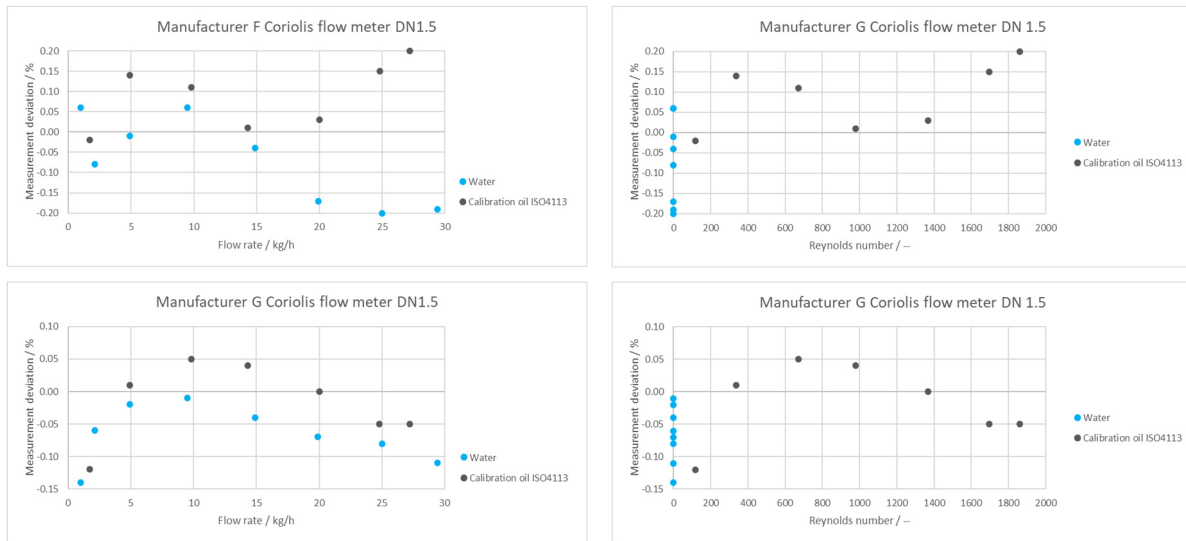


Fig. 13. Measurement deviations obtained for Coriolis flow meters of two different manufacturers for water and calibration oil ISO4113 depending on flow rate and Reynolds number; liquid temperature: 20 °C.

The influence of the differences in transport properties on the measurement deviation is shown in Figures 12 and 13, which summarise the deviations of Coriolis flow meters from three different manufacturers for a liquid temperature of 20 °C. Depending on the flow rate and test liquid, the measurement deviations are up to -0.3 % for flow rates above or equal 50 l/h. The sections where the measurement deviations change linearly with the Reynolds number can be used to transfer the calibration from one liquid to another, provided that the liquid properties of the target liquid are between those for which measurement results are available. Moreover, measuring ranges must be chosen appropriately so that a transfer of a calibration becomes possible.

The diagrams shown in Figures 12 and 13 demonstrate that results obtained for one flow meter cannot be transferred to another one even if it is based on the same measuring principle. How a combination of temperature effects and fluid properties can affect the  $k$ -factor is shown by the example of two identical screw spindle flow meters whose housings are made of different materials (Figure 14). Three different hydrocarbons were used as test liquids. Additionally, the liquid temperature was changed. It can be seen that the  $k$ -factor increases systematically with decreasing viscosity for the same liquid temperature. However, due to the different housing materials and their different thermal expansion coefficients, the screw spindle meter with a stainless-steel housing shows a wider spread in the  $k$ -factor (0.7 % at 6000 l/h) than the meter with a carbon steel housing (0.4 % at 6000 l/h). In addition, changes in the  $k$ -factor for flow rates above 2000 l/h also become non-linear for this meter for Exxol D40, the liquid with the lowest viscosity. The plots also show that the curves overlap only to a limited extent when the

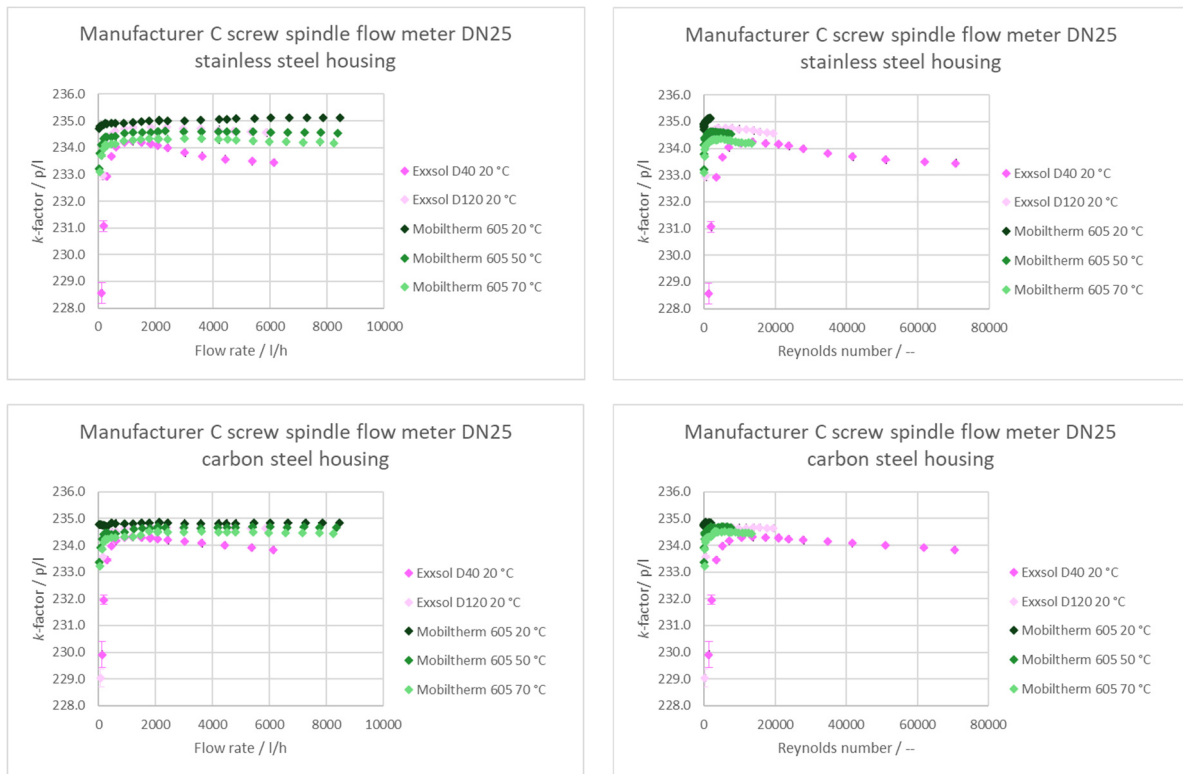


Fig. 14.  $k$ -factors obtained for two screw spindle flow meters with different housings in dependence of the flow rate and the Reynolds number for three different hydrocarbons and at different liquid temperatures;  $u_{\text{test benches}} (k=2)$ : 0.1 % - 0.08 % for measurements at 20 °C, 0.2 % - 0.05 % for measurements at 50 °C and 70 °C. For viscosities see Figure 2.

differences in viscosity between the test liquids are too great. This limits their use for the transfer of the  $k$ -factors to a liquid with an in-between viscosity.

As the examples show, a transfer of the  $k$ -factor based on the Reynolds number only makes sense for measurement ranges where the factor changes only insignificantly with the Reynolds number and relevant, additional effects can be excluded. This would mean that the  $k$ -factors would have to be corrected in advance for the effect of thermal expansion, or at least all factors would have to be determined for the same liquid temperature so that the temperature effect inherent in the device occurs in the same way. Further examples for the effect of liquid properties on flow meters are shown in Figure 20 for a servo-controlled PD flow meter and for a turbine in Annex IV.

### 3.5.2 Strouhal number

A method to gain more insight into the dependencies in the case of volumeters such as PD flow meters is to look at the Strouhal number, kinematic viscosity and temperature-corrected Reynolds number (Paton et al., 2008, Chen et al., 2011). By

$$d = d_0 (1 + \alpha (T - T_0))$$

with

- $d$ : diameter of flow meter pipe at test temperature
- $d_0$ : diameter of flow meter pipe at reference temperature
- $\alpha$ : linear thermal expansion coefficient of pipe material
- $T$ : test temperature

$T_0$ : reference temperature

a temperature correction is applied.

The Strouhal number is obtained from

$$\text{Strouhal} = k\text{-factor} \cdot d^3.$$

In Figure 15 the data of Figure 14 are plotted as Strouhal number versus Reynolds number. The curves in the two figures differ only slightly. For further investigations only the part of the curves where the flow meters show the typical Reynolds number dependent characteristic is relevant. This is the case for several liquids / liquid temperatures at a Reynolds number of 13360. A plot of the Strouhal number versus kinematic viscosity at 13360 for different test liquids and liquid temperatures can provide further information on the dependencies. Figure 16 shows a significantly different behaviour of the two flow meters. For both meters the Strouhal number increases with kinematic viscosity, but the rate is slightly different for different liquid temperatures. If sufficient data is available and other dominant effects can be excluded, the relationship between Strouhal number and kinematic viscosity can be used to derive a viscosity correction for a flow meter if needed.

In general, liquid effects cannot be ruled out even with flow meters that measure the mass flow directly. This was demonstrated as part of the international key comparison CCM.FF-K2.2011,

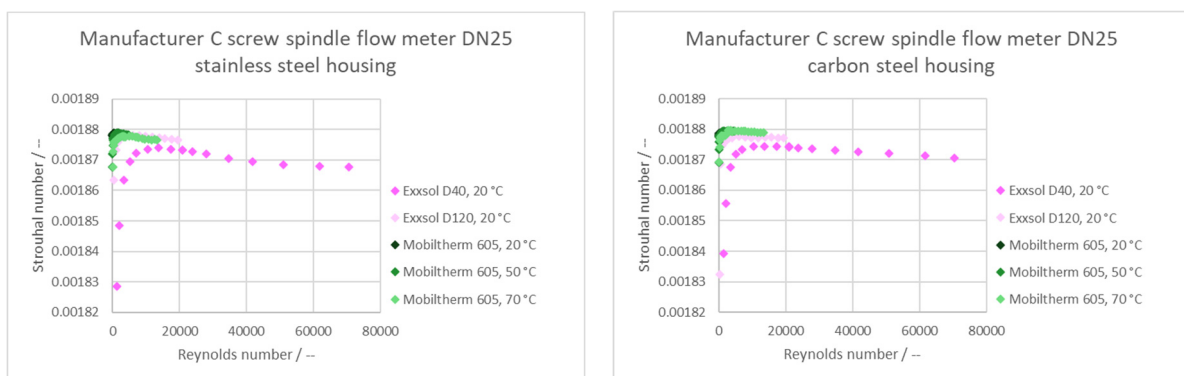


Fig. 15. Strouhal number versus Reynolds number of the data shown in Figure 14 of two screw spindle flow meters with different housings for different test liquids and different liquid temperatures.

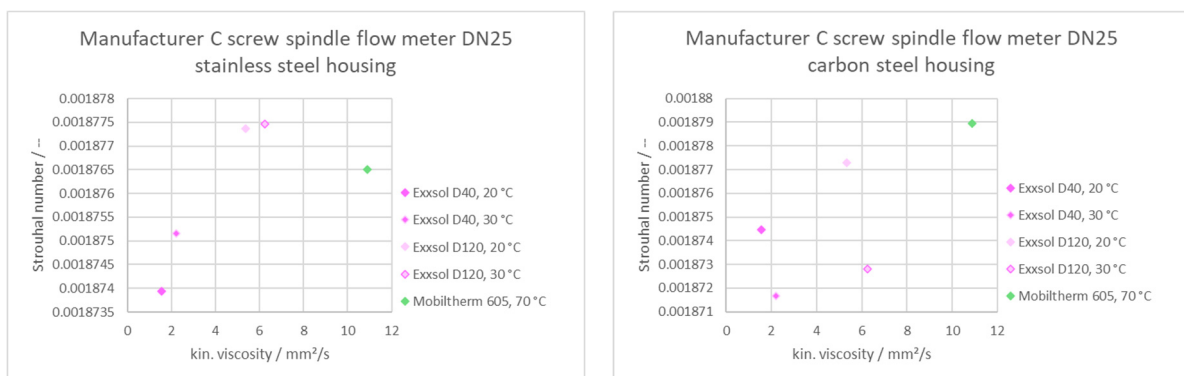


Fig. 16. Strouhal number versus kinematic viscosity of two screw spindle meters for different test liquids and at different liquid temperatures for a Reynolds number of ~13360.

in which various hydrocarbons were measured in addition to water. Measurement deviations of 0.1 % and deviations between the flow meters of the same order of magnitude were found for the hydrocarbon-based liquids.

### 3.5.3 Uncertainty contribution associated with the Reynolds number

Since, in addition to the test bench uncertainty, all measurement errors are themselves subject to standard deviations. These are added quadratically in the measurement uncertainty analysis (cp. 4). The uncertainty components of the individual measured variables of the Reynolds number must therefore also be taken into account, the uncertainty contribution of a calibration calculated in this way is always greater than for measured values.

The uncertainty contribution of the Reynolds number can be derived from:

$$u_{Re} = \sqrt{\left(\frac{Q_m}{Re} \frac{\delta Re}{\delta Q_m} u_{Q_m}\right)^2 + \left(\frac{\eta}{Re} \frac{\delta Re}{\delta \eta} u_{\eta}\right)^2 + \left(\frac{d}{Re} \frac{\delta Re}{\delta d} u_d\right)^2}$$

with

$$\frac{Q_m}{Re} \frac{\delta Re}{\delta Q_m} = 1$$

$$\frac{\eta}{Re} \frac{\delta Re}{\delta \eta} = -1$$

$$\frac{d}{Re} \frac{\delta Re}{\delta d} = -1$$

using the definition of the Reynolds number. Furthermore,

$$u_{Q_m} = \frac{dQ_m}{Q_m}$$

and  $dQ_m$  as standard deviation of the flow measurement,

$$u_d = \frac{dd}{d}$$

$dd$  as accuracy of the pipe diameter determination,

$$u_{\eta} = \sqrt{u_{\eta_m}^2 + \left(\frac{1}{\eta} \frac{d\eta}{dT} \delta T\right)^2}$$

$u_{\eta_m}$ : accuracy of the viscosity measurement itself

$\delta T$ : accuracy of the temperature measurement at which the viscosity is measured.

### 3.6 Performance assessments at dynamic flows versus stable flows

At operating conditions flow meters seldom experience constant flow rates as during laboratory calibrations. Typically, they are exposed to variable and often irregular flow changes. The question is, if and how these dynamic flow changes affect the measurement performance of the meters. Different approaches are followed here. To gain insights into the response function of a meter, for instance tests with periodic or cyclic flow changes with varying repetition times are carried out. Furthermore, information about the response behaviour of meters can be obtained by measuring step responses, i.e. abrupt flow changes such as an ideally instantaneous change from constant flow rate to zero flow or vice versa. More extensive tests comprise the realization of flow changes typical for the application of interest. Some more information on the different tests can be found for instance in Wiklund and Peloso (2002), Shinder and Moldover (2009), Warnecke et al. (2022) or Yoshida and Furuichi (2023). In the EMPIR project Safest measurement infrastructure was developed which is capable to generate dynamic flow changes according to the Worldwide harmonized Light Duty Test Cycle (WLTC) or the World Harmonized Transient Cycle (WHTC). More information can be found in D1 “*Report on the use of a new infrastructure for assessing the measurement performance of the flow meters and other systems, which are used in fuel consumption measurements of passenger cars and trucks, under dynamic load changes*”. In D8 “*Technical guide for the assessment of flow meter performance under dynamic load changes*” details on the analysis of flow meter data at dynamic load changes are given. Which of the above-mentioned tests is appropriate depends on the application and the area of interest. When it comes to precise dosing, delay times of flow meters may become relevant. If the measurement of both rapid flow changes and zero flows is essential, then a test regime should reflect this accordingly. Depending on the type of dynamic flow change different results are to be expected for a flow meter.

A dynamic test regime enables a much more comprehensive characterisation of the measuring behaviour of a flow meter and yields information about the uncertainty component arising from the use of a fixed  $k$ -factor or a flow rate-dependent  $k$ -factor, either of the two derived from measurements at constant flow rates. Deviations in accumulated mass or volume obtained from the different measurement regimes could mean that the  $k$ -factor(s) applied do not take sufficiently well into account their flow rate dependency for the application of interest or that there is an additional dynamic component in the meter response. So far, no indications for a dynamic response component of a flow meter were found.

Figures 18 and 19 show some first results obtained for four different test profiles given in Figure 17 for a Coriolis flow meter. In Table 1 the values of the accumulated mass derived from the measurement and when using a nominal  $k$ -factor are compared for the example of the Coriolis flow meter. In case of water as test liquid a general good agreement between the  $k$ -factors derived from the profile measurements and the static measurements exist. There also is a good to very good agreement between measured and determined accumulated mass values. A comparable result is obtained when realizing the profiles using calibration oil which has a significantly higher viscosity (Figure 2). Maximum difference between flow meter-derived total mass and reference is 0.5 %. The shorter the profile and/or the smaller the flow rates, the greater the differences become. To a greater extent this can be explained by auto zero effects (cp. 3.7, Figure 19). When comparing the  $k$ -factors of the measurements at constant flow rates in the flow rate ranges of the test profiles with those derived from profile measurements a good agreement is found (Figure 19).

Similar principal variations in the  $k$ -factor are observed for a servo-controlled PD flow meter installed in a conditioned system and the test liquids calibration oil and cold cleaner (Figure



20). For comparison, the results of the calibration at constant flow rates are given. Again, a clear similarity between the  $k$ -factors obtained for the individual test profiles and the  $k$ -factors derived from the static measurements for the main flow rate ranges of the test profiles is found. In consequence, negligible differences in accumulated mass obtained from the flow meter measurement and the references occur. In this case, the fact that a flow rate-dependent  $k$ -factor is typically used also has an effect.

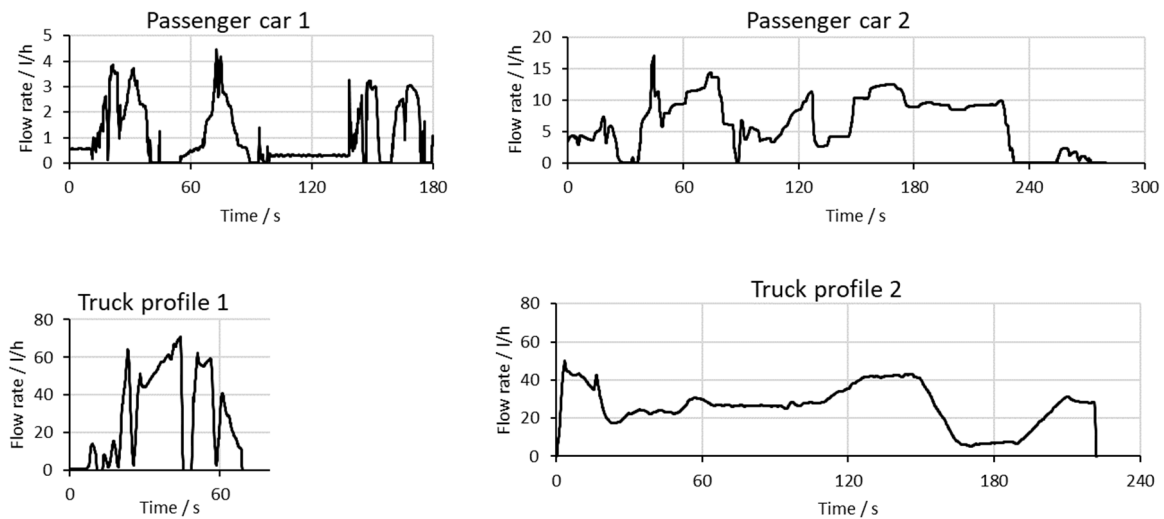


Fig. 17. Vehicle test profiles derived from the WLTC and the WHTC.

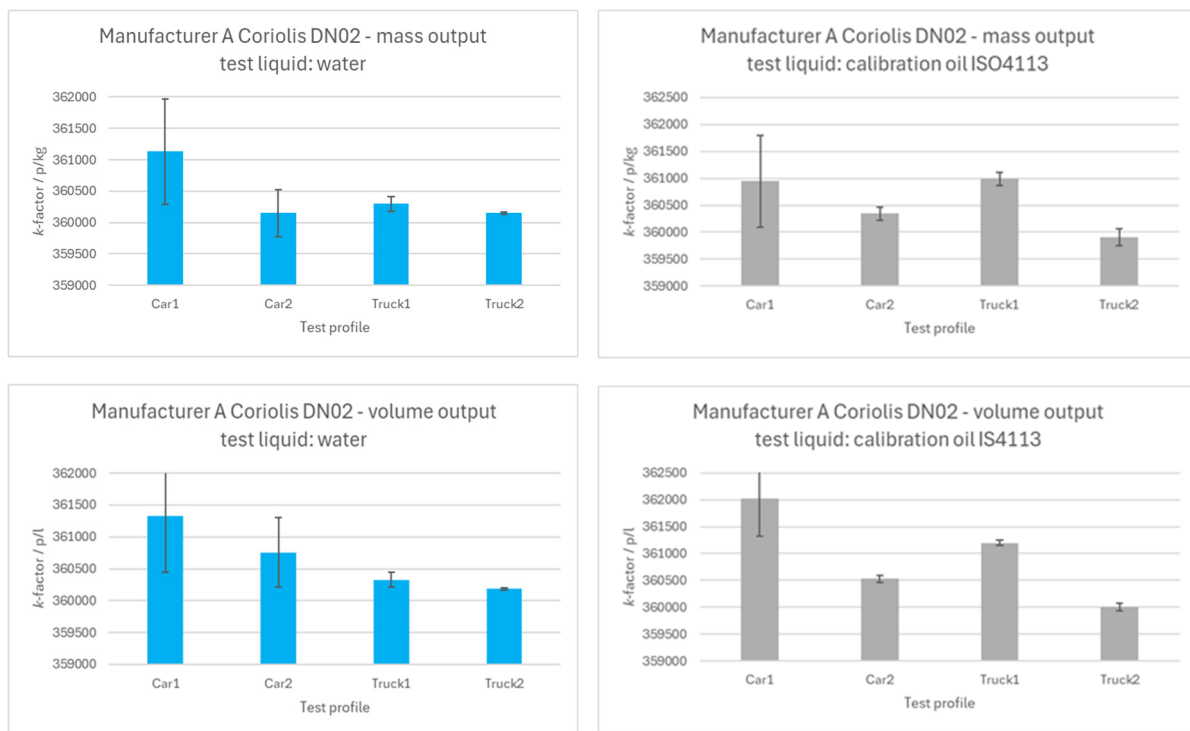


Fig. 18.  $k$ -factors obtained for a Coriolis flow meter DN02 (top: mass output, bottom: volume output, pulse value 360000 p/kg resp. 360000 p/l) for the test profiles of Figure 17 and two different test liquids.  $u_{\text{test benches}} (k=2)$ : 0.1 % - 0.05 %.

Tab. 1. Comparison of accumulated mass measured by the reference and obtained based on the nominal  $k$ -factor of the Coriolis flow meter of Fig. 18.  $u_{\text{test rigs}} (k=2)$ : 0.1 % - 0.05 %.

Profile	Water		Calibration oil ISO4113	
	mass <sub>ref</sub> / kg	deviation from mass <sub>ref</sub> when nominal $k$ -factor is used / %	mass <sub>ref</sub> / kg	deviation from mass <sub>ref</sub> when nominal $k$ -factor is used / %
Car1	0.0537 ± 0.0002	0.199	0.0449 ± 0.0001	0.507
Car2	0.4884 ± 0.0003	0.061	0.3896 ± 0.0003	0.148
Truck1	0.5554 ± 0.0007	0.761	0.3784 ± 0.0004	0.332
Truck2	1.6219 ± 0.0020	0.041	0.9438 ± 0.0018	0.002

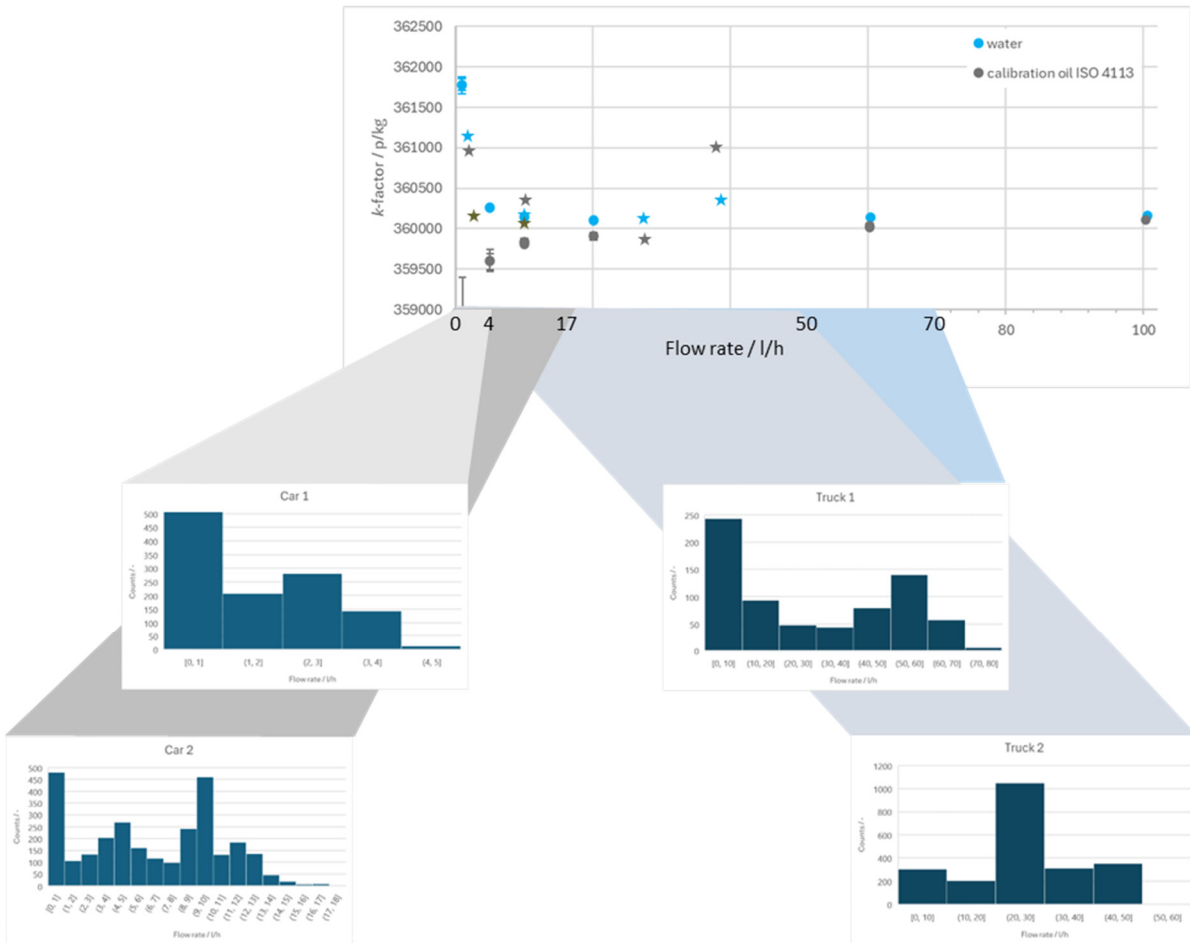


Fig. 19. Top: ●  $k$ -factors derived from measurements at constant flow rates of a Coriolis DN02 flow meter (pulse value 360000 p/kg) and ★  $k$ -factors derived from profile measurements and two different test liquids.  $u_{\text{test benches}} (k=2)$ : 0.1 % - 0.05 %.; bottom: Histograms of the flow rates of the test profiles of Figure 17.

A largely comparable result is obtained if a considerably longer test profile with different flow characteristics is considered. The test profile shown in Figure 21 was based on the fuel demand of the engine control unit of a ferry navigating in the harbour area.

Figure 22 shows some initial results obtained for the test profile given in Figure 21, obtained for a Coriolis flow meter, a servo-controlled PD flow meter and a screw spindle flow meter.

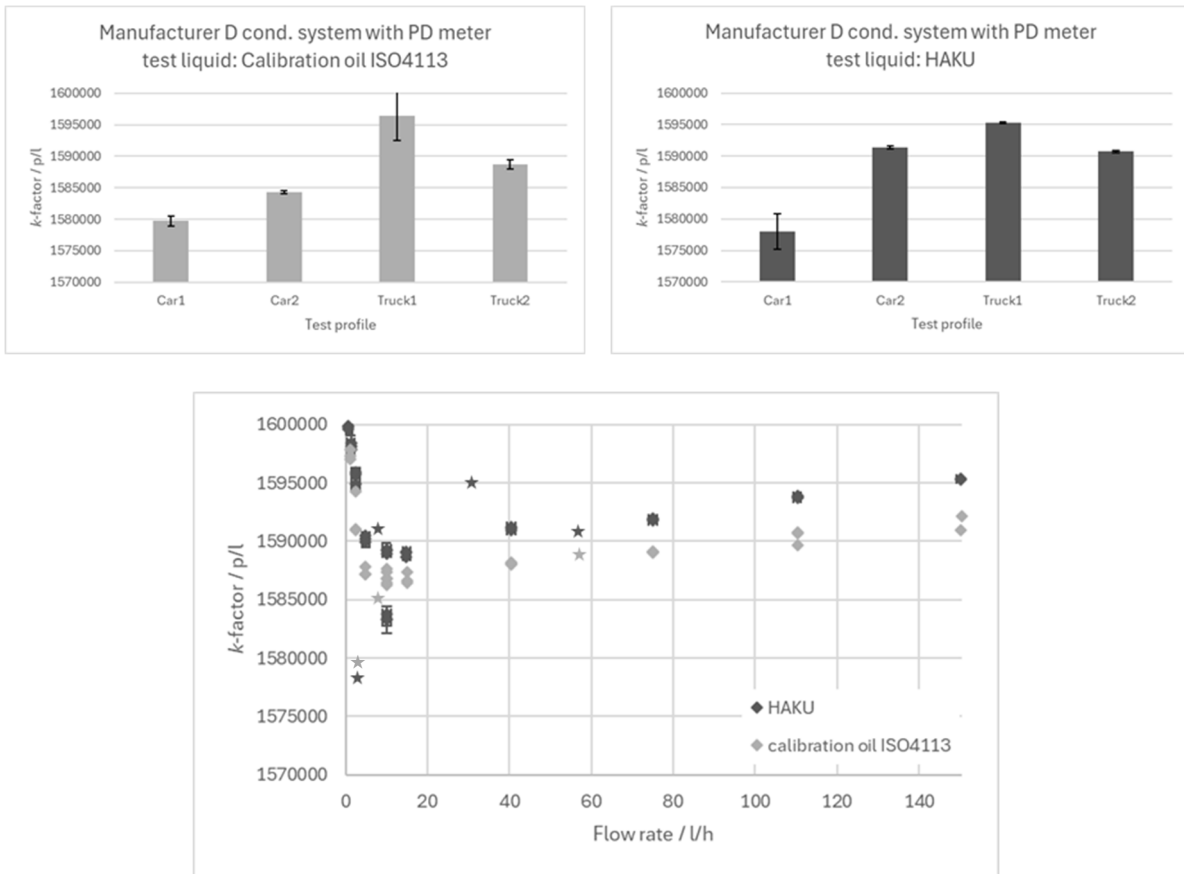


Fig. 20. Top:  $k$ -factors obtained for a conditioned system with a servo-controlled PD flow meter for the test profiles of Figure 17 and two different test liquids, bottom: ●  $k$ -factors derived from measurements at constant flow rates and ★ from profile measurements for the same test liquids.  $u_{\text{test benches}} (k=2)$ : 0.1 % - 0.05 %. Density and viscosity data of HAKU and calibration oil ISO4113 are listed in Annex V.

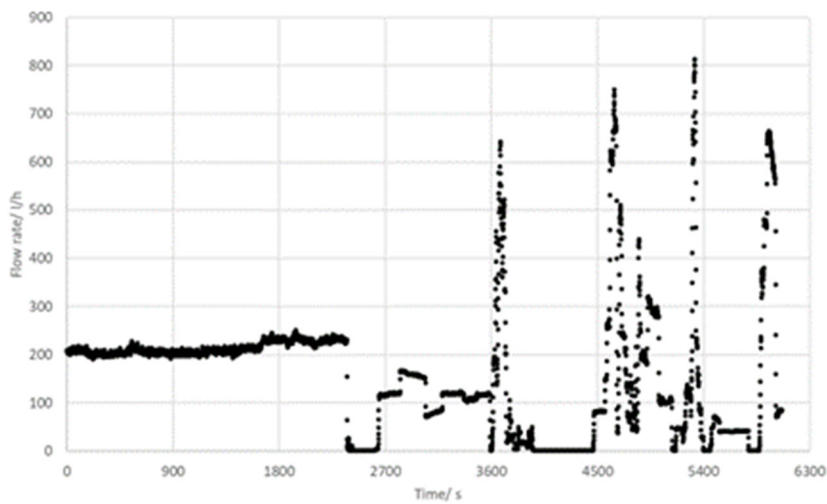


Fig. 21. Fuel demand of the engine control unit of a ferry navigating in a harbour area.

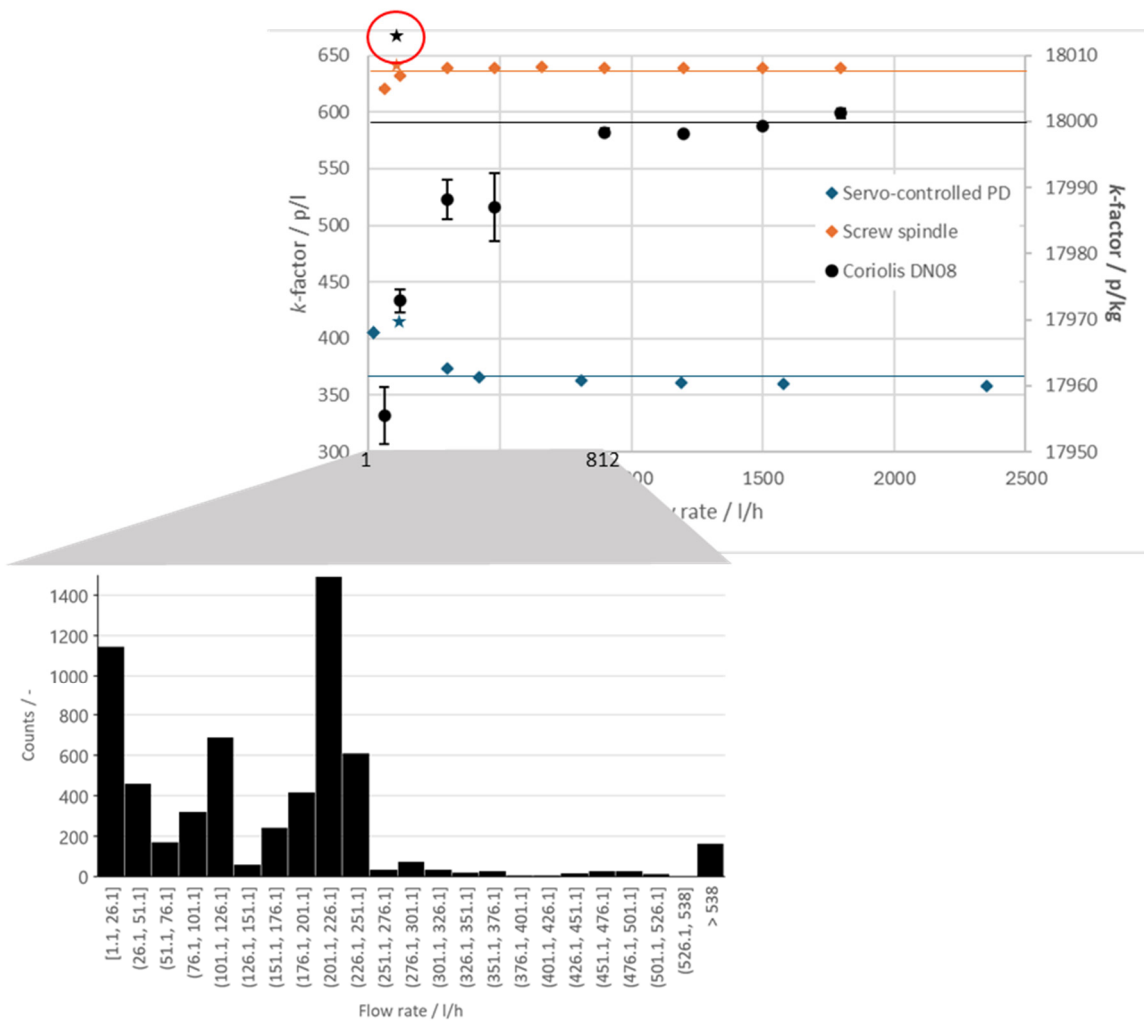


Fig. 22. Top:  $k$ -factors of a Coriolis DN08, a screw spindle DN20 and a servo-controlled flow meter obtained from measurements at constant flows ( $\bullet$  p/kg,  $\blacklozenge$  p/l) and  $k$ -factors derived from profile measurements ( $\star$ ); bottom: Histogram of the flow rates of the test profile shown in Figure 21. Test liquid: white spirit 180/210ea, liquid temperature: 20 °C – 22 °C,  $U_{\text{test bench}} (k=2)$ : 0.1 % - 0.05 %.

Again, a good basic agreement exists between  $k$ -factors derived from the measurements at constant flow rates and the profile derived  $k$ -factors when the flow rate range of the test profile is taken into account. The discrepancy of  $\sim 0.6$  % between the nominal  $k$ -factor of the Coriolis flow meter of 18000 p/kg and the actually obtained is clearly larger. The fact that the  $k$ -factor obtained from the profile measurements is higher than the  $k$ -factors derived from the static calibrations is also striking. Additional measurements showed that auto zero related effects can be excluded as explanation. Cause for the deviation is likely the reduced measurement performance of Coriolis flow meters in their low flow rate range. The performance decrease is due to the increased effect of the instability inherent of the instrument due to its measuring principle when approaching zero flow. If a cutoff flow rate of 14 l/h is set in this example and for the meter considered here, the  $k$ -factor then obtained from the profile measurements agrees with the nominal  $k$ -factor. Also, the accumulated mass from the flow meter measurement and that of the reference are nearly identical. The measurements were repeated with a second, identical Coriolis flow meter leading to comparable results. An inappropriately selected cutoff flow rate can lead to a deviation of several tenths of a percent between the total mass recorded by the reference and the flow meter.

In principle, this results in an additional or revised measurement uncertainty contribution.

### 3.7 Other effects

Depending on the measuring device, the output used and operating conditions, other factors can also have an effect on flow measurements. It is difficult to quantify these influences, as they depend on the individual operating conditions and how a meter was installed. In some instances, mitigation measures can be done. These include for instance

- transfer of mechanical stresses to the flow meter by the installation in the pipe section;
- influence of mechanical and hydraulic vibrations on the flow meter performance;
- effects originating from the internal processing of the raw data, use of digital components/ discreet data.

For Coriolis flow meters an additional effect may arise from auto zero effects in the lower flow rate range. For measurements in this range to be as accurate as possible, the zero point must be set for the measurement conditions relevant to the subsequent application. In Figure 23 an example of the change in the  $k$ -factor at flow rates below 200 l/h is shown.

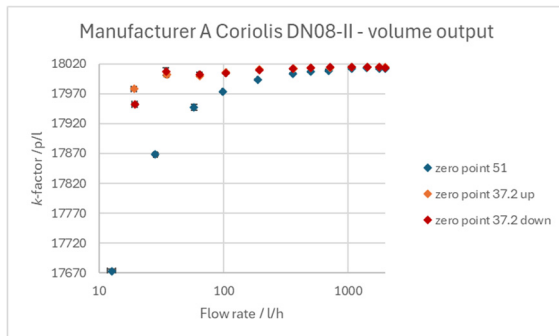


Fig. 23. Example of the effect of the auto zero set on the  $k$ -factor of a Coriolis flow meter (pulse value: 18000 p/l); test liquid: water; ambient/liquid temperature: 20 °C;  $u_{\text{test bench}} (k=2) = 0.05\%$ .

Usually, manufacturers of flow meters provide a recommendation for the auto zero setting of their devices. An alternative route for an optimized auto zero determination which has proved to be very reliable has been proposed by Frahm et al. (2024). The procedure is described in Annex I. Moreover, it must also be ensured that the meter is earthed.

When a flow meter has both, a mass and a volume output, the results for one output typically cannot be transferred to the other given usual quality demands. This means that the output that will be used later should be calibrated. In Figure

24 an example is given of the  $k$ -factors obtained for the mass output and the volume output of a Coriolis flow meter. The  $k$ -factors differ with regard to their average value, but also the spread of the values and therefore the measurement deviations are different. Whether this is relevant or not depends on the application. Another example of how the outputs can differ in their behaviour can be seen in Figure 25. While in the  $k$ -factor of the mass output an effect of the liquid temperature is visible, the  $k$ -factor of the volume output does not show this kind of effect.

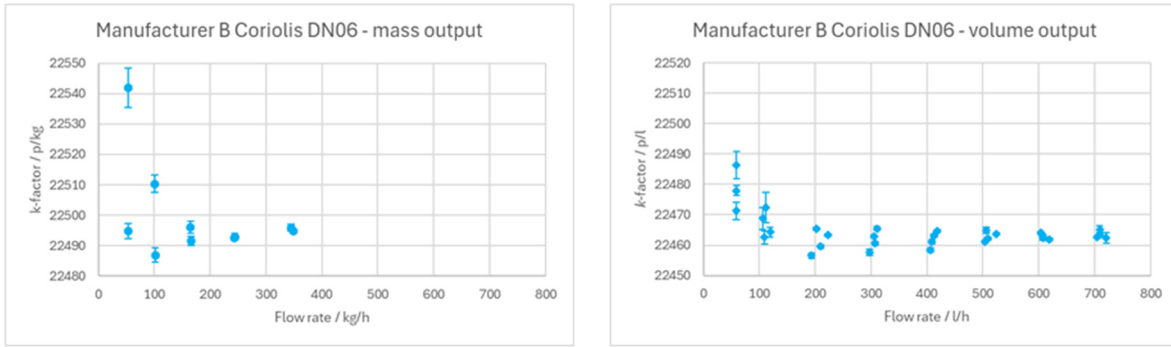


Fig. 24. Example of the  $k$ -factors obtained for the mass output (left) and the volume output (right) of a Coriolis flow meter (pulse value: 22500 p/kg resp. p/l); test liquid: water; ambient/liquid temperature: 20 °C;  $u_{\text{test bench}} (k=2) = 0.05 \%$ .

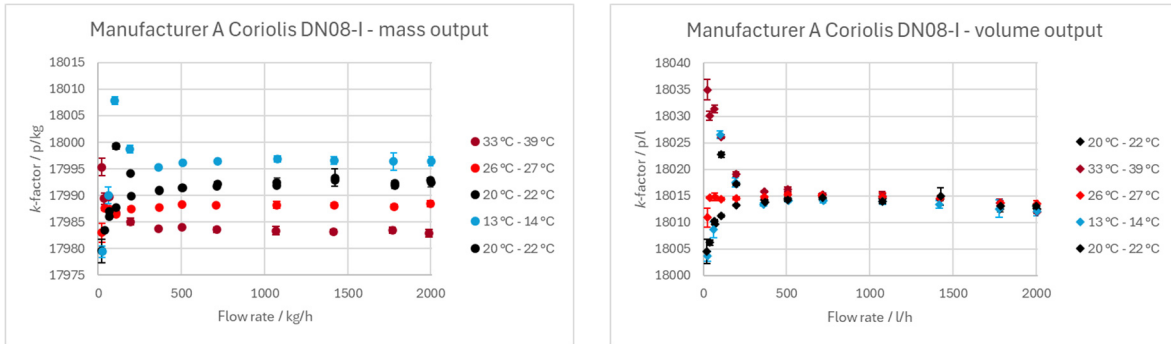


Fig. 25. Example of the  $k$ -factors obtained for the mass output (left) and the volume output (right) of a Coriolis flow meter at different liquid temperatures (pulse value: 18000 p/kg resp. p/l); test liquid: water; ambient temperature: 20 °C;  $u_{\text{test bench}} (k=2) = 0.05 \%$ .

## 4. Combined measurement uncertainty

The result of a calibration of a flow meter is reported in some form of “performance indicator” (PI) (CCM WGFF, 2013). When the uncertainty of the mean performance indicator for a device  $U_{PI}$  is reported, it will include additional components over the base uncertainty due to:

- instrumentation and characteristics associated with the MUT  $u_{AI}$  (e.g. associated with liquid temperature, liquid pressure, long-term stability, ...)
- liquid properties (if applicable)  $u_{prop}$ , and
- repeatability or short-term reproducibility for the MUT  $u_{repeat}$  or  $u_{reprod.}$

The uncertainty of the performance indicator can be expressed as:

$$U_{PI} = 2 u_{PI} = 2 \sqrt{u_{base}^2 + u_{AI}^2 + u_{prop}^2 + u_{repeat\ or\ reprod.,MUT}^2}$$

with

$u_{base}$  as the type B uncertainty of the reference standard obtained by using the law of propagation of uncertainty as described in the GUM. This uncertainty component comprises all uncertainty contributions of the test bench at the flow rate and conditions considered. “2” corresponds to the coverage factor corresponding to a 95 % confidence interval if a sufficient number of observations is available. If this is not the case a revised coverage factor is needed (Annex III).

The reported uncertainty of the performance indicator must not be less than the uncertainty stated for the standard providing traceability at that point in the operating range. The uncertainty components that apply in the associated instrumentation and properties categories must be considered on a case-by-case basis (= MUT specific) and quantified by an appropriate uncertainty analysis, following the GUM corroborated by measurements or simulations.





## Literature

### References

- CCM WGFF, 2013. [WGFF Guidelines for CMC Uncertainty and Calibration Report Uncertainty](#). Evaluation of measurement data — Guide to the expression of uncertainty in measurement. (GUM) JCGM 100:2008, [https://www.bipm.org/documents/20126/2071204/JCGM\\_100\\_2008\\_E.pdf](https://www.bipm.org/documents/20126/2071204/JCGM_100_2008_E.pdf)
- Frahm, E., Borchling, A., Mills, Ch., 2024. The use of Coriolis meter as transfer standards in the field of fluid flow. Measurement: Sensors, [https://conferences.imeko.org/event/9/contributions/384/attachments/247/1202/30\\_Fullpaper.pdf](https://conferences.imeko.org/event/9/contributions/384/attachments/247/1202/30_Fullpaper.pdf)
- [ILAC P14:09/2020 ILAC Policy for Measurement Uncertainty in Calibration](#)
- ISO 5168(2005): Measurement of fluid flow — Procedures for the evaluation of uncertainties
- Ouverney Costa, F., Pope, J. G., Gillis, K. A., 2020. Modeling temperature effects on a Coriolis mass flowmeter. Flow Meas. Instr., 76, 101811, <https://doi.org/10.1016/j.flowmeasinst.2020.101811>
- Shinder, I., Moldover, M., 2009. Dynamic gravitational standard for liquid flow: Model and measurements. 7<sup>th</sup> ISFF, Fairbanks, Aug. 10-14, 2009
- Wang, T., Hussain, Y., 2010. Pressure effects on Coriolis mass flow meters. Flow Meas. and Instr., vol. 21, no. 4, 504-510
- Warnecke, H, Kroner, C, Ogheard, F, Kondrup, J B, Christoffersen, N, Benková, M, Bükér, O, Haack, S, Huovinen, M, Unsal, B, 2022. New metrological capabilities for measurements of dynamic liquid flows. Metrologia, vol 59(2), <https://doi.org/10.1088/1681-7575/ac566e>
- Wiklund D., Peluso, M., 2002. Quantifying and specifying the dynamic response of flowmeters. White paper. ISA 2002 Technical Conference. <https://www.emerson.com/documents/automation/white-paper-quantifying-specifying-dynamic-response-of-flowmeters-rosemount-en-77342.pdf>
- Wolf, H., 2014. Transporteigenschaften von Kraftstoffen. PTB-Report, Th-5
- Yoshida, T., Furuichi, N. 2023. Development of controllable volumetric prover for evaluating responsiveness of flowmeter under controlled-transient flows. Measurement, vol 208, <https://doi.org/10.1016/j.measurement.2023.112456>

### Further reading

Available for downloading at: <https://www.ptb.de/empir2020/safest/information-communication/downloads/>

- D1 - Report on the use of a new infrastructure for assessing the measurement performance of the flow meters and other systems, which are used in fuel consumption measurements of passenger cars and trucks, under dynamic load changes (e.g. in the WLTC or WHTC)
- D3 - Report on the impact of the properties of alternative and synthetic fuels (i.e. densities > 400 kg/m<sup>3</sup>, viscosities > 0.01 mPa·s, pressures ≤ 300 bar, and flow rates 0.0001 m<sup>3</sup>/h – 8 m<sup>3</sup>/h) on the performance of flow meters that are used for fuel consumption measurements
- D4 - Report on the simulation of the interaction between the test liquid and the flow meter and its impact on the quality of the measurement of fuel consumption
- D5 - Technical guide for the comprehensive uncertainty assessment of flow meters, which are used for fuel consumption measurements in the maritime sector, using hydrocarbons as test liquids (densities > 400 kg/m<sup>3</sup>, viscosities > 0.01 mPa·s, pressures ≤ 300 bar, and flow rates 0.0001 m<sup>3</sup>/h – 8 m<sup>3</sup>/h) with consideration of their performance under dynamic flows
- D6 - Fuel property matrix of the transport properties of alternative (e.g. bio diesel) and synthetic fuels with the following densities > 400 kg/m<sup>3</sup>, viscosities > 0.01 mPa·s, temperatures ≤ 620 K, pressures ≤ 300 bar
- D7 - Report on the advancement of in line measurements (using a new sensor system) of the density and viscosity of alternative and synthetic fuels with the following densities > 400 kg/m<sup>3</sup>, viscosities > 0.01 mPa·s, temperatures ≤ 620 K, pressures ≤ 300 bar and flow rates < 2 m<sup>3</sup>/h
- D8 - Technical guide for the assessment of flow meter performance under dynamic load changes. The pre normative research in this guide will be provided as metrological input for the further development of international standards with a focus on close to real world flow calibrations in the transport sector



## Annex I

### Proposed procedure for determining an optimized zero point of a Coriolis flow meter

1. Set the desired conditions (pressure, temperature)
2. Ensure that there is no flow
3. Set automatic zero point
4. Measure at least two small flow points (e.g. 50 l/h and 100 l/h)
5. Determine the measurement deviations  $\varepsilon$  at the respective flow points
6. Calculate the difference in the measurement deviation between the two flow points (gradient) (e.g.:  $\varepsilon_{100}-\varepsilon_{50}$ )
7. Measure the same flow points with two manually set zero points (e.g.  $ZP_{\text{auto}} = 5$ , manual  $ZP1 = 2$  and manual  $ZP2 = 8$ )
8. Determine the measurement deviation for each flow point
9. Calculate the difference in the measurement deviations in the same way as in step 6
10. Plot the results: x-axis: zero point, y-axis: difference of the measurement deviations
11. Determine a linear regression
12. Determine the point of intersection with the x-axis
- Point of intersection with the x-axis corresponds to the optimum zero point

For verification:

13. Repeat measurement with optimum zero point
14. Determine the measurement deviation

## Annex II

### Student's distribution

<b>Degrees of freedom</b>	<b>Confidence level</b>										
	<b>Two-sided</b>	<b>50 %</b>	<b>60 %</b>	<b>70 %</b>	<b>80 %</b>	<b>90 %</b>	<b>95 %</b>	<b>98 %</b>	<b>99 %</b>	<b>99.5 %</b>	<b>99.8 %</b>
<b>1</b>	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
<b>2</b>	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
<b>3</b>	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
<b>4</b>	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
<b>5</b>	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
<b>6</b>	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
<b>7</b>	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
<b>8</b>	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
<b>9</b>	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
<b>10</b>	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
<b>11</b>	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
<b>12</b>	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
<b>13</b>	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
<b>14</b>	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
<b>15</b>	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
<b>16</b>	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
<b>17</b>	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
<b>18</b>	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
<b>19</b>	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
<b>20</b>	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
<b>21</b>	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
<b>22</b>	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
<b>23</b>	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
<b>24</b>	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
<b>25</b>	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
<b>26</b>	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
<b>27</b>	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
<b>28</b>	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
<b>29</b>	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
<b>30</b>	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
<b>40</b>	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
<b>50</b>	0.679	0.849	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261	3.496
<b>60</b>	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
<b>80</b>	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	2.887	3.195	3.416
<b>100</b>	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	2.871	3.174	3.390
<b>∞</b>	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

A t-distribution with infinitely many degrees of freedom is a normal distribution. The number of degrees of freedom for a *t*-distribution is equal to the number of observations minus one.

## Annex III

### Revised coverage factors

Only with a sufficient number of observations a coverage factor  $k=2$  will mean that the expanded uncertainty provides a confidence level close to 95 %. However, where either of these assumptions is not applicable, a revised coverage factor and expanded uncertainty need to be determined.

The effective degree of freedom  $\nu_{\text{eff}}$  needs to be determined from the Welch-Satterthwaite equation

$$\nu_{\text{eff}} = \frac{u_{PI}^4(y)}{\sum_{i=1}^N \frac{u_i^4(y)}{\nu_i}}$$

with

$u_{PI}$ : combined uncertainty

$y$ : output value

$u_i$ : individual uncertainty components

$N$ : number of observations.

For Type A evaluations  $\nu_i$  is obtained from

$$\nu_i = N - 1.$$

For Type B evaluations  $\nu_i$  is given by

$$\nu_i \approx \frac{1}{2} \frac{1}{\frac{\Delta u_i^2(y)}{u_i^2(y)}}$$

with  $\frac{\Delta u_i}{u_i}$  based on experience.

When lower and upper limits are used in Type B evaluations (e.g. when using information of a calibration certificate) and the probability of the quantity lying outside this interval is negligible, the degrees of freedom are infinite

$$\nu_i \rightarrow \infty.$$

With the thus obtained  $\nu_{\text{eff}}$  the appropriate coverage factor  $k$  can be obtained from the Student's distribution in Annex III for a confidence level of 95 %.

## Annex IV

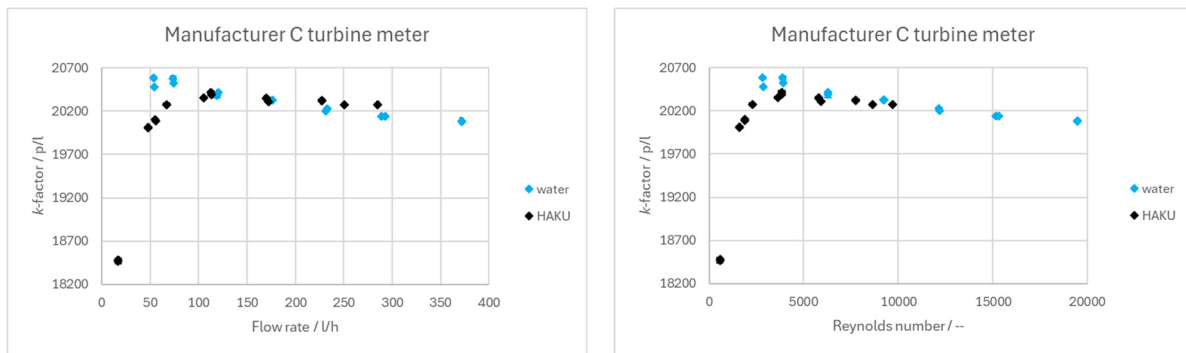


Fig. A1.  $k$ -factors obtained for a turbine measured with water and HAKU as test liquids; liquid temperature: 20.5 °C;  $u_{\text{test benches}}$  ( $k=2$ ): 0.05 %. The maximum difference in the  $k$ -factor for the two liquids is in the range of 0.6 %.

## Annex V

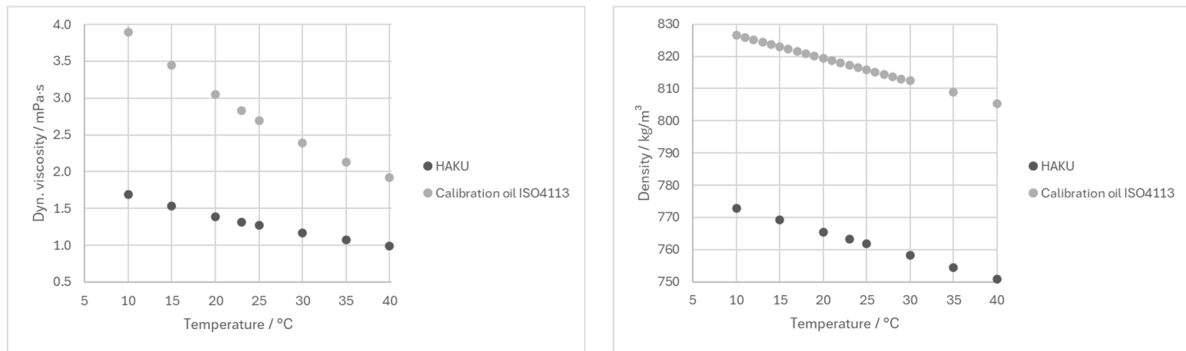


Fig. A2. Transport properties of calibration oil (Castor) according to ISO 4113 and HAKU (cold cleaner, Kluth);  $u_{\text{visc}} (k=2)$ : 1 % - 2 %,  $u_{\text{dens}} (k=2)$ : 0.1 kg/m<sup>3</sup>.









Bundesministerium  
für Wirtschaft  
und Klimaschutz

Die Physikalisch-Technische Bundesanstalt, das nationale Metrologieinstitut, ist eine wissenschaftlich-technische Bundesoberbehörde im Geschäftsbereich des Bundesministeriums für Wirtschaft und Klimaschutz.



**Physikalisch-Technische Bundesanstalt**  
Nationales Metrologieinstitut  
ISNI: 0000 0001 2186 1887

Bundesallee 100  
38116 Braunschweig

Presse- und Öffentlichkeitsarbeit

Telefon: (0531) 592-93 21  
Fax: (0531) 592-30 08  
E-Mail: [presse@ptb.de](mailto:presse@ptb.de)  
[www.ptb.de](http://www.ptb.de)